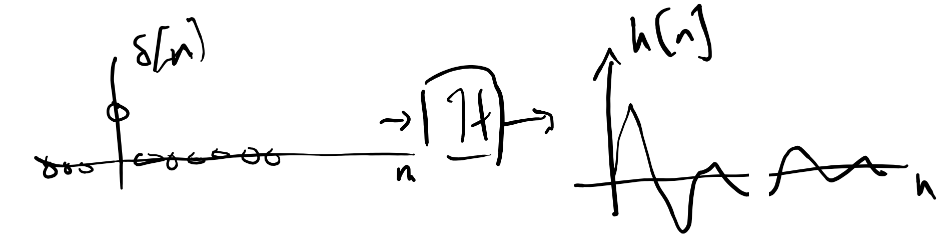
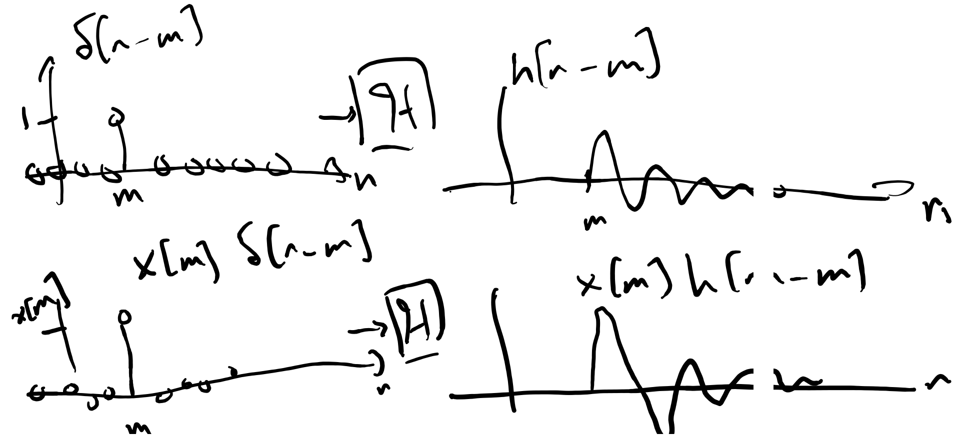
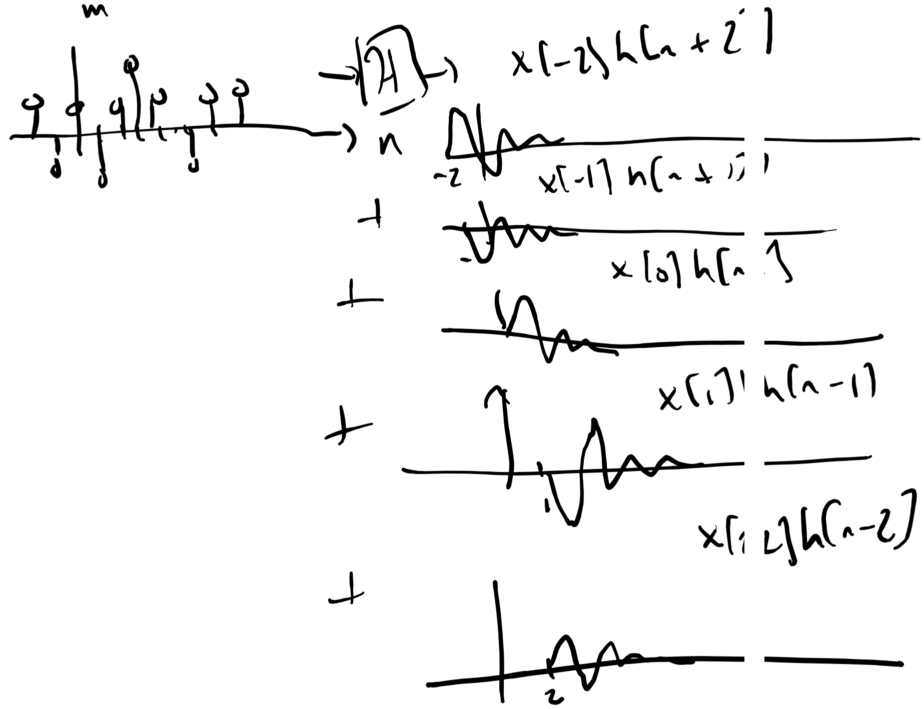
2022oct05

