2022dec12

Monday, December 12, 2022 17:57

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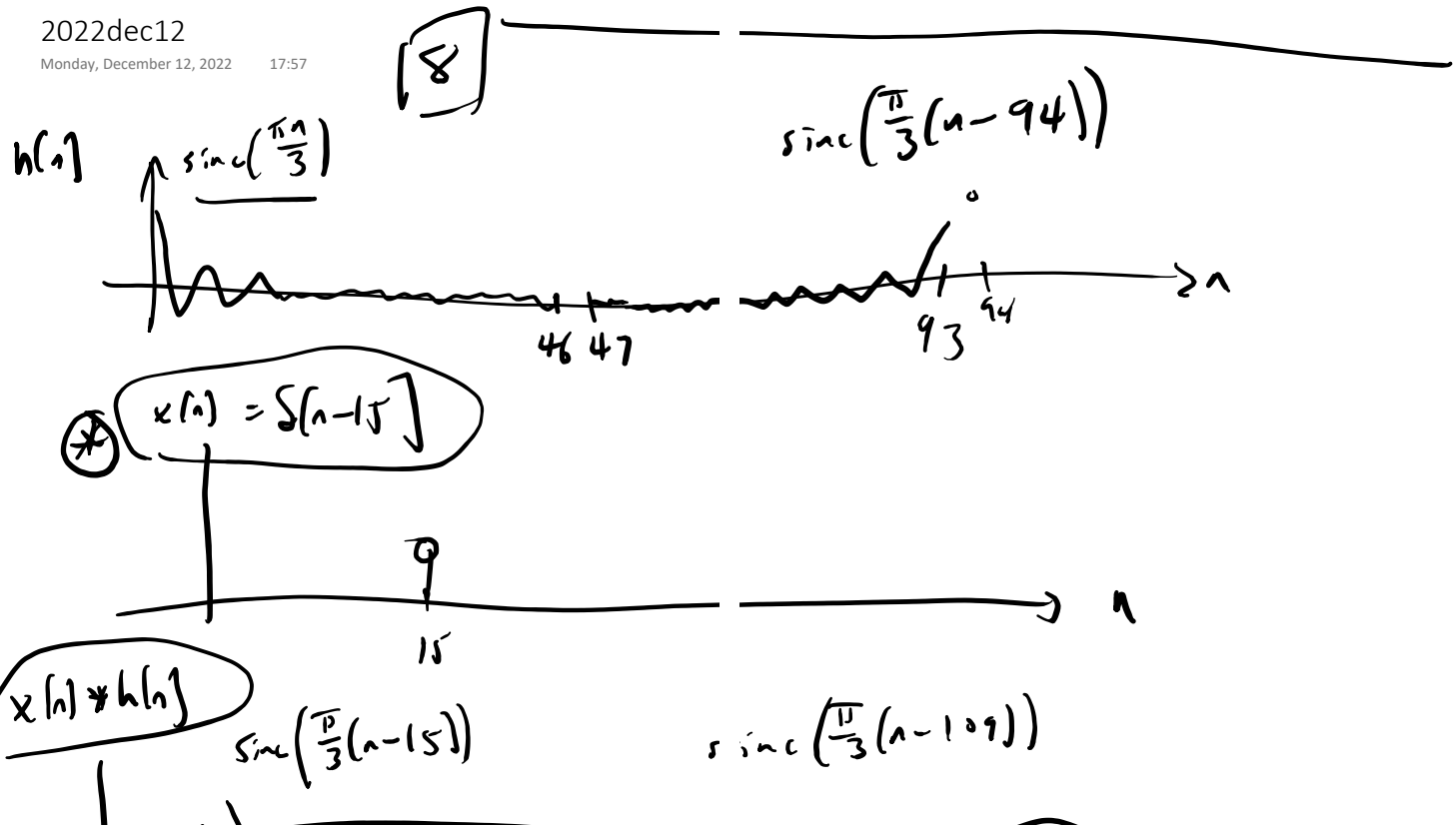
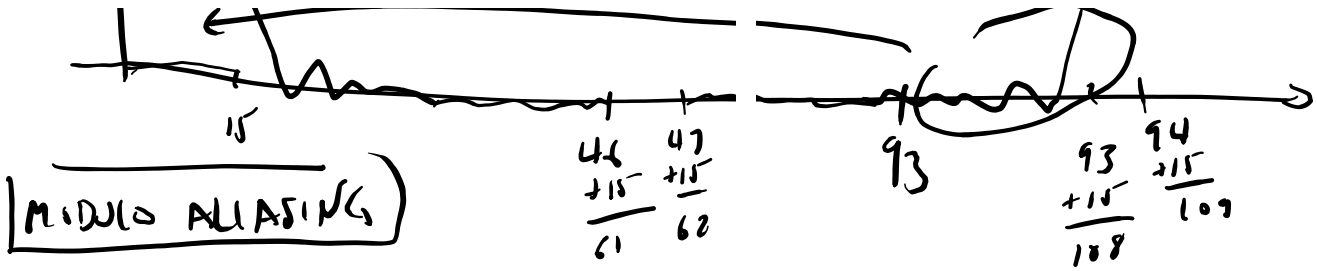
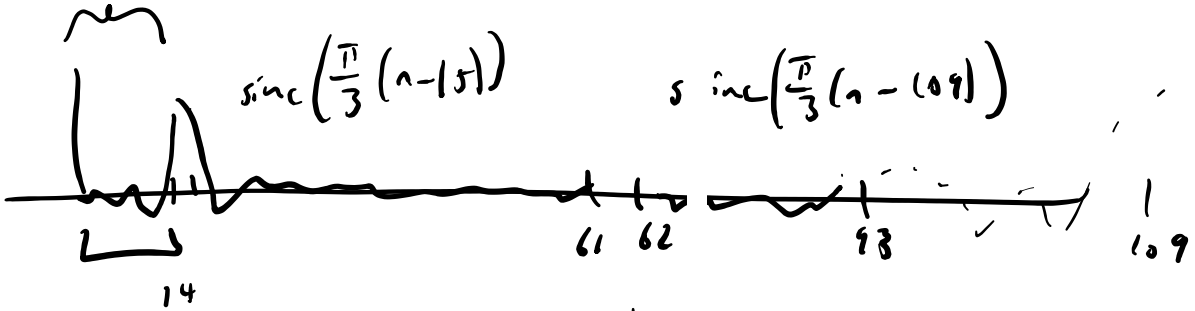




