2022nov09

Wednesday, November 9, 2022 13:09

$$v[n] = e^{j\omega_0 n} w[n]$$

$$w[n] = \begin{cases} 1 & 0 \le n \le N-1 \\ 0 & \text{else} \end{cases}$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} = \sum_{n=0}^{N-1} e^{j\omega_0 n} e^{-j\omega n}$$

$$= \sum_{n=0}^{N-1} e^{-j(\omega-\omega_0)n} = W(\omega - \omega_0)$$

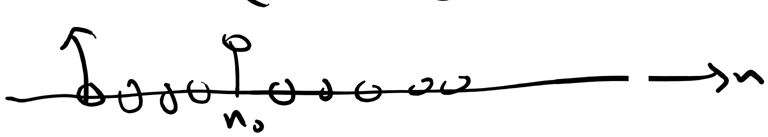
$$W(\omega) = \sum_{n=0}^{N-1} e^{-j\omega n} = e^{-j\omega\left(\frac{N-1}{2}\right)} \frac{\sin(\omega N/2)}{\sin(\omega/2)}$$

FREQ SHIFT PROPERTY OF DTFT

$$x[n] = e^{j\omega_0 n} w[n] \longleftrightarrow X(\omega) = W(\omega - \omega_0)$$

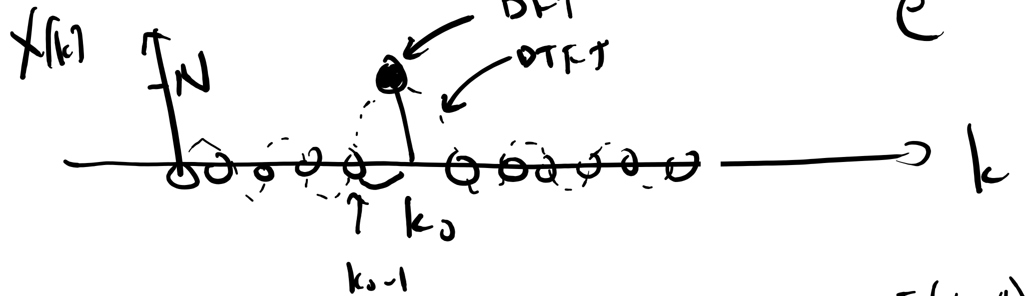
DISCRETE-TIME IMPULSE

$$\delta[n-n_0] = \begin{cases} 1 & n = n_0 \\ 0 & \text{else} \end{cases}$$



DISCRETE-FREQUENCY IMPULSE

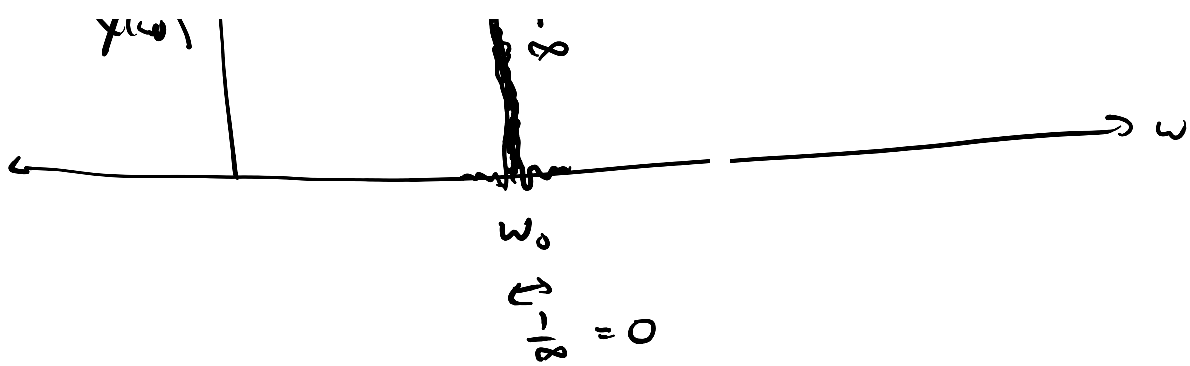
$X[k] = \delta[k-k_0] = \begin{cases} 1 & k = k_0 \\ 0 & \text{else} \end{cases}$ = SPECTRUM OF $e^{j\omega_0 n}$ FOR $\omega_0 = \frac{2\pi k_0}{N}$



$\omega_0 - \frac{2\pi}{N} = \frac{2\pi(k_0 - 1)}{N}$

$$x[n] = e^{j\omega_0 n} w[n] \longleftrightarrow X(\omega) = e^{-j(\omega-\omega_0)\frac{N-1}{2}} \frac{\sin\left(\frac{N}{2}(\omega-\omega_0)\right)}{\sin\left(\frac{1}{2}(\omega-\omega_0)\right)}$$

