## Lecture 23: Vector Semantics

Mark Hasegawa-Johnson 3/2023
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## Outline

- What is a word? wordforms vs. lemmas vs. word senses
- What is meaning? synonymy, similarity, and relatedness
- Vector semantics: CBOW and skip-gram
- Generative training vs. Contrastive training
- Visualizations
- Bias


## What is a word?



## What is a word?

Is this a word?


## What is a word?

Is this a word?


## word (1 )untable and uncountable, pl al words)

## Wordform

A wordform is a unique sequence of characters.

- Wordforms are much easier for computers to find than lemmas, therefore most automatic processing deals with wordforms.
- ...however, we lose something. "dog" and "dogs" become completely unrelated - as unrelated as "dog" and "exaggerate."

1. The smallest unit of language that has a particular meaning and can be expressed by itself; the smallest discrete, meaningful unit of language. (contrast morpheme.) [quotations $\mathbf{V}$ ]
2. The smallest discrete unit of spoken language with a particular meaning, composed of one or more phonemes and one or more morphemes [quotations $\boldsymbol{V}$
3. The smallest discrete unit of written language with a particular meaning, composed of one or more letters or symbols and one or more morphemes [quotations $\mathbf{V}$
4. A discrete, meaningful unit of language approved by an authority or native speaker (compare nonword). [quotations $\mathbf{V}$ ]
5. Something like such a unit of language:
6. A sequence of letters, characters, or sounds, considered as a discrete entity, though it does not necessarily belong to a language or have a meaning [quotations $\mathbf{V}$ ]

## Lemma

A lemma is what humans usually think of as a "word." It is defined to be the form of the word that appears in a dictionary.

- Other wordforms that can be easily predicted from the lemma need not be listed.
word (i )untable and uncountable, plural words)

1. The smallest unit of language that has a particular meaning and can be expressed by itself; the smallest discrete, meaningful unit of language. (contrast morpheme.) [quotations $\mathbf{V}$ ]
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4. A discrete, meaningful unit of language approved by an authority or native speaker (compare nonword). [quotations V]
5. Something like such a unit of language:
6. A sequence of letters, characters, or sounds, considered as a discrete entity, though it does not necessarily belong to a language or have a meaning [quotations $\mathbf{V}$ ]


## Word sense

Often, a word has different meanings that are completely unrelated. We think of them as different words, that just happen to be spelled and pronounced the same way.
We say that these are different "senses" of the same word.


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## Wordform, lemma, and word sense

- wordform
- easy for a computer to work with: just look for space-bounded sequences of characters
- lemma
- This is what humans think of as a word. A cluster of wordforms whose spellings, pronunciations, and meanings can all be derived from one another by applying simple rules.
- word sense
- A meaning so distinct from the other meanings of the word that it's hard to consider them the same word.


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## Synonymy and similarity




## SimLex-999

SimLex-999 is a gold standard resource for the evaluation of models that learn the meaning of words and concepts.
SimLex-999 provides a way of measuring how well models capture similarity, rather than relatedness or association. The scores in SimLex-999 therefore differ from other well-known evaluation datasets such as WordSim-353 (Finkelstein et al. 2002). The following two example pairs illustrate the difference - note that clothes are not similar to closets (different materials, function etc.), even though they are very much related:

| Pair | Simlex-999 rating | WordSim-353 rating |
| :--- | :--- | :--- |
| coast - shore | 9.00 | 9.10 |
| clothes - closet | 1.96 | 8.00 |

- Algorithms that try to estimate the similarity of two wordforms can be tested on databases such as SimLex-999.
- Humans rated the similarity of each word pair on a 10 -point scale.


## Similarity vs. Relatedness

Similar: words can be used interchangeably in most contexts

Related: there is some connection between the two words, such that they tend to appear in the same documents.