MODULO ALIASING



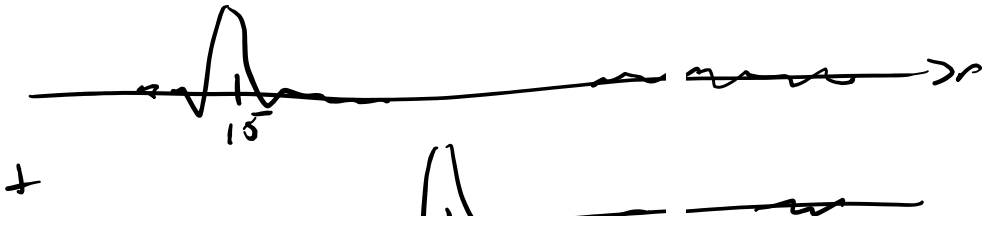
$$y[n] = \begin{cases} \text{sinc}\left(\frac{\pi}{3}(n-(109-64)\right) & 0 \le n \le 14 \\ \text{sinc}\left(\frac{\pi}{3}(n-15)\right) & 14 \le n \le 61 \end{cases}$$

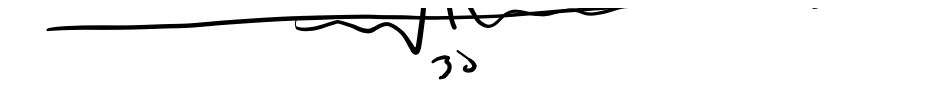
$\left| \text{sinc}\left(\frac{\pi}{3}(n-109)\right) \right| \qquad 62 \le n \le 93$

$\text{sinc}\left[\frac{\pi}{3}(n-(109-94))\right]$

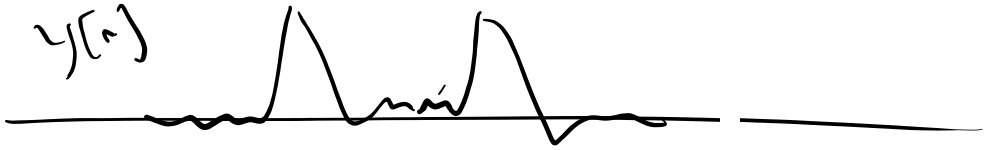
$= \text{sinc}\left[\frac{\pi}{3}(n-15)\right]$