IF $x[n] = \cos(\omega_0 n)$ $(N \to \infty)$, THEN

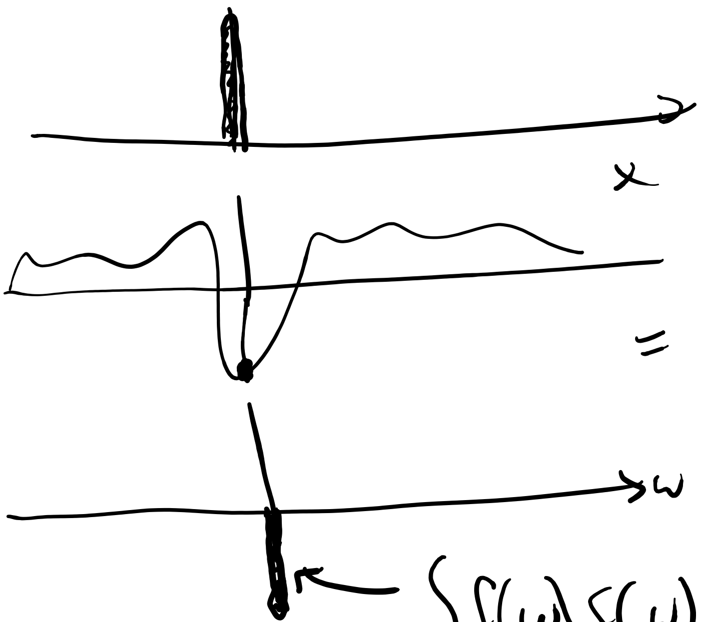


CONTINUOUS-FREQUENCY IMPULSE

$\delta(\omega-\omega_0) = \begin{cases} 0 & \omega \neq \omega_0 \\ \infty & \omega = \omega_0 \end{cases}$

AND $\int^{\pi} \delta(\omega-\omega_0)\, d\omega = 1$

SAMPLING PROPERTY



$\delta(\omega)$

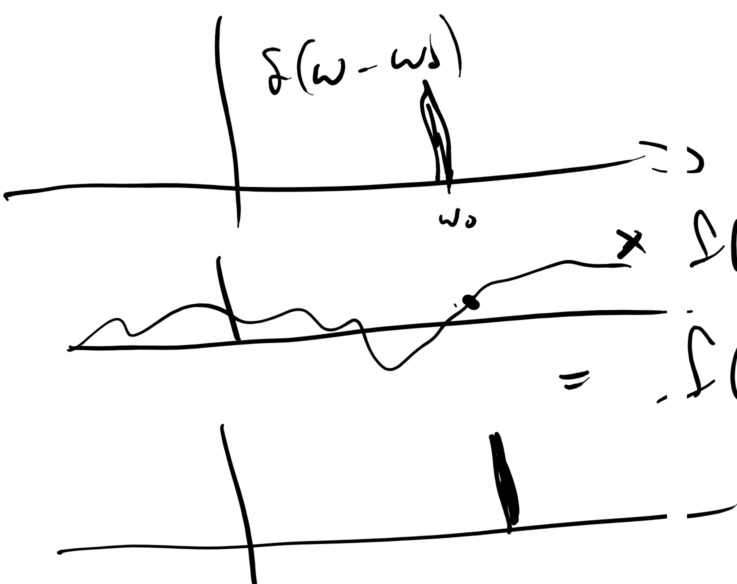
$f(\omega)$

$= f(0)\,\delta(\omega)$

$\int f(\omega)\,\delta(\omega)\,d\omega$

$= \int f(0)\,\delta(\omega)\,d\omega =$

$f(0) \int \delta(\omega)\,d\omega = f(0)$



$\times\ f(\omega)$

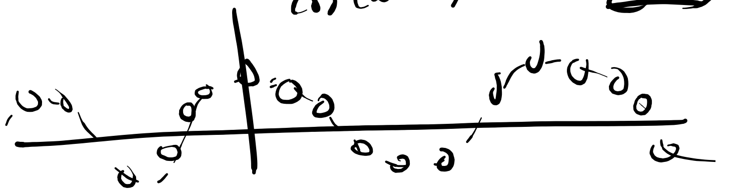
$= f(\omega_0)\,\delta(\omega - \omega_0)$

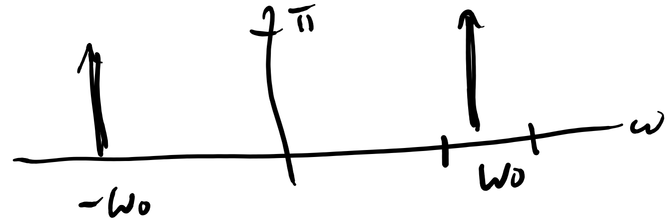
DFFT PAIRS

$\pi\,\delta(\omega - \omega_0) + \pi\,\delta(\omega + \omega_0)$

DTFT THM ... $\pi\delta(\omega-\omega_0)$

$\cos(\omega_0 n) \longleftrightarrow$





$$\int_{\omega_0-\epsilon}^{\omega_0+\epsilon} X(\omega)\,d\omega = \pi$$

$$X(\omega) * W(\omega) = \int_{-\pi}^{\pi} X(\theta)\, W(\omega-\theta)\,d\theta$$

