2022oct07

Friday, October 7, 2022 13:34

$x[n] \longrightarrow \boxed{H} \longrightarrow y[n] = x[n - n_0]$

$\delta[n] \longrightarrow \boxed{H} \longrightarrow h[n] = \delta[n - n_0]$

$$y[n] = \sum_m h[m]\, x[n-m]$$

$$h[m] = \begin{cases} 1 & m = n_0 \\ 0 & \text{else} \end{cases}$$

$\cos(\omega n) \longrightarrow \boxed{H} \longrightarrow y[n] = ?$

$$= |H(\omega)| \cos(\omega n + \angle H(\omega))$$

$$H(\omega) = \sum_{m=-\infty}^{\infty} h[m]\, e^{-j\omega m}$$

$h[m] = 0$ except when $m = n_0$

$$= e^{-j\omega n_0}$$

$e^{j\omega n} \rightarrow \boxed{H} \rightarrow H(\omega) e^{j\omega n}$

$= e^{j\omega(n-n_0)}$

$|H(\omega)| = |e^{-j\omega n_0}|$

$= \sqrt{\cos^2(\omega n_0) + \sin^2(\omega n_0)} = 1$

$\angle H(\omega) = \tan^{-1}\left(\frac{\sin(-\omega n_0)}{\cos(-\omega n_0)}\right)$

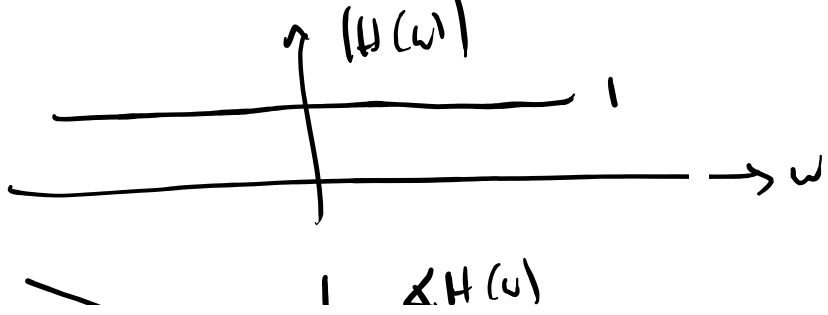
$= \tan^{-1}(\tan(-\omega n_0)) = -\omega n_0$

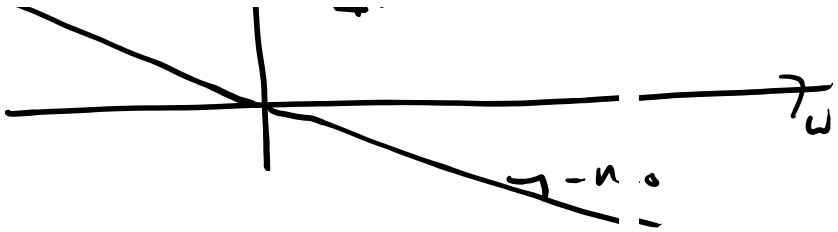
$H(\omega) = |H(\omega)| e^{j \angle H(\omega)}$

$= e^{-j\omega n_0}$

$|H(\omega)| = 1$

$\angle H(\omega) = -\omega n_0$





$$\cos(\omega n) \rightarrow \boxed{H} \rightarrow |H(\omega)| \cos(\omega n + \angle H(\omega))$$

$$= 1 \cdot \cos(\omega n - \omega n_0)$$

$$= \cos(\omega (n - n_0))$$