2022oct28

Friday, October 28, 2022 12:58

2021, exam 2, prob 2 one possibility

$f[n] = g[n] * h[n]$, FIND $g[n]$ s.t.

Desire $F(\omega)$ real

($\measuredangle F(\omega) = 0$ or $\measuredangle F(\omega) = \pi$ $\forall \omega$)

$e^{j\pi} = -1$

$$F(\omega) = G(\omega) H(\omega)$$

$$= G(\omega) |H(\omega)| e^{j \measuredangle H(\omega)}$$

$-\measuredangle H(\omega)$

$$G(\omega) = e^{-j \measuredangle H(\omega)}$$

$$H(\omega) = \sum_{n=-\infty}^{\infty} h[n] e^{-j\omega n}$$

$$= \sum_{n=-6}^{2} e^{-j\omega n}$$

Dummy var

$m = n + 6$

$n = m - 6$

$$= \sum_{m=0}^{8} e^{-j\omega(m-6)}$$

$$= e^{j\omega 6} \sum_{m=0}^{8} e^{-j\omega m}$$

$L = 9$

$$= e^{j\omega 6} \frac{1 - e^{-j\omega 9}}{1 - e^{-j\omega}}$$

$$1 - e^{-j\omega}$$

$$= e^{j\omega 6} \, e^{-j\omega\left(\frac{L-1}{2}\right)} \frac{\sin(\omega L/2)}{\sin(\omega/2)}$$

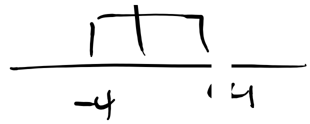
$$= e^{j\omega 6} \, e^{-j\omega 4} \frac{\sin(9\omega/2)}{\sin(\omega/2)}$$

$$= e^{2j\omega} \frac{\sin(\omega 9/2)}{\sin(\omega/2)}$$

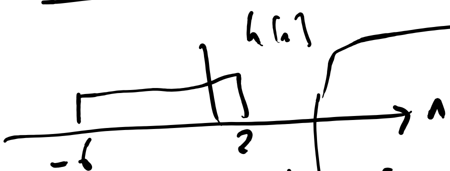
$$F(\omega) = G(\omega)H(\omega)$$

$$= \frac{\sin(\omega 9/2)}{\sin(\omega/2)} \quad \text{if } G(\omega) = e^{-2j\omega}$$

$$\boxed{g[n] = \delta[n-2]}$$



another possibility



$$* \quad g[n] = \begin{cases} 1 & -2 \le n \le 6 \\ 0 & \text{else} \end{cases}$$

$f[n]$ — symmetric in time

$\Leftrightarrow F(\omega)$ real

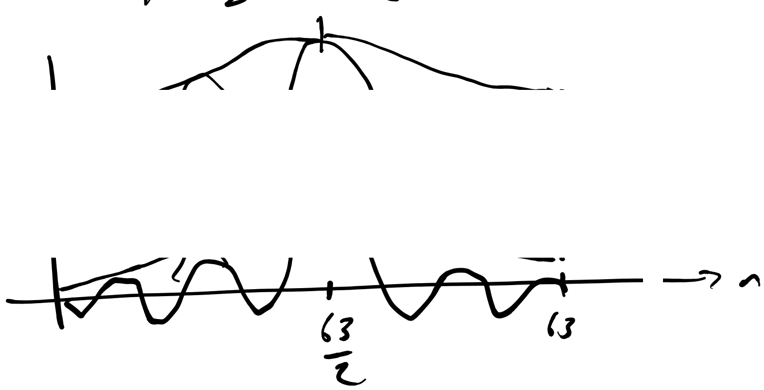
-4 4

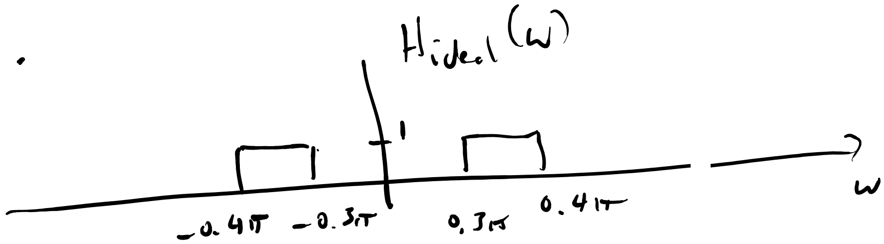
$\boxed{\text{Fall 2021, exam 2, problem 5}}$