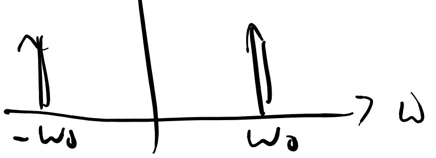
$$X(\omega) = \delta(\omega-\omega_0)$$

$$X(\omega) * W(\omega) = \int \delta(\theta-\omega_0)\, W(\omega-\theta)\,d\theta$$

$$X(\omega) * W(\omega) = \int \delta(\theta-\omega_0)\, \underbrace{W(\omega-\theta)}_{\text{PLUG } \theta=\omega_0 \text{ IN HERE}}\,d\theta$$

$$= W(\omega-\omega_0)$$

$X(\omega)$



$*$

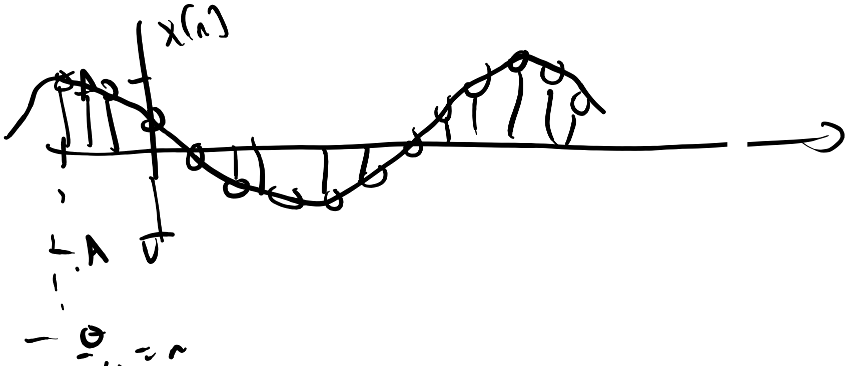
$W(\omega)$

$=$

WRITTEN EXAMPLE

GIVEN $\boxed{e^{j\omega_0 n} \longleftrightarrow 2\pi\delta(\omega-\omega_0)}$ Definition

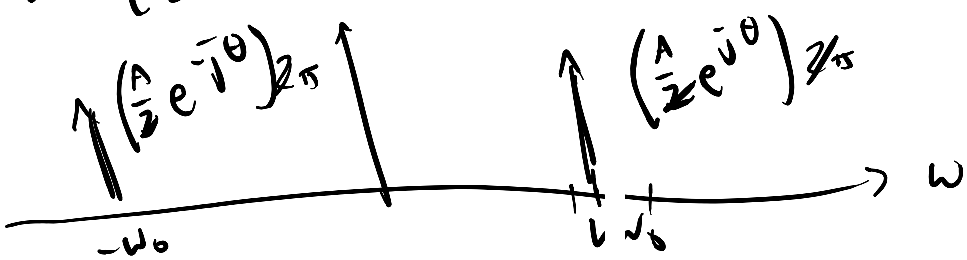
WHAT IS $X(\omega)$ IF $x[n] = A\cos(\omega_0 n + \theta)$



$$x[n] = \frac{A}{2} e^{j(\omega_0 n + \theta)} + \frac{A}{2} e^{-j(\omega_0 n + \theta)}$$

$$= \underline{\left(\frac{A}{2} e^{j\theta}\right)} \underline{e^{j\omega_0 n}} + \underline{\left(\frac{A}{2} e^{-j\theta}\right)} \underline{e^{-j\omega_0 n}}$$

$$X(\omega) = 2\pi\left(\frac{A}{2} e^{j\theta}\right) \delta(\omega - \omega_0) + 2\pi\left(\frac{A}{2} e^{-j\theta}\right) \delta(\omega + \omega_0)$$



$$\int_{\omega_0 - \epsilon}^{\omega_0 + \epsilon} X(\omega)\, d\omega = \cancel{2}\pi \frac{A}{\cancel{2}} e^{j\theta} = \pi A e^{j\theta}$$

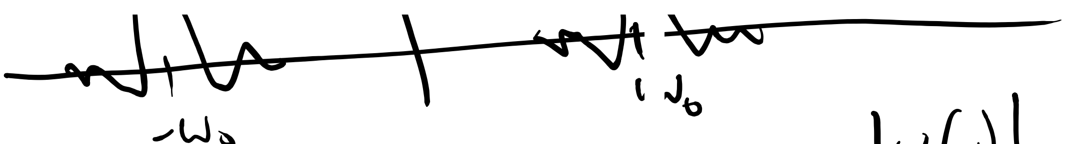
(b) $y[n] = x[n]\,w[n]$

$$Y(\omega) = \frac{1}{2\pi}\left(X(\omega) * W(\omega)\right)$$

$X(\omega)$

$* \; W(\omega)$

$$= \pi A e^{j\theta}\, W(\omega - \omega_0) + \pi A e^{-j\theta}\, W(\omega + \omega_0)$$



$$|X(\omega)| = \pi A\,|W(\omega - \omega_0)| + \pi A\,|W(\omega + \omega_0)|$$