UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
CS440/ECE448 Artificial Intelligence
Conflict Exam 2
Spring 2023

April 3, 2023

Your Name: ______________________________

Your NetID: ______________________________

Instructions

- Please write your name on the top of every page.
- Have your ID ready; you will need to show it when you turn in your exam.
- This will be a CLOSED BOOK, CLOSED NOTES exam. You are permitted to bring and use only one 8.5x11 page of notes, front and back, handwritten or typed in a font size comparable to handwriting.
- No electronic devices (phones, tablets, calculators, computers etc.) are allowed.
- SHOW YOUR WORK. Correct answers derivation may not receive full credit if you don't show your work.
- Make sure that your answer includes only the variables that it should include. Solve integrals and summations. After that is done, do not further simplify explicit numerical expressions. For example, the answer $x = \frac{1}{1+\exp(-0.1)}$ is MUCH preferred (much easier for us to grade) than the answer $x = 0.524979$.

Possibly Useful Formulas

Consistent Heuristic: $h(p) \leq d(p,r) + h(r)$

Alpha-Beta Max Node: $v = \max(v, \text{child})$; $\alpha = \max(\alpha, \text{child})$

Alpha-Beta Min Node: $v = \min(v, \text{child})$; $\beta = \min(\beta, \text{child})$

Variance Network: $\mathscr{L} = \frac{1}{n-1}\sum_{i=1}^{n}\left(f_2(x_i) - (f_1(x_i) - x_i)^2\right)^2$

Unification: $U = S(P) = S(Q)$; $U \Rightarrow \exists x : Q$; $U \Rightarrow \exists x : P$

Bayes Rule: $P(Y=y|X=x) = \frac{P(X=x|Y=y)P(Y=y)}{\sum_{y'} P(X=x|Y=y')P(Y=y')}$

Unnormalized Relevance: $\tilde{R}(f_c, x_d) = \frac{\partial f_c}{\partial x_d} x_d f_c$

Normalized Relevance: $R(f_c, x_d) = \frac{\frac{\partial f_c}{\partial x_d} x_d}{\sum_{d'} \frac{\partial f_c}{\partial x_{d'}} x_{d'}} f_c$

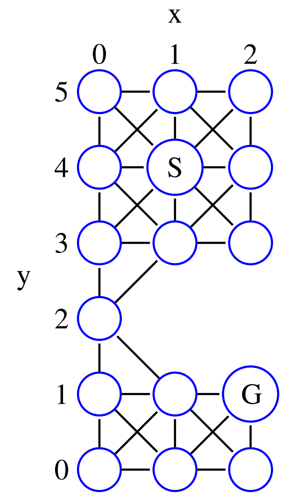
Softmax: $\underset{j}{\text{softmax}}(e) = \frac{\exp(e_j)}{\sum_k \exp(e_k)}$

Softmax Deriv: $\frac{\partial\, \text{softmax}_m(e)}{\partial e_n} = \underset{m}{\text{softmax}}(e)\delta[m-n] - \underset{m}{\text{softmax}}(e)\, \underset{n}{\text{softmax}}(e)$, $\delta[m-n] = \begin{cases} 1 & m = n \\ 0 & m \neq n \end{cases}$

Viterbi: $v_t(j) = \max_i v_{t-1}(i) a_{i,j} b_j(x_t)$

Transformer: $c_i = \text{softmax}(q_i @ k^T) @ v$

Question 1 ***(8 points)***



In the maze shown above, nodes are named by their (x,y) coordinates, where x is the column number (starting from the left), and y is the row number (starting from the bottom), A robot is trying to find a path from the start node, $(1,4)$ (labeled "S"), to the goal node, $(2,1)$ (labeled "G"). Every horizontal or vertical step has a cost of 1 unit; every diagonal step has a cost of $\sqrt{2}$ units. Prove that Manhattan distance is not a consistent heuristic for this problem.

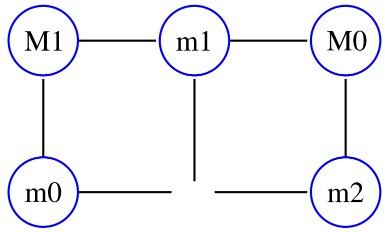
Solution: We can prove that Manhattan distance is not consistent by finding any nodes p and r such that

