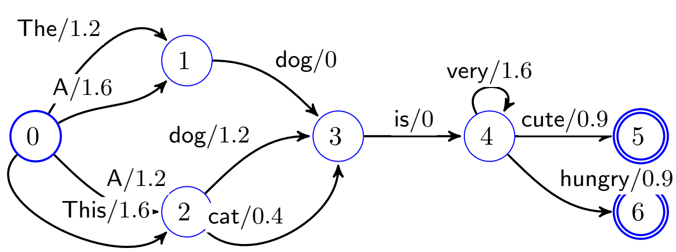
ECE 417 Multimedia Signal Processing
Homework 5

UNIVERSITY OF ILLINOIS
Department of Electrical and Computer Engineering

Assigned: Monday, 11/2/2020; Due: Monday, 11/9/2020
Reading: Mohri, Pereira & Riley, Weighted Finite State Transducers in Speech Recognition, 2001

Problem 5.1



The best-path algorithm for a WFSA is

- **Initialize:**

$$\delta_0(i) = \begin{cases} \bar{1} & i = \text{initial state} \\ \bar{0} & \text{otherwise} \end{cases}$$

- **Iterate:**

$$\delta_k(j) = \underset{t:n[t]=j,\ell[t]=s_k}{\text{best}} \delta_{k-1}(p[t]) \otimes w[t]$$
$$\psi_k(j) = \underset{t:n[t]=j,\ell[t]=s_k}{\text{argbest}} \delta_{k-1}(p[t]) \otimes w[t]$$

- **Backtrace:**

$$t_k^* = \psi(q_{k+1}^*), \qquad q_k^* = p[t_k^*]$$

where k is the number of input words that have been observed, and j is the state index. Unlike an HMM, $\delta_k(j) = \bar{0}$ for most states at most times. We only need to keep track of $\delta_k(j)$ and $\psi_k(j)$ for (k, j) at which $\delta_k(j) \neq \bar{0}$.

Create a table:

- with columns indexed by k, $0 \leq k \leq 5$,
- for the utterance $[s_1, \ldots, s_5] = [\text{A}, \text{dog}, \text{is}, \text{very}, \text{hungry}]$,
- for the FSA shown above, whose transition weights are given in surprisal form.
- In each column: list the states j for which $\delta_k(j) \neq \bar{0}$ ($\delta_k(j) < \infty$, since we're using surprisals).

- For each such state, list its $\delta_k(j)$ (as a surprisal), and
- list its backpointer, $\psi_k(j)$, which should be a transition, in the format $t = (p, \ell, w, n)$ showing the previous state, label, weight, and next state.

Solution:

k	0	1	2	3	4	5
j	0	1	3	4	4	6
$\delta_k(j)$	0	1.6	1.6	1.6	3.2	4.1
$\psi_k(j)$	-	$(0, \text{A}, 1.6, 1)$	$(1, \text{dog}, 0, 3)$	$(3, \text{is}, 0, 4)$	$(4, \text{very}, 1.6, 4)$	$(4, \text{hungry}, 0.9, 6)$
j		2				
$\delta_k(j)$		1.2				
$\psi_k(j)$		$(0, \text{A}, 1.2, 1)$				

Problem 5.2

Show that if u, v, x, y, z are surprisals, then

$$\min(u, v, x, y, z) - \ln(5) \le u \oplus v \oplus x \oplus y \oplus z \le \min(u, v, x, y, z)$$

Specify the values of u, v, x, y, z that cause the lower bound to be met with equality. Specify the values of u, v, x, y, z that cause the upper bound to be met with equality.

Solution:

Without loss of generality, assume that $u = \min(u, v, x, y, z)$. Then

$$\begin{aligned} u \oplus v \oplus x \oplus y \oplus z &= -\ln\left(e^{-u} + e^{-v} + e^{-x} + e^{-y} + e^{-z}\right) \\ &= u - \ln\left(e^{0} + e^{u-v} + e^{u-x} + e^{u-z}\right) \end{aligned}$$