Wednesday, October 5, 2022 13:08





$$\sum_m x[m]\,\delta[n-m]$$



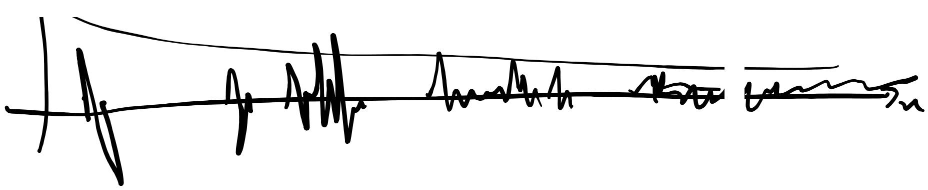
$$\sum_m x[m]\, h[n-m]$$

$=$

$$\sum_m h[m]\, x[n-m]$$

$h[m]$

h



$$x[n] = \sum_{m=-\infty}^{\infty} x[m]\, \delta[n-m]$$

$$\downarrow$$

$$\boxed{\mathcal{H}}$$

$$\downarrow$$

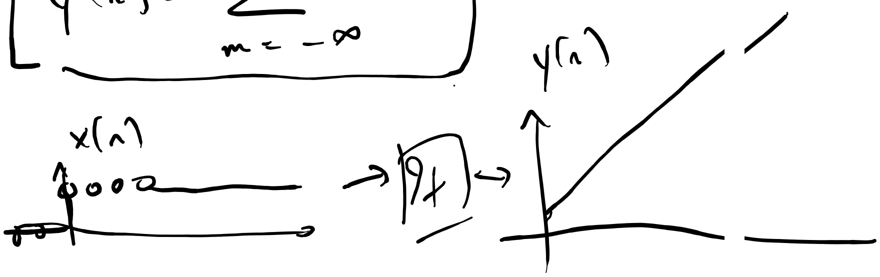
$$y[n] = \sum_{m=-\infty}^{\infty} x[m]\, h[n-m]$$

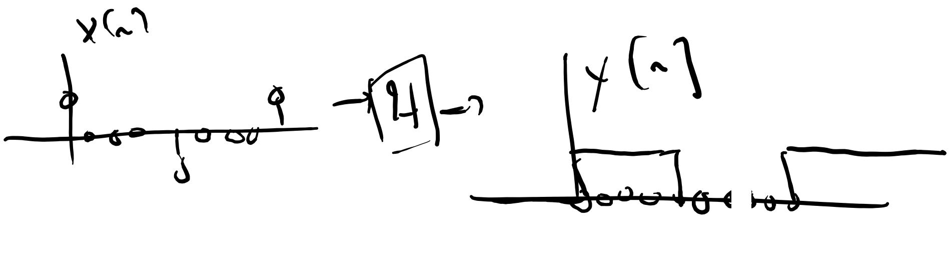
$$= \sum_{m=-\infty}^{\infty} h[m]\, x[n-m]$$

$m = \cdots$

Written Example

$$\boxed{y[n] = \sum_{m=-\infty}^{n} x[m]}$$





IS IT LINEAR?

$x_1[n] \longrightarrow y_1[n] = \sum_{m=-\infty}^{n} x_1[n]$

$x_2[n] \longrightarrow y_2[n] = \sum_{m=-\infty}^{\infty} x_2[n]$

$x[n] = x_1[n] + x_2[n]$

$\longrightarrow \boxed{H} \longrightarrow y[n]$

ASK: is $y[n] = y_1[n] + y_2[n]$?

$y[n] = \sum^{n} x[m]$

$$y[n] = \sum_{m=-\infty} x[m]$$

$$= \sum_{m=-\infty}^{n} \left(x_1[m] + x_2[m]\right)$$

$$= \sum x_1[m] + \sum x_2[m]$$

$$= y_1[n] + y_2[n] \quad \checkmark$$

IT IS LINEAR

<u>IS IT SHIFT-INVARIANT?</u>

$$x_1[n] \longrightarrow y_1[n] = \sum^{n} x_1[m]$$

$$y_1[n] = \sum_{m=-\infty} x_1[m]$$

$$x[n] = x_1[n-d]$$

$$\longrightarrow \boxed{H} \longrightarrow y[n]$$

IS $y[n] = y_1[n-d]$?

$$y[n] = \sum_{m=-\infty}^{n} x[m]$$

$$= \sum_{m=-\infty}^{n} x_1[m-d]$$

$p = m - d \qquad m = n \ \boxed{\text{AT}} \ p = n - d$

$m = p + d \qquad m = -\infty \ \boxed{\text{AT}} \ p = -\infty$

$$y[n] = \sum_{p=-\infty}^{n-d} x_1[p]$$

$$= y_1[n-d] \quad ✓$$

H IS SHIFT-INVARIANT

$$y[n] = h[n] * x[n]$$

$$= \sum_{m=-\infty}^{\infty} h[m]\, x[n-m]$$

$$= \sum_{m=-\infty}^{\infty} x[m]\, h[n-m]$$

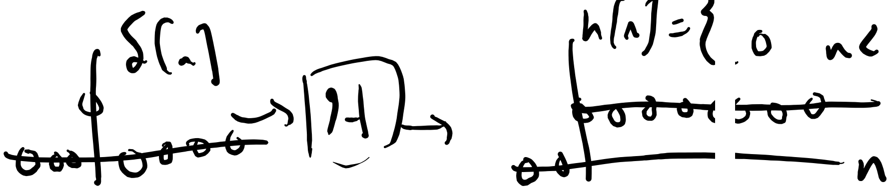
WHAT IS $h[m]$?

$h[m]$ IS THE IMPULSE RESPONSE!

$\delta[n] \rightarrow \boxed{H} \rightarrow h[n]$!!

$$h[n] = \sum_{m=-\infty}^{n} \delta[n]$$

$$h[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$



$$y[n] = h * x$$

$$= \sum_{m=-\infty}^{\infty} x[m]\, h[n-m]$$

$$= \sum_{m=-\infty}^{n} x[m]$$ ← ORIGINAL DEFINITION

EQUAL IFF

$$h[n-m] = \begin{cases} 1 & m \leq n \\ 0 & m > n \end{cases}$$

$$h[n-m] = \begin{cases} 1 & n-m \geq 0 \\ 0 & n-m < 0 \end{cases}$$