$$h(p) \geq d(p,r) + h(r)$$

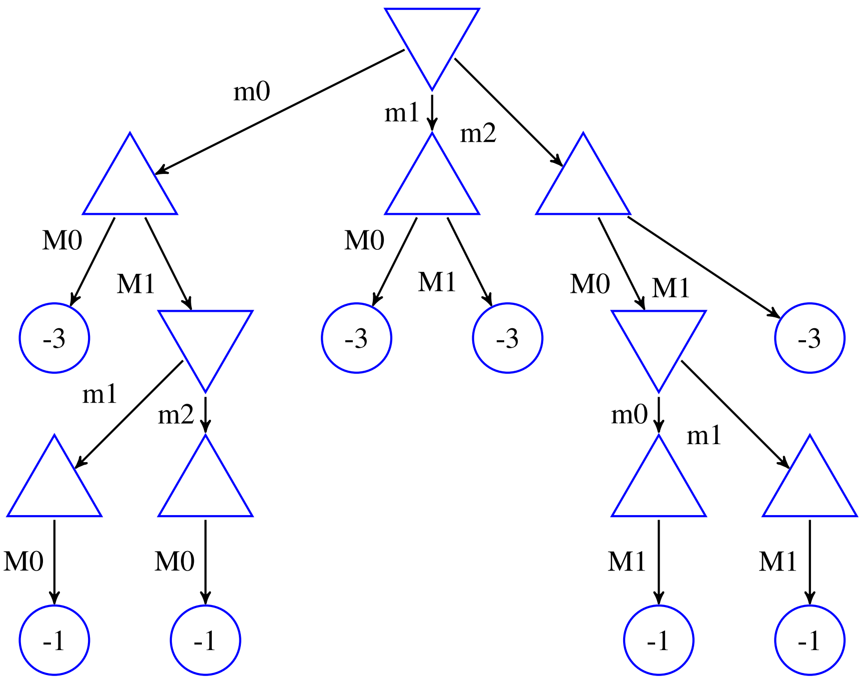
An example would be nodes $p = (0,2)$ and $r = (1,1)$. The distance between these nodes is $d(p,r) = \sqrt{(}2)$. If Manhattan distance is the heuristic, then $h((1,1)) = 1$, while $h((0,2)) = 3$, so

$$h((0,2)) \geq d((0,2),(1,1)) + h((1,1))$$

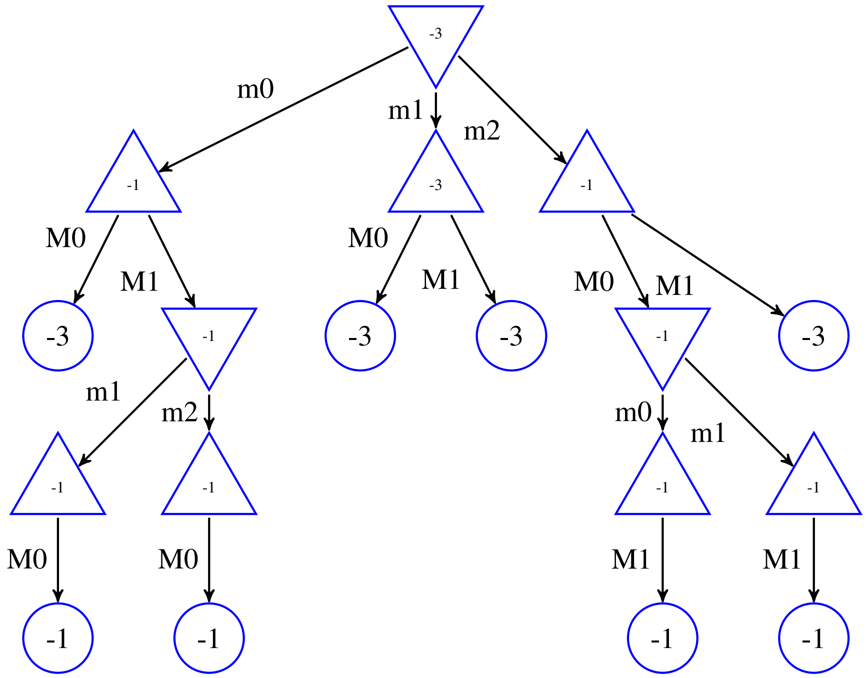
Question 2 *(7 points)*



Consider a game in which Min starts with 3 stones on the edges of a square (shown above as "m0,m1" and "m2"), and Max starts with 2 stones on the corners of the square (shown above as "M0" and "M1"). Min plays first. On their turn, each player removes one of their own stones. The game ends when one player has any stone with no remaining neighbors; that player wins a number of points equal to the number of stones they still have on the board. The figure below shows the game tree for this game. In each triangle (each Min or Max node), enter a number specifying the value of that node. Specify the minimax value of the game. Indicate a minimax move sequence (there are several different sequences that a pair of optimal players might play; specify one of those sequences).



Solution:



The minimax value is -3. The two optimal sequences are (m1,M1) and (m1,M0); note that it is not optimal for Min to start with m0 or m2.