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## Review: Naïve Bayes: the "Bag-of-words" model

We can estimate the likelihood of an e-mail by pretending that the e-mail is just a bag of words (order doesn't matter).
With only a few thousand spam e-mails, we can get a pretty good estimate of these things:

- $P(W=$ "hi" $\mid Y=$ spam $), P(W=$ "hi" $\mid Y=$ ham $)$
- $P(W=$ "vitality" $\mid Y=$ spam $), P(W=$ "vitality" $\mid Y=$ ham $)$
- $P(W=$ "production" $\mid Y=$ spam $), P(W=$ "production" $\mid Y=$ ham $)$

Then we can approximate $P(X \mid Y)$ by assuming that the words, $W$, are conditionally independent of one another given the category label:

$$
P(X=x \mid Y=y) \approx \prod_{i=1}^{n} P\left(W=w_{i} \mid Y=y\right)
$$



## Similarity: The Internet is the database

Similarity = words can be used interchangeably in most contexts
How do we measure that in practice?
Answer: extract examples of word $w_{1},+/-\mathrm{N}$ words ( $\mathrm{N}=2$ or 3 ):
...hot, although iced coffee is a popular... ...indicate that moderate coffee consumption is benign...
...and of $w_{2}$ :
...consumed as iced tea. Sweet tea is... ...national average of tea consumption in Ireland...

The words "iced" and "consumption" appear in both contexts, so we can conclude that $s$ (coffea, tea) $>0$. No other words are shared, so we can conclude $s$ (coffee, tea) $<1$.

## skip-gram context probability

Consider the "...hot although iced coffee is a popular...".
Define the target word to be $w_{0}=$ coffee.
Define the context words $w_{-3}=$ hot, $w_{-2}=$ although,..,$w_{3}=$ popular. The skip-gram probability is a naïve Bayes model of the context:

$$
p\left(w_{-3}, \ldots, w_{3} \mid w_{0}\right)=\prod_{\substack{i \neq 0 \\ i=-3}}^{3} p\left(w_{i} \mid w_{0}\right)
$$

## The skip-gram model



- Skip-gram is a model of word meaning:
- The meaning of a word is defined to be the distribution of context words that it can predict.
- We find out which words $w_{t}$ can predict by learning neural nets that predict its context words $w_{t+j}$ :

$$
\mathcal{L}=-\frac{1}{T} \sum_{t=0}^{T-1} \sum_{j=-c, j \neq 0}^{c} \ln P\left(w_{t+j} \mid w_{t}\right)
$$

The "continuous bag of words" model (CBOW)

$\frac{1}{T} \sum_{t=1}^{T} \sum_{-c \leq j \leq c, j \neq 0} \log p\left(w_{t} \mid w_{t+j}\right)$

$\frac{1}{T} \sum_{t=1}^{T} \sum_{-c \leq j \leq c, j \neq 0} \log p\left(w_{t+j} \mid w_{t}\right)$

- CBOW is a similar model of word meaning:
- The meaning of a word is defined to be the distribution of context words that predict it the best.
- We find out which words predict $w_{t}$ by learning neural nets that predict $w_{t}$ given its context words, $w_{t+j}$, for $-c \leq j \leq c$ :

$$
\mathcal{L}=-\frac{1}{T} \sum_{t=0}^{T-1} \sum_{j=-c, j \neq 0}^{c} \ln P\left(w_{t} \mid w_{t+j}\right)
$$

## "Probability," for a NN, means softmax

- What does it mean that we train a neural net to compute $P\left(w_{t} \mid w_{t+j}\right)$ ?
- It's a probability, so it must mean a softmax:

$$
P\left(W_{t}=m \mid W_{t+j}=n\right)=\frac{\exp \left(e_{m, n}\right)}{\sum_{m^{\prime}} \exp \left(e_{m \prime, n}\right)}
$$

- But what are the inputs to the neural net? What is $e_{m, n}$ ?


## Vector Semantics

- The simplest useful assumption is this: a word is a vector.

$$
P\left(W_{t}=m \mid W_{t+j}=n\right)=\frac{\exp \left(v_{m} @ v_{n}\right)}{\Sigma_{m^{\prime}} \exp \left(v_{m}, @ v_{n}\right)}
$$

-...where $v_{m}$ is a d-dimensional vector, $v_{m}=\left[v_{m, 0}, \ldots, v_{m, d-1}\right]$

