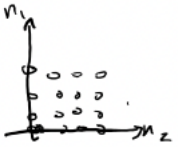
2021sep09

Tuesday, September 7, 2021 10:56

$$h[n_1, n_2] = \begin{cases} 1 & 0 \le n_1, n_2 \le N-1 \\ 0 & \text{else} \end{cases}$$



$h[n_1, n_2] = h_1[n_1]\, h_2[n_2]$?

$$h_1[n_1] = \begin{cases} 1 & 0 \le n_1 \le N-1 \\ 0 & \text{else} \end{cases}$$

$$H(\omega_1, \omega_2) = H_1(\omega_1) H_2(\omega_2)$$

$$H(\omega_1, \omega_2) = \sum_{n_1} \sum_{n_2} h[n_1, n_2]\, e^{-j(\omega_1 n_1 + \omega_2 n_2)}$$

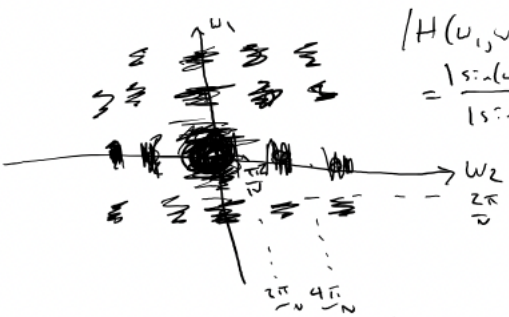
$$= \sum_{n_1} \sum_{n_2} h_1[n_1] h_2[n_2]\, e^{-j\omega_1 n_1} e^{-j\omega_2 n_2}$$

$$= \sum_{n_1} h_1[n_1] e^{-j\omega_1 n_1} \sum_{n_2} h_2[n_2] e^{-j\omega_2 n_2}$$

$$H_1(\omega_1) = \sum_{n_1} h_1[n_1] e^{-j\omega_1 n_1}$$

$$= \sum_{n_1=0}^{N-1} e^{-j\omega_1} = \frac{1}{1 - e^{-j\omega_1 N_1}}$$

$$= e^{-j\omega_1 \left(\frac{N_1 - 1}{2}\right)} \frac{\sin(\omega_1 N/2)}{\sin(\omega_1/2)}$$



$$|H(\omega_1, \omega_2)| = \frac{|\sin(\omega_1 N/2)| \cdot |\sin(\omega_2 N/2)|}{|\sin(\omega_1/2)| \cdot |\sin(\omega_2/2)|}$$