NON-CAUSAL B/C WHY NOT RIGHT-SIDED?

STABLE B/C \( \sum_{n=\infty}^{\infty} |x[n]| \) FINITE

1 Solve by convolution

\[ y[n] = \sum_{m=-\infty}^{\infty} h[m] \cdot x[n-m] \]
\[ y[n] = \sum_{m=-5}^{n+5} x[n-m]h[m] \]

\[ \begin{align*}
  & \quad = \sum_{m=-5}^{n+5} \frac{1}{m} = \sum_{m=0}^{n+10} \frac{1}{m} \\
  & \quad = n + 11 \quad \text{for} \quad n + 5 \geq -5 \quad n + 5 \leq 15 \\
  & \quad \text{for} \quad n \geq -10 \quad n \leq 0
\end{align*} \]

**Case** \( n < -10 \):

\[
\begin{array}{c}
h[m] \\
\hline
-5 \quad 5 \\
\end{array}
\]

\[ x[n-m]h[m] = 0 \quad \text{everywhere} \]

so \( y[n] = \sum_{m} x[n-m]h[m] = 0 \)

**Case** \( n > 0 \):

\[
\begin{array}{c}
h[m] \\
\hline
-5 \quad 5 \\
\end{array}
\]

\[ x[n-m]h[m] \]

\[ x[n-m]h[m] = 0 \quad \text{everywhere} \]

so \( y[n] = \sum_{m} x[n-m]h[m] = 0 \)
\[ y(m) = \sum_{m=-\infty}^{\infty} x(n-m) \]
\[ = \sum_{n=-10}^{0} 1 = 0 \]
\[ = 0 - (n - 10) + 1 \]
\[ = 11 - n \]
\[ y(n) = \begin{cases} 
0 & n < -10 \\
11-n & -10 \leq n \leq 0 
\end{cases} \]

\[ \text{\(\hat{y}(\omega)\)} \]
\[ \text{\(\int_{-\infty}^{\infty} y(n) e^{-j\omega n} dn\)} \]

\(\bigcirc:\ \text{Solve } \ Y(\omega) = H(\omega) X(\omega) \)

\[ X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} \]
\[ \sum_{n=0}^{\infty} e^{-jn} = \sum_{m=0}^{\infty} e^{-j(m-n)} = \sum_{m=0}^{\infty} e^{-jwm} = e^{-jw} \sum_{m=0}^{\infty} (e^{-jw})^m = e^{-jw} \frac{1}{1-e^{-jw}} \]

**Useful Fact**

\[ \sum_{m=0}^{\infty} a^m = \frac{1}{1-a} \]

\[ \sum_{n=0}^{\infty} a^n - \sum_{m=0}^{\infty} a^m = \frac{1}{1-a} - a^n \sum_{m=0}^{\infty} a^m = \frac{1}{1-a} - a^n \frac{1}{1-a} = \frac{1-a^n}{1-a} \]
\[
\sum_{n=0}^{\infty} a^n = \frac{1 - a}{1 - a}
\]

\[
X(\omega) = e^{+j\omega} \frac{1 - e^{-j\omega}}{1 - e^{-j\omega}} = H(\omega)
\]

\[
Y(\omega) = H(\omega)X(\omega)
\]

\[
= e^{+j\omega} \left( \frac{1 - e^{-j\omega}}{1 - e^{-j\omega}} \right)^2
\]

\[
= e^{+j\omega} \left( \frac{1 - e^{-j\omega}}{1 - e^{-j\omega}} \right)
\]