- The only trainable parameters in this model are the word vectors!
- The dictionary, $v$, is a matrix, with as many rows as there are words in the vocabulary:

$$
v=\left[\begin{array}{c}
v_{\mathrm{a}} \\
\vdots \\
v_{\mathrm{zzZ}}
\end{array}\right]=\left[\begin{array}{ccc}
v_{\mathrm{a}, 0} & \cdots & v_{\mathrm{a}, d-1} \\
\vdots & \ddots & \vdots \\
v_{\mathrm{zzZ}, 0} & \cdots & v_{\mathrm{zzz}, d-1}
\end{array}\right]
$$

## cosine similarity

If words $w_{1}$ and $w_{2}$ are similar, $w_{1}$ is represented by vector $\vec{v}_{1}$, and $w_{2}$ by vector $\vec{v}_{2}$, then the angle between the two vectors should be small.

Angle between two vectors can be measured by their dot product:

$$
\cos \theta=\frac{\vec{v}_{1} \cdot \vec{v}_{2}}{\left|\vec{v}_{1}\right|\left|\vec{v}_{2}\right|}
$$



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where

$$
\vec{v}_{1} \cdot \vec{v}_{2}=\sum_{i=0}^{d-1} v_{1, i} v_{2, i}, \quad\left|\vec{v}_{1}\right|=\sqrt{\sum_{i=0}^{d-1} v_{1, i}^{2}}
$$

## Vector Semantics

There are many ways to make this model more flexible. For example:

- Every word could have two different vectors: one ( $v_{m}$ ) for when it's being predicted, one $\left(c_{n}\right)$ for when it is predicting, thus $e_{m, n}=v_{m} @ c_{n}$.
- We could put a delay-weight matrix, $w_{j}$, in between the word vectors, thus $e_{m, j, n}=v_{m} @ w_{j} @ c_{n}$.
- We could even use an MLP to calculate the similarity, for example, $e_{m, n}=$ $\operatorname{ReLU}\left(\left[v_{m}, v_{n}\right] @ w_{0}\right) @ w_{1}$.
- ...but notice, all these methods are based on the idea of a matrix as a dictionary:

$$
v=\left[\begin{array}{c}
v_{\mathrm{a}} \\
\vdots \\
v_{\mathrm{zzz}}
\end{array}\right]=\left[\begin{array}{ccc}
v_{\mathrm{a}, 0} & \cdots & v_{\mathrm{a}, d-1} \\
\vdots & \ddots & \vdots \\
v_{\mathrm{zzz}, 0} & \cdots & v_{\mathrm{zzz}, d-1}
\end{array}\right]
$$

## Vector Semantics

The CBOW probability is now:

$$
P\left(W_{t}=m \mid W_{t+j}=n\right)=\frac{\exp \left(v_{m} @ v_{n}\right)}{\sum_{m^{\prime}} \exp \left(v_{m^{\prime}} @ v_{n}\right)}
$$

Remember the derivative of a softmax:

$$
\frac{\partial \operatorname{softmax}_{m}(e)}{\partial e_{k}}=\left\{\begin{array}{cc}
\operatorname{softmax}_{m}(e)\left(1-\operatorname{softmax}_{m}(e)\right) & m=k \\
-\operatorname{softmax}_{m}(e) \operatorname{softmax}_{k}(e) & m \neq k
\end{array}\right.
$$

## Training a CBOW model



$$
\frac{1}{T} \sum_{t=1}^{T} \sum_{-c \leq j \leq c, j \neq 0} \log p\left(w_{t} \mid w_{t+j}\right) \quad \frac{1}{T} \sum_{t=1}^{T} \sum_{-c \leq j \leq c, j \neq 0} \log p\left(w_{t+j} \mid w_{t}\right)
$$

In order to find the parameters, we use gradient descent:

$$
\begin{aligned}
& \nabla_{v_{m}} \mathcal{L}=-\frac{1}{T} \sum_{t: w_{t}=m} \sum_{j=-c, j \neq 0}^{c} \nabla_{v_{m}} \ln P\left(W_{t}=m \mid w_{t+j}\right) \\
& =-\frac{1}{T} \sum_{t: w_{t}=m} \sum_{j=-c, j \neq 0}^{c}\left(1-P\left(W_{t}=m \mid w_{t+j}\right)\right) v_{w_{t+j}}
\end{aligned}
$$