- First sidelobe (stop-band ripple) $\underbrace{< -40 \text{ dB relative to main lobe}}$
  - o Rect
  - o Hamming ←

- Length = 64

  Delay: $\frac{L-1}{2} = \frac{63}{2}$





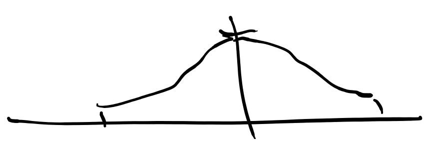
$$H_{ideal}(\omega) = H_{LPF}(\omega; 0.4\pi) - H_{LPF}(\omega; 0.3\pi)$$

$$h_{ideal}[n] = 0.4 \operatorname{sinc}(0.4\pi n) - 0.3 \operatorname{sinc}(0.3\pi n)$$

$$h_{ideal}\left[n-\frac{63}{2}\right] = 0.4\,\mathrm{sinc}\left(0.4\pi\left(n-\frac{63}{2}\right)\right) - 0.3\,\mathrm{sinc}\left(0.3\pi\left(n-\frac{63}{2}\right)\right)$$

$$w[n] = 0.54 - 0.4\cos\left(2\pi\frac{n}{63}\right)$$

$$\boxed{h[n] = \left(0.54-0.4\cos\left(2\pi\frac{n}{63}\right)\right)\cdot\left(0.4\,\mathrm{sinc}\left(0.4\pi\left(n-\frac{63}{2}\right)\right) - 0.3\,\mathrm{sinc}\left(0.3\pi\left(n-\frac{63}{2}\right)\right)\right)}$$



n

$$w(n) = 0.54 + 0.46\cos\left(2\pi\frac{ }{L-1}\right)$$

2017, problem 3

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{j2\pi k t/T_0}$$

LPF AT 3kHz, sampled at

$F_s = 6\,kHz$

$T_0 = 0.001$ seconds

$X_k \Rightarrow 0$ after LPF if

$\frac{k}{T_0} > 3\text{kHz}$, i.e., $1000k > 3\text{kHz}$

after filtering

$$X_k = \begin{cases} 0 & |k| > 3 \\ X_k \text{ original} & |k| < 3 \\ ? & |k| = 3 \end{cases}$$

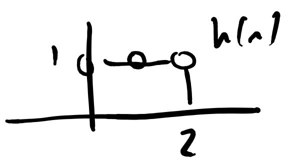
$$x[n] = \sum_{k=-3}^{3} X_k e^{j2\pi k n / T_0 F_s} = \sum_{k=-3}^{3} X_k e^{j2\pi k n/6}$$

$t \to nT_s = \frac{n}{F_s}$

$T_0 F_s = (0.001)(6000) = 6$

$$y[n] = \frac{1}{3} \sum_{m=0}^{2} x[n-m]$$

$= h[n] * x[n]$



$$H(\omega) = e^{-j\left(\frac{L-1}{2}\right)} \frac{\sin(\omega L/2)}{\sin(\omega/2)}$$

