2022nov07

Monday, November 7, 2022 13:34

1 1,000,000,000 audio samples

convolved with $h[n]$, $L = 1025$

($h[0]$ thru $h[1024]$ nonzero)

1a Time-domain convolution

$$y[n] = \sum_{m=-\infty}^{\infty} h[m]\,x[n-m]$$

$$= \sum_{m=0}^{1024} h[m]\,x[n-m]$$ — multiplications / output sample

Complexity: $(1025) \times (1{,}000{,}000{,}000)$ multiplications

$\approx 10^{12}$ multiplications

1b <u>overlap-add</u>

rule of thumb: $M \approx L$

$N = M + L - 1$ should be a power of 2

$= M + 1025 - 1 \Rightarrow N = 2048, \; M = 1024$

Per 1024 samples:

① Compute $X[k] = FFT\{x[n]\}$ w/ $N = 2048$

$\Rightarrow N \log_2 N = 2048 \times 11$ multiplications

② Compute $Y[k] = H[k] \cdot X[k]$

$\Rightarrow N = 2048$ multiplications

③ Compute $y[n] = FFT^{-1}\{Y[k]\}$

$\Rightarrow N \log_2 N = 2048 \times 11$ multiplications

④ Add $y_{t+1}[n]$ to $y_t[n] \Rightarrow M$ additions / M samples

Total: $\dfrac{N(2\log_2 N + 1)}{M}$ $\dfrac{\text{multiplications}}{\text{sample}}$

For this example, $N = 2M$

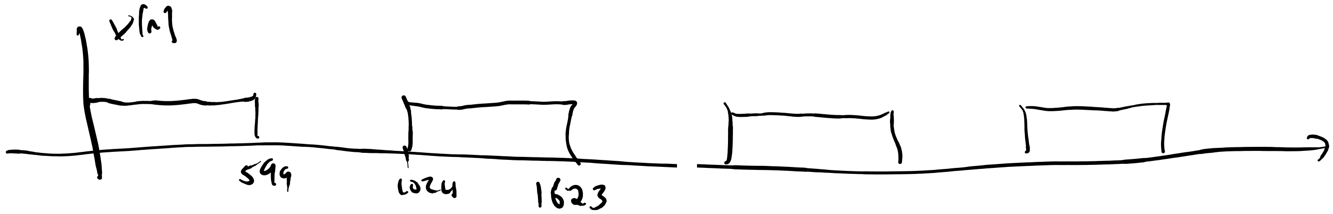
$\Rightarrow 2(2\log_2 N + 1)$ $\dfrac{\text{multiplications}}{\text{sample}}$

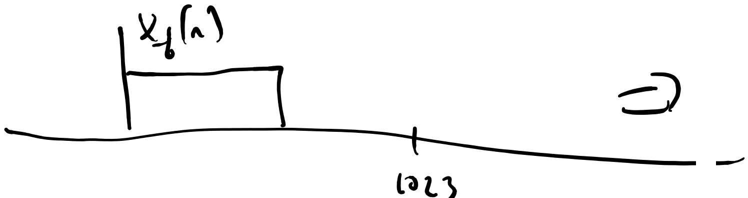
$= 2 \cdot 23 = 46$ or $41 \dfrac{\text{additions}}{\text{sample}}$

Total complexity:

$46{,}000{,}000{,}000 = 4.6 \times 10^{10}$ total multiplications

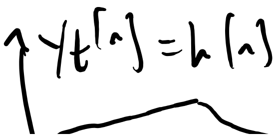
$\boxed{2}$ $x[n] = \begin{cases} 1 & 0 \le n \le 599 \\ 0 & 600 \le n \le 1023 \end{cases}$

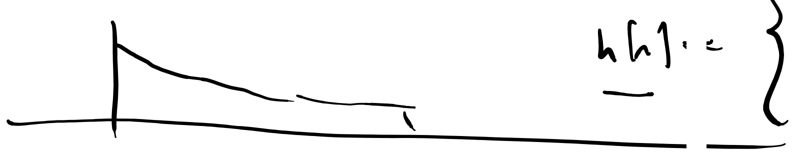




$\Rightarrow$ $X[k]$

$y_t[n] = h[n] \circledast x_t[n]$ $\Leftarrow$ $Y[k] = H[k]X[k]$





$\underline{h[n]} = \begin{cases} a^n & 0 \le n \le 1024 \\ 0 & \text{else} \end{cases}$

$N = 2048 = M + L - 1$ $\quad$ $M = 1024$, $L = 1025$

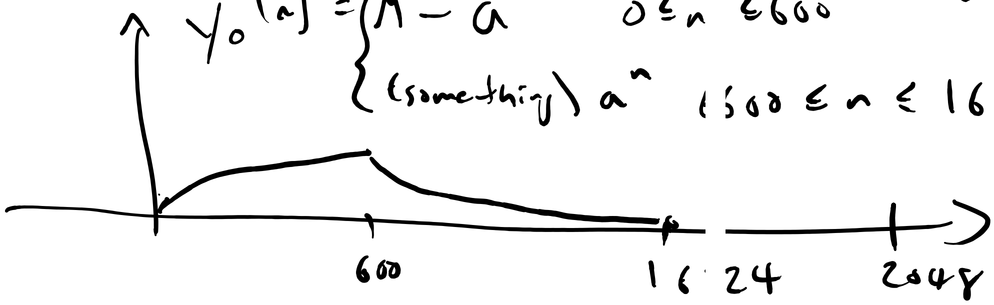
COULD DO THIS:

$$X[k] = e^{-j\omega\left(\frac{M-1}{2}\right)} \frac{\sin(\omega M/2)}{\sin(\omega/2)} \Bigg|_{\omega = \frac{2\pi k}{N}}$$

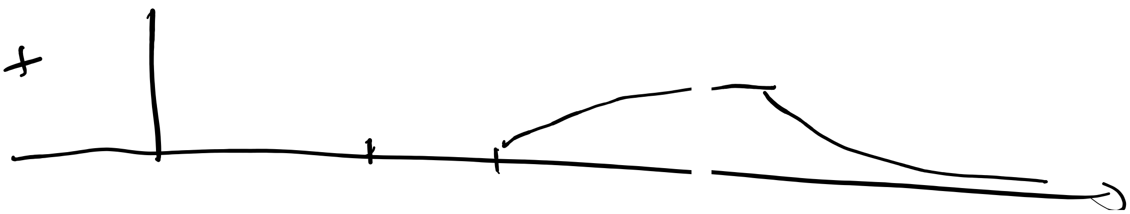
INSTEAD LET'S COMPUTE: $y[n] = x[n] * h[n]$ ...

$$= x[n] \circledast h[n]_{L+M-1}^{N\geq}$$

$$y_0[n] = \begin{cases} (1 - a^n) & 0 \leq n \leq 600 \\ (\text{something})\, a^n & 600 \leq n \leq 1624 \end{cases} \qquad = FFT^{-1}(X[k]H[k])$$



$+$ $y_1[n - 1024]$



$+$ $y_2[n - 2048]$ 1024

$=$ $y[n] \approx$