## Training a CBOW model



$$
\frac{1}{T} \sum_{t=1}^{T} \sum_{-c \leq j \leq c, j \neq 0} \log p\left(w_{t} \mid w_{t+j}\right) \quad \frac{1}{T} \sum_{t=1}^{T} \sum_{-c \leq j \leq c, j \neq 0} \log p\left(w_{t+j} \mid w_{t}\right)
$$

The CBOW model is trained by setting every vector equal to a weighted average of the words that occurred near it!

$$
\begin{gathered}
v_{m} \leftarrow v_{m}-\eta \nabla_{v_{m}} \mathcal{L} \\
v_{m} \leftarrow v_{m}+\frac{\eta}{T} \sum_{t: w_{t}=m} \sum_{j=-c, j \neq 0}^{c}\left(1-P\left(W_{t}=m \mid w_{t+j}\right)\right) v_{w_{t+j}}
\end{gathered}
$$

## Try the quiz!

- Try the quiz: https://us.prairielearn.com/pl/course_instance/129874/assessment/ 2337428
- $\mathrm{Vm}=[1,0,0,0]+(\mathrm{eta} / \mathrm{T})^{*}\left((1-\mathrm{P}(\text { coffee } \mid \text { smells }))^{*}[0,0,0,1]+(1-\right.$ $P($ coffee $\mid$ hot $\left.))^{*}[0,0,1,0]\right)$
- P(coffeelsmells) = exp(vcoffee@vsmells)/sum(exp(vsmells@v))
$=\exp (0) / \operatorname{sum}(\exp (0)+\exp (0)+\ldots+\exp (0)+\exp (v s m e l l s @ v s m e l l s))$
$=1 /((N-1)+\exp (1))$


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## Contrastive loss vs. Generative loss

- A generative loss is one like this:

$$
\mathcal{L}=-\frac{1}{T} \sum_{t=0}^{T-1} \sum_{j=-c, j \neq 0}^{c} \ln \frac{\exp \left(v_{w_{t}} @ v_{w_{t+j}}\right)}{\sum_{m^{\prime}} \exp \left(v_{m^{\prime}} @ v_{w_{t+j}}\right)}
$$

- Notice that this loss term compares each word, $w_{t}$, to every other word in the dictionary.
- Sometimes, generative training can take a very long time to converge.
- Sometimes, we get faster training using contrastive loss.


## Contrastive loss

We train the neural network by listing, as positive examples, the words that occur in the context of " $w=$ coffee," e.g.,
$\mathcal{D}_{+}(w)=\{$ hot, although, iced, moderate, hot, consumption $\}$

Create a contrastive database by choosing the same number of words, at random, from among the words that never appeared in the context of "coffee:"
$\mathcal{D}_{-}(w)=\{$ aardvark, dog, gazebo, actor, precipitates, iceberg $\}$

## Training with contrastive loss

The coefficients $\vec{v}_{i}=\left[v_{i, 0}, \ldots, v_{i, d-1}\right]$ for each vector are then learned in order to maximize the log probability of the dataset:

$$
\begin{aligned}
\mathcal{L} & =\ln p(\text { Data })=\sum_{w \in \mathcal{V}} \ln p\left(\mathcal{D}_{+}(w) \mid w\right)+\sum_{w \in \mathcal{V}} \ln p\left(\mathcal{D}_{-}(w) \mid w\right) \\
& =\sum_{w \in \mathcal{V}} \sum_{c \in \mathcal{D}_{+}(w)} \ln p(c \mid w)+\sum_{w \in \mathcal{V}} \sum_{c \in \mathcal{D}_{-}(w)} \ln (1-p(c \mid w)) \\
& =\sum_{\vec{v} \in \mathcal{V}} \sum_{\vec{c} \in \mathcal{D}_{+}(w)} \ln \frac{1}{1+e^{-\vec{c} \cdot \vec{v}}}+\sum_{\vec{v} \in \mathcal{V}} \sum_{\vec{c} \in \mathcal{D}_{-}(w)} \ln \left(1-\frac{1}{1+e^{-\vec{c} \cdot \vec{v}}}\right)
\end{aligned}
$$

