UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 401 Signal Processing
EXAM 2

November 2, 2026

- This is a CLOSED BOOK exam.
- You are permitted one sheet of handwritten notes, 8.5x11.
- Calculators and computers are not permitted.
- Do not leave any unresolved integrals or summations in your answer. Other than that, however, you should not simplify explicit numerical expressions.
- If you're taking the exam online, you will need to have your webcam turned on. Your exam will appear on Gradescope at exactly 1:00pm; you will need to photograph and upload your answers by exactly 2:00pm.
- There will be a total of 100 points in the exam. Each problem specifies its point total. Plan your work accordingly.
- You must SHOW YOUR WORK to get full credit.

Name: _____

NetID: _____

DTFT and DFT

$$h[n] * \cos(\omega n) = |H(\omega)| \cos\left(\omega n + \angle H(\omega)\right)$$

$$H(\omega) = \sum_{n=-\infty}^{\infty} h[n] e^{-j\omega n}, \quad h[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(\omega) e^{j\omega n} d\omega$$

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N}, \quad x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j2\pi kn/N}$$

DTFT and DFT Properties

1. Periodicity: $X(\omega + 2\pi) = X(\omega)$, $X[k+N] = X[k]$
2. Linearity: $z[n] = ax[n] + by[n] \leftrightarrow Z(\omega) = aX(\omega) + bY(\omega), \quad Z[k] = aX[k] + bY[k]$
3. Time Shift: $x[n - n_0] \leftrightarrow e^{-j\omega n_0} X(\omega), \quad e^{-j\frac{2\pi k n_0}{N}} X[k]$ if $x[n]$ periodic
4. Frequency Shift: $e^{j\omega_0 n} x[n] \leftrightarrow X(\omega - \omega_0), \quad X\left[k - \frac{N\omega_0}{2\pi}\right]$ if $N\omega_0/2\pi$ is an integer
5. Linear Convolution in Time: $y[n] = h[n] * x[n] \leftrightarrow Y(\omega) = H(\omega)X(\omega)$
6. Circular Convolution in Time: $y[n] = h[n] \circledast x[n] \leftrightarrow Y[k] = H[k]X[k]$
7. Convolution in Frequenty: $y[n] = w[n]x[n] \leftrightarrow Y(\omega) = \frac{1}{2\pi} W(\omega) * X(\omega)$

Rectangular & Hamming Windows, Ideal LPF, and Tones

$$w_R[n] = \begin{cases} 1 & 0 \le n \le N-1 \\ 0 & \text{otherwise} \end{cases} \leftrightarrow W_R(\omega) = e^{-\frac{j\omega(N-1)}{2}} \frac{\sin(\omega N/2)}{\sin(\omega/2)}$$

$$w_H[n] = 0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right) w_R[n] \leftrightarrow W_H(\omega) = 0.54 W_R(\omega) - 0.23 W_R\left(\omega - \frac{2\pi}{N-1}\right) - 0.23 W_R\left(\omega + \frac{2\pi}{N-1}\right)$$

$$h_{\text{ideal}}[n] = \frac{\omega_c}{\pi} \text{sinc}(\omega_c n) \leftrightarrow H_{\text{ideal}}(\omega) = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & \text{otherwise} \end{cases}$$

$$e^{j\omega_0 n} \leftrightarrow 2\pi\delta(\omega - \omega_0)$$

$$\frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j\frac{2\pi kn}{N}} \leftrightarrow \frac{2\pi}{N} \sum_{k=0}^{N-1} X[k] \delta\left(\omega - \frac{2\pi kn}{N}\right)$$

Overlap-Add

$$N \ge M + L - 1$$
$$x_t[n] = x[n + tM] w[n]$$
$$Y_t[k] = X_t[k] H[k]$$
$$y[n] = \sum_t y_t[n - tM]$$

1. (25 points) Suppose $x[n] = r[n]e^{j\theta[n]}$ is the inverse DTFT of the following function, where $r[n]$ and $\theta[n]$ are both real-valued functions of n:

$$X(\omega) = \begin{cases} e^{-17j\omega} & -0.1\pi \leq \omega \leq 0.5\pi \\ 0 & \text{otherwise} \end{cases}$$

Find $r[n]$ and $\theta[n]$.

Solution:

$$r[n] = 0.3\text{sinc}\left(0.3\pi(n-17)\right) = \frac{\sin(0.3\pi(n-17)}{\pi(n-17)}$$

$$\theta[n] = 0.2\pi(n-17)$$

2. (25 points) Suppose that $F(\omega)$ is the DTFT of the following signal:

$$f[n] = \begin{cases} \cos\left(\frac{17\pi n}{37}\right) & -18 \leq n \leq 18 \\ 0 & \text{otherwise} \end{cases}$$

(a) What is $\max F(\omega)$, the maximum amplitude of $F(\omega)$?

Solution:

$$\max F(\omega) = \frac{37}{2}$$

(b) What are the two frequences at which $F(\omega)$ reaches its maximum?

Solution:

$$\omega = \pm\frac{17\pi}{37}$$

(c) At what frequencies does $F(\omega) = 0$? List all such frequencies (or give an equation that specifies all of them).

Solution:

$$\omega = \frac{17\pi}{37} + k\frac{2\pi}{37}$$

for all integer values $1 \leq k \leq 36$ except $k = 20$.

3. (25 points) Suppose that $X(\omega)$ is the DTFT of a periodic signal $x[n]$ with a period of 71 samples:

$$x[n] = \begin{cases} 1 & 0 \leq n \leq 16 \\ 0 & 17 \leq n \leq 70 \\ x[n-71] & \text{otherwise} \end{cases}$$

What is $X(\omega)$?

Solution:

$$X(\omega) = \sum_{k=0}^{71} \frac{2\pi}{71} e^{-j\frac{16\pi k}{142}} \frac{\sin(17\pi k/142)}{\sin(\pi k/142)} \delta\left(\omega - \frac{2\pi k}{71}\right)$$

4. (25 points) Suppose $x[n]$ is a periodic square wave with a period of 72:

$$x[n] = \begin{cases} 1 & 0 \leq n \leq 16 \\ 0 & 17 \leq n \leq 71 \\ x[n-72] & \text{otherwise} \end{cases}$$

which has been divided into frames according to:

$$x_t[n] = \begin{cases} x[n+100t] & 0 \leq n \leq 99 \\ 0 & \text{otherwise} \end{cases}$$

Suppose $h[n]$ is given by:

$$h[n] = \begin{cases} (0.3)^n & 0 \leq n \leq 43 \\ 0 & \text{otherwise} \end{cases}$$

Suppose that $Y_1[k] = H[k]X_1[k]$, where $Y_1[k]$, $H[k]$, and $X_1[k]$ are the 256-point discrete Fourier transforms of $y_1[n]$, $h[n]$, and $x_1[n]$, respectively. Sketch $y_1[n]$. Specify values of n for which $y_1[n]$ is an increasing nonzero function of n, and values of n for which $y_1[n]$ is a decreasing nonzero function of n, and values of n for which $y_1[n] = 0$.

Solution:

- $y_1[n]$ is a nonzero positive increasing function of n for $44 \leq n \leq 60$.
- $y_1[n]$ is a nonzero positive decreasing function of n for $60 \leq n \leq 103$
- $y_1[n] = 0$ for all other values of n.

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