Question 3 ***(7 points)***

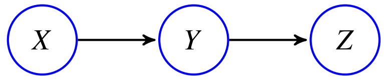
u, v, w, x, y, and z are variables. You are trying to determine whether or not it's possible to perform a step of backward-chaining using the rule $T = \text{Sings}(u, \text{folkmusic}) \Rightarrow \text{Plays}(u, \text{guitar})$. Your goalset, $\mathscr{G}$, currently includes the following goals:

$$\mathscr{G} = \left\{ \begin{array}{c} P = \text{Eats}(\text{tiger}, \text{cellphone}) \\ Q = \exists v : \text{Plays}(\text{anne}, v) \\ R = \exists w : \text{Plays}(w, \text{flute}) \\ S = \exists x : \text{Zambonis}(x, \text{icerink}) \end{array} \right\}$$

Which proposition (P, Q, R, or S) can be unified with the consequent of T? What is the resulting unified proposition, what is the resulting substitution dictionary, and what new fact is added to the goalset?

Solution: Q can be unified to the consequent of T, producing $\text{Plays}(\text{anne}, \text{guitar})$, $S = \{u : \text{anne}, v : \text{guitar}\}$, and adding $\text{Sings}(\text{anne}, \text{folkmusic})$ to the goalset.

Question 4 *(7 points)*



X, Y, and Z are random variables, each of which can take the values -1, 0, or 1. Their causal dependencies are shown in the Bayes network above. The parameters of this model are:

	x = -1	0	1
$P(X=x)$	a	b	c
$P(Y=-1\|X=x)$	d	e	f
$P(Y=0\|X=x)$	g	h	i
$P(Y=1\|X=x)$	j	k	l

	y = -1	0	1
$P(Z=-1\|Y=y)$	m	n	o
$P(Z=0\|Y=y)$	p	q	r
$P(Z=1\|Y=y)$	s	t	u

In terms of the parameters a through u, what is $P(Y=0|Z=1)$?

Solution:

$$\begin{aligned}
P(Y=0|Z=1) &= \frac{P(Y=0,Z=1)}{P(Z=1)} \\
&= \frac{\sum_x P(X=x,Y=0,Z=1)}{\sum_x \sum_y P(X=x,Y=y,Z=1)} \\
&= \frac{agt+bht+cit}{agt+bht+cit+ads+bes+cfs+aju+bku+clu}
\end{aligned}$$

Question 5 ***(7 points)***

Suppose $f_j = \text{softmax}_j(e)$, where $e = [e_0, \ldots, e_{N-1}]$ is a vector. In terms of e_m and f_n for any useful values of m and n, what is the unnormalized relevance of e_3 to the output f_5?

Solution:

$$\tilde{R}(f_5, e_e) = \frac{\partial f_5}{\partial e_3} e_3 f_5$$

Let's define $e = w@h$, then we can write:

$$\frac{\partial f_5}{\partial e_3} = -\underset{5}{\text{softmax}}(e)\,\underset{3}{\text{softmax}}(e) = -f_5 f_3$$

Therefore

$$\tilde{R}(f_5, e_3) = -f_5^2 f_3 e_3$$

Question 6 ***(7 points)***

Your friend is sending you a message by flashing red and green lights in a coded pattern. The first light is red or green with equal probability. After that, each time, the lights stay the same with probability a:

$$P(Y_t = \text{green}|Y_{t-1} = \text{green}) = P(Y_t = \text{red}|Y_{t-1} = \text{red}) = a,$$

otherwise they change color. Because of the fog, you only see the correct color with probability b:

$$P(X_t = \text{green}|Y_t = \text{green}) = P(X_t = \text{red}|Y_t = \text{red}) = b,$$

otherwise you mistakenly see the wrong color. Suppose you see the sequence $X_0 = R\text{red}, X_1 = \text{green}$. In terms of the parameters a and b, what is the joint probability $P(Y_0 = \text{red}, Y_1 = \text{green}, X_0 = \text{red}, X_1 = \text{green})$?

Solution:

$$\begin{aligned}
&P(Y_0 = \text{red}, X_0 = \text{red}, Y_1 = \text{green}, X_1 = \text{green}) \\
&= P(Y_0 = \text{red})P(X_0 = \text{red}|Y_0 = \text{red})P(Y_1 = \text{green}|Y_0 = \text{red})P(X_1 = \text{green}|Y_1 = \text{green}) \\
&= \frac{1}{2}(1-a)b^2
\end{aligned}$$

Question 7 ***(7 points)***

Suppose the the input to a transformer is the sequence of scalar values $v_t = \left(\frac{t}{1000}\right)^2$, where $0 \le t \le 999$. You are trying to find the context, c_i, for a query, q_i, whose inner product with the keys is:

$$q_i @ k_t = \begin{cases} 0 & t \in \{0,1,2\} \\ -\infty & \text{otherwise} \end{cases}$$

Find the numerical value of c_i.

Solution:

$$c_i = \text{softmax}(q_i @ k^T) @ v$$

$$\text{softmax}(q_i @ k^T) = \begin{cases} \frac{1}{3} & t \in \{0,1,2\} \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned} c_i &= \frac{1}{3}(v_0 + v_1 + v_2) \\ &= \frac{1}{3}\left(0 + \left(\frac{1}{1000}\right)^2 + \left(\frac{2}{1000}\right)^2\right) \end{aligned}$$

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