$h(n) * \left(\delta[n-15] + \delta(n-30)\right)$





$y[n]$



[4]

$x[n] = \text{sinc}(0.5\pi n)$

$h[n] = \frac{1}{3}\text{sinc}(\pi n/3)$

$y[n] = x[n] * h[n]$

$Y(\omega) = X(\omega)H(\omega)$

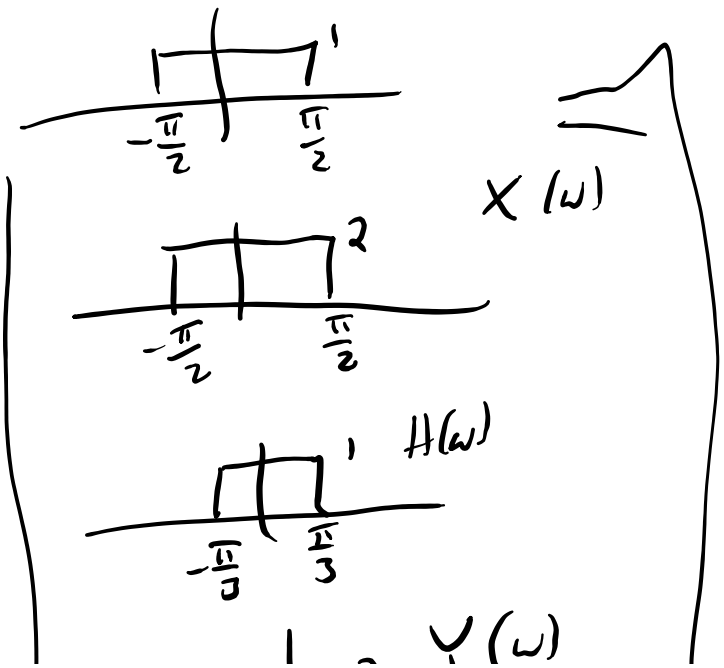
$\begin{cases} 1 & |\omega| < \omega_c \end{cases}$

$\frac{W_c}{\pi}\,\mathrm{sinc}(W_c n) \longleftrightarrow$ $\begin{cases} 1 & |\omega| < \omega_c \\ 0 & \omega_c < |\omega| < \pi \end{cases}$

$0.5\,\mathrm{sinc}(0.5\pi n) \longleftrightarrow$

$x[n] = \mathrm{sinc}(0.5\pi n) \longleftrightarrow$

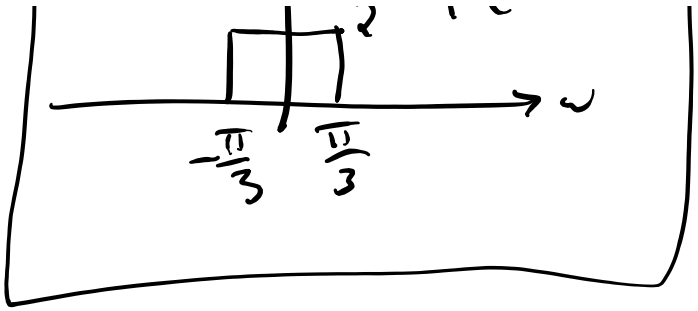
$h[n] = \frac{1}{3}\mathrm{sinc}\left(\frac{\pi n}{3}\right) \longleftrightarrow$



$y[n] \longleftrightarrow$

$= 2\cdot\left(\frac{\omega_c}{\pi}\right)\mathrm{sinc}(W_c n)$

$= \frac{2}{3}\,\mathrm{sinc}\left(\frac{\pi n}{3}\right)$



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f) $x(n) = \cos(0.32\pi n)\,W_H[n]$

EVERYWHERE ELSE

$W_R(n)$

$W_H(n) = \begin{cases} 0.54 - 0.46\cos\left(\frac{2\pi n}{100}\right) & 0 \le n \le 99 \\ 0 & \text{else} \end{cases}$