$$= e^{-j\omega} \frac{\sin(3\omega/2)}{\sin(\omega/2)}$$

NULLS AT

$$\frac{3\omega}{2} = m\pi \qquad \text{FOR NONZERO INTEGERS } m$$

$$\omega = m\frac{2\pi}{3} = \pm\frac{2\pi}{3}$$

$$\frac{2\pi k}{6} = \pm\frac{2\pi}{3} \quad \text{AT } k = \pm 2$$

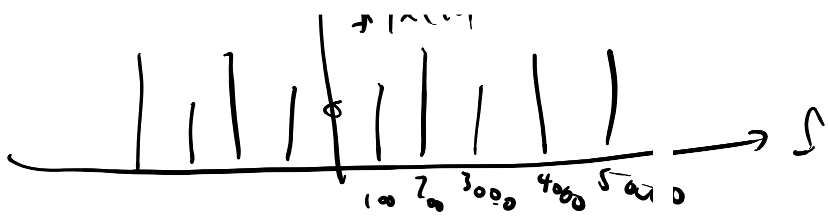
(a) $Y_k = 0$ FOR $|k| > 3$

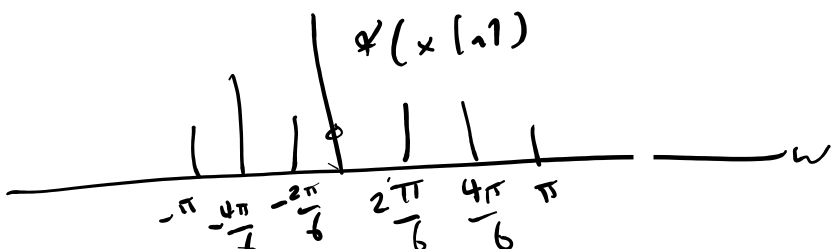
BECAUSE OF ANTI-ALIASING FILTER

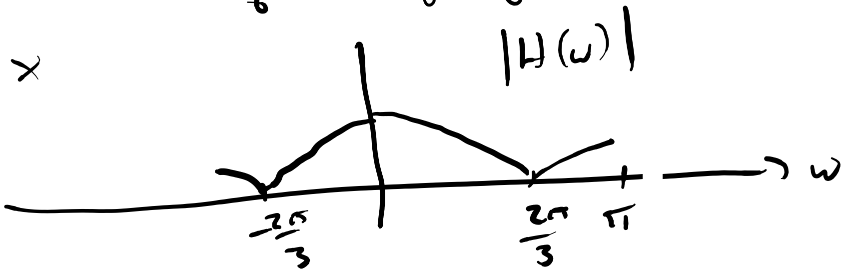
$Y_k = 0$ FOR $k = \pm 2$

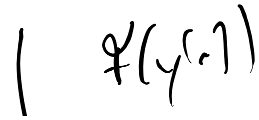
BECAUSE OF AVERAGING

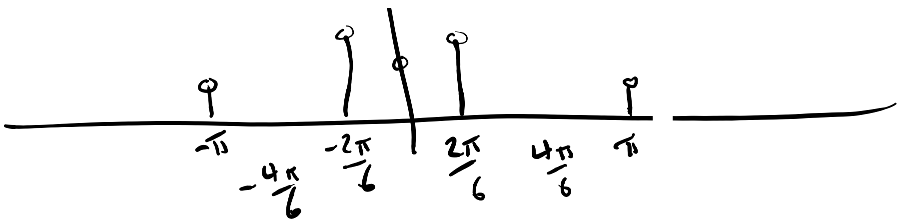
$|W(f)|$











Fourier series of $x[n]$

has $X_k \begin{cases} = \text{those of } x(t) & |k| < 3 \\ = 0 & |k| > 3 \end{cases}$

$$y[n] = h[n] * x[n]$$

$$Y_k = H(k\omega_0) X_k$$

$$= H\left(\frac{2\pi k}{6}\right) X_k$$

$$= e^{-j\left(\frac{2\pi k}{6}\right)} \frac{\sin\left(\frac{3}{2}\left(\frac{2\pi k}{6}\right)\right)}{\sin\left(\frac{1}{2}\left(\frac{2\pi k}{6}\right)\right)} X_k$$

$$|Y_k| = \begin{cases} 3 X_k & k = 0 \\ \cancel{\frac{\sin\left(\frac{3}{2}\left(\frac{2\pi}{6}\right)\right)}{\sin\left(\frac{1}{2}\left(\frac{2\pi}{6}\right)\right)}}^{\,2} X_k & k = \pm 1 \\ 0 & \end{cases}$$

$$\left\{ \left| \cancel{\frac{\sin\left(\frac{3}{2}\left(\frac{6\pi}{6}\right)\right)}{\sin\left(\frac{1}{2}\left(\frac{6\pi}{6}\right)\right)}} \right| X_k \right. \qquad k = \pm 3$$

$$0 \qquad \text{otherwise}$$

$\boxed{\text{2014, exam 1, problem 4}}$

$$x[n] = \begin{cases} \left(\frac{1}{2}\right)^n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