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## Visualizations: Similarity

Mikolov et al. (2013) tested word2vec on SimLex-999, and had better results than previously published baselines. Here are some examples from their paper. Notice that not all of their "similar words" are really similar - some are just related. I'll talk more about that next time.

| Model <br> (training time) | Redmond | Havel | ninjutsu | graffiti | capitulate |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Collobert (50d) (2 months) | conyers <br> lubbock <br> keene | plauen dzerzhinsky osterreich | reiki <br> kohona <br> karate | cheesecake gossip dioramas | abdicate accede rearm |
| Turian (200d) (few weeks) | McCarthy <br> Alston <br> Cousins | Jewell Arzu Ovitz |  | gunfire emotion impunity |  |
| Mnih (100d) (7 days) | Podhurst <br> Harlang <br> Agarwal | Pontiff <br> Pinochet <br> Rodionov |  | anaesthetics monkeys Jews | Mavericks <br> planning <br> hesitated |
| Skip-Phrase (1000d, 1 day) | Redmond Wash. Redmond Washington Microsoft | Vaclav Havel president Vaclav Havel Velvet Revolution |  | $\begin{aligned} & \text { spray paint } \\ & \text { grafitti } \\ & \text { taggers } \end{aligned}$ | capitulation capitulated capitulating |

Table 6: Examples of the closest tokens given various well known models and the Skip-gram model trained on phrases using over 30 billion training words. An empty cell means that the word was not in the vocabulary.

## Visualizations: Relatedness



Christian S. Perone, "Voynich Manuscript: word vectors and t-SNE visualization of some patterns," in Terra Incognita, 16/01/2016, http://blog.christianperone.com/2016/01/voynich-manuscript-word-vectors-and-t-sne-visualization-of-some-patterns/.

Mikolov (2013) showed that word2vec captures similarity relationships among words. For example, the difference between the vectors for "woman" and "man" is roughly the same as the difference between the vectors for "queen" and "king." Perone (2016) showed that this effect works differently depending on the training corpus: in his blog post, he looks at word relatedness in the $15^{\text {th }}$ century Voynich manuscript.

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## Learning biased analogies from data

- It's useful that algorithms like word2vec learn appropriate analogies, like "Paris $\rightarrow$ France as Tokyo $\rightarrow$ Japan" and "kings $\rightarrow$ king as queens $\rightarrow$ queen."
- Unfortunately, it also learns other analogies that were implied in the training corpus, but that are invalid analogies.
- The paper that first demonstrated that problem was named after one of the worst such discovered analogies:
> "Man is to Computer Programmer as Woman is to Homemaker? Debiasing Word Embeddings," Bolukbasi et al., 2016


## Biased analogies

Bolukbasi et al. defined a "male-female" continuum by subtracting vec(female)-vec(male), vec(woman)-vec(man), and so on, then averaging these difference vectors.
They then took all of the words whose dictionary definitions included gender-specific language (man, woman), and considered those to be the gender-specific words (words for which a gender difference is appropriate).
All other words were considered gender-neutral (any difference on the male-female dimension is inappropriate).
The result is a second dimension: the appropriateness of a gender bias.

## The Male-Female vs. Neutral-Specific Space

Here's the resulting 2D space, from Bolukbasi et al., 2016:


## Outline

- What is a word? Lemmas, wordforms, and word sense
- Synonymy, similarity, and relatedness
- Word2vec: maximize

$$
\mathcal{L}=\sum_{\vec{v} \in \mathcal{V}} \sum_{\vec{c} \in \mathcal{D}_{+}(w)} \ln \frac{1}{1+e^{-\vec{c} \cdot \vec{v}}}+\sum_{w \in \mathcal{V}} \sum_{\vec{c} \in \mathcal{D}_{-}(w)} \ln \frac{1}{1+e^{\vec{c} \cdot \vec{v}}}
$$

- Visualizations
- Similarity: list the K-nearest neighbors, show that they are similar
- Relatedness: analogies are shown as directions in the vector space!
- Bias
- Bias can be reduced by learning a direction that should not depend on the female-male axis, and then squashing the female-male axis to zero for words that should be gender-neutral.