FIND $X[k]$

$X[k] = X(w)\Big|_{w=\frac{2\pi k}{N}}$ IF $x(n)$ LENGTH $= N$

IT IS WINDOWED, SO FIND $X(w)$

$$x[n] = \frac{1}{2} e^{j0.32\pi n} w_H[n] + \frac{1}{2} e^{-j0.32\pi n} w_H[n]$$

FREQ SHIFT PROPERTY

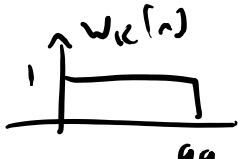
$$X(w) = \frac{1}{2} W_H(w - 0.32\pi) + \frac{1}{2} W_H(w + 0.32\pi)$$

$\Rightarrow$ FIND $W_H(w)$!

$$w_H[n] = w_R[n] \left(0.54 - 0.46 \cos\left(\frac{2\pi n}{100}\right)\right)$$

$$= 0.54 w_R[n] - 0.23 e^{j\frac{2\pi n}{100}} w_R[n] - 0.23 e^{-j\frac{2\pi n}{100}} w_R[n]$$

$$W_H(w) = 0.54 W_R(w) - 0.23 W_R\left(w - \frac{2\pi}{100}\right) - 0.23 W_R\left(w + \frac{2\pi}{100}\right)$$



$\longleftrightarrow$ $W_R(w) = e^{-jw\left(\frac{N-1}{2}\right)} \frac{\sin(wN/2)}{\sin(w/2)}$

$$V[k] = X(w)\big|_{w=\frac{2\pi k}{100}} \quad (1)$$

(2)

WHERE

$$X(w) = \frac{1}{2} W_H(w - 0.32\pi) + \frac{1}{2} W_H(w + 0.32\pi)$$

WHERE

$$W_H(w) = 0.54 W_R(w) - 0.23 W_R\left(w + \frac{2\pi}{100}\right) - 0.23 W_R\left(w - \frac{2\pi}{100}\right)$$

WHERE

$$W_R(w) = e^{-jw\left(\frac{99}{2}\right)} \left(\frac{\sin(100w/2)}{\sin(w/2)} \right)$$

COMBINE (1) & (2)

$$V[k] = \frac{1}{2} W_H\left(\frac{2\pi k}{100} - \frac{32\pi}{100}\right) + \frac{1}{2} W_H\left(\frac{2\pi k}{100} + \frac{32\pi}{100}\right)$$

$$= \frac{1}{2} W_H\left(\frac{2\pi (k-16)}{100}\right) + \frac{1}{2} W_H\left(\frac{2\pi (k+16)}{100}\right)$$

$$= \frac{1}{2} W_H[k-16] + \frac{1}{2} W_H[k+16] = \begin{cases} 0 & \text{UNLESS} \\ k = 15, 16, 17 & \text{OR} \\ & N-15, N-16, \\ & N-17 \end{cases}$$

COMBINE (1) & (3)

$$W_H[k] = 0.54 W_R\left(\frac{2\pi k}{100}\right) - 0.23 W_R\left(\frac{2\pi (k-1)}{100}\right) - 0.23 W_R\left(\frac{2\pi (k+1)}{100}\right)$$

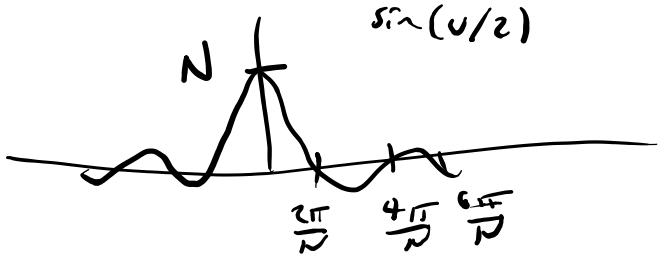
$$= 0.54 W_R[k] - 0.23 W_R[k+1] - 0.23 W_R[k-1] = \begin{cases} 0 & \text{unless} \\ k = 0, \pm 1 \end{cases}$$

COMBINE ① & ④

$$W_K[k] = e^{-j\left(\frac{2\pi k}{100}\right)\left(\frac{99}{2}\right)} \frac{\sin\left(\frac{100}{2}\frac{2\pi k}{100}\right)}{\sin\left(\frac{1}{2}\frac{2\pi k}{100}\right)} \qquad \frac{\sin(100