Each of the terms is $0 \le e^{u-v} \le 1$, where the upper and lower bounds correspond to the values $v = u$ and $v = \infty$, respectively. Therefore

$$u - \ln(5) \le u \oplus v \oplus x \oplus y \oplus z \le u - \ln(1)$$

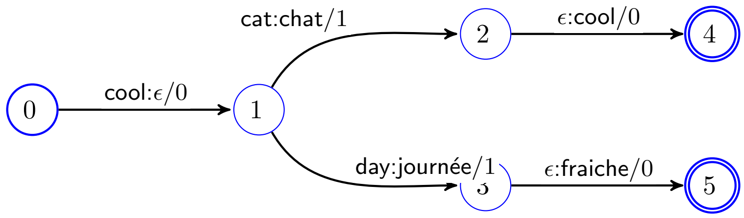
The lower bound is satisfied if

$$u = v = x = y = z$$

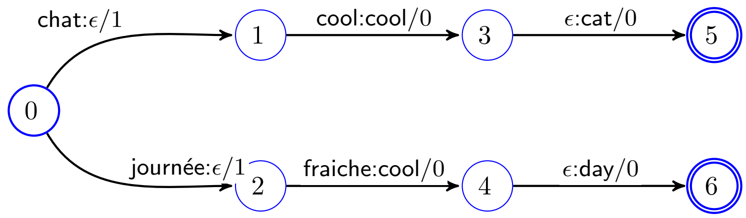
The upper bound is satisfied if one of the variables has a finite value, and all of the others are $+\infty$.

Problem 5.3

Consider the problem of translating from English into French, and then back into English again. The English-to-French WFST is called E2F. With its edge weights written as surprisals (in this case, $-\log_2 p(t)$), it is written as

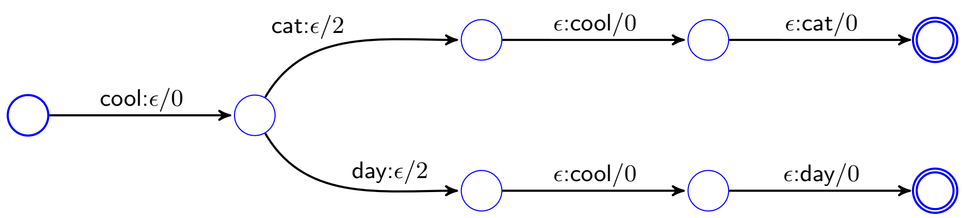


The French-to-English WFST is called F2E. With its edge weights written as surprisals (in this case, $-\log_2 p(t)$), it is written as



Find the WFST E2E = E2F ∘ F2E. You do not need to show the disconnected transitions (the transitions that can't be reached from the start state).

Solution: A valid solution will only accept two input sentences: either "cool cat" or "cool day." In response to either such sentence, it will produce the same sentence as output. If you follow the `fstcompose` algorithm given in lecture, you'll wind up with a lot of disconnected transitions, but the important connected transitions should show as follows:



Problem 5.4

Suppose you have two WFSTs, $A = \{\Sigma_A, \Omega_A, Q_A, E_A, i_A, F_A\}$ and $B = \{\Sigma_B, \Omega_B, Q_B, E_B, i_B, F_B\}$. Suppose we want to create $C = A \circ B = \{\Sigma_A, \Omega_B, Q_A \times Q_B, E_C, i_A \times i_B, F_A \times F_B\}$, where $Q_C = Q_A \times Q_B$ means that the states Q_C are tuples of the form $q_C = (q_A, q_B)$. Let the transitions be defined in the standard way,

$$t_A = (p[t_A], i[t_A], o[t_A], w[t_A], n[t_A])$$
$$t_B = (p[t_B], i[t_B], o[t_B], w[t_B], n[t_B])$$
$$t_C = (p[t_C], i[t_C], o[t_C], w[t_C], n[t_C])$$

In each of the following cases, you're considering a pair of transitions t_A and t_B, and deciding how to create one or more transitions t_C. Specify:

- the previous state, $p[t_C]$, as a tuple: one state from Q_A, and one from Q_B (for example, you might specify $p[t_C] = (p[t_A], p[t_B])$).
- Specify $n[t_C]$ in the same way.
- Specify also the input string $i[t_C]$, output string $o[t_C]$, and weight $w[t_C]$.

Specify $(p[t_C], i[t_C], o[t_C], w[t_C], n[t_C])$ under each of the following three cases:

(a) t_A has an ϵ output string ($o[t_A] = \epsilon$).

Solution: $((p[t_A], p[t_B]), (n[t_A], p[t_B]), i[t_A], \epsilon, w[t_A])$

(b) t_B has an ϵ input string ($i[t_B] = \epsilon$).

Solution: $((p[t_A], p[t_B]), (p[t_A], n[t_B]), \epsilon, o[t_B], w[t_B])$

(c) t_A and t_B have matching non-epsilon strings ($i[t_B] = o[t_A] \neq \epsilon$).

Solution: $((p[t_A], p[t_B]), (n[t_A], n[t_B]), i[t_A], o[t_B], w[t_A] \otimes w[t_B])$