(a) $$X(\omega) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2} e^{-j\omega}\right)^n$$

$$= \frac{1}{1 - \frac{1}{2} e^{-j\omega}}$$

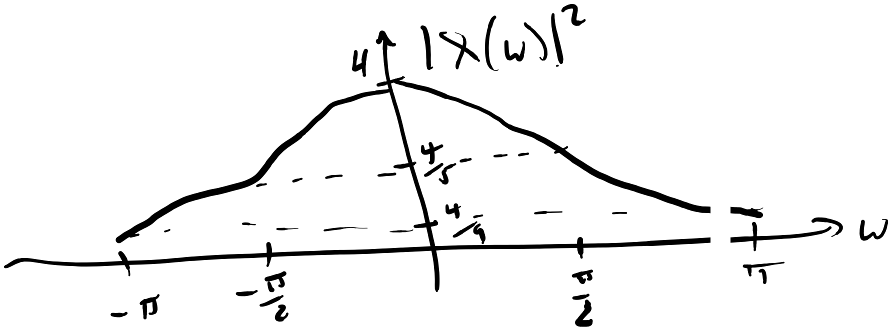
(b) $|X(\omega)|^2 = X(\omega)\, X^*(\omega)$

$$= \left(\frac{1}{1 - \frac{1}{2}e^{-j\omega}}\right)\left(\frac{1}{1 - \frac{1}{2}e^{j\omega}}\right)$$

$$= \frac{1}{1 - \frac{1}{2}e^{-j\omega} - \frac{1}{2}e^{j\omega} + \frac{1}{4}}$$

$$= \frac{1}{\frac{5}{4} - \frac{1}{2}\left(e^{j\omega} + e^{-j\omega}\right)}$$

$$= \frac{1}{\frac{5}{4} - \cos(\omega)}$$



$$\left(\frac{1}{\frac{5}{4} - 1} = \frac{1}{\frac{1}{4}} = 4 \qquad |\,\omega = 0\right.$$

π

$$|X(\omega)|^2 = \begin{cases} \dfrac{1}{\frac{5}{4}} = \dfrac{4}{5} & \omega = \pm\frac{\pi}{2} \\[2ex] \dfrac{1}{\frac{5}{4}+1} = \dfrac{4}{9} & \omega = \pm\pi \end{cases}$$

$$\frac{1}{1-\frac{1}{2}e^{-j\omega}} = \frac{1}{1-\frac{1}{2}\cos(-\omega) - \frac{1}{2}j\sin(-\omega)}$$

$$= \frac{1}{\left(1-\frac{1}{2}\cos(\omega)\right) + j\frac{1}{2}\sin(\omega)}$$

$$\measuredangle \text{DENOM} = \operatorname{atan}\left(\frac{\frac{1}{2}\sin(\omega)}{1-\frac{1}{2}\cos(\omega)}\right)$$

$$\measuredangle X(\omega) = -\operatorname{atan}\left(\frac{\frac{1}{2}\sin(\omega)}{1-\frac{1}{2}\cos(\omega)}\right)$$

$$|X(\omega)| = \frac{1}{\sqrt{\left(1-\frac{1}{2}\cos(\omega)\right)^2 + \frac{1}{4}\sin^2\omega}}$$

$$|X(\omega)|^2 = \frac{1}{\left(1-\frac{1}{2}\cos(\omega)\right)^2 + \frac{1}{4}\sin^2(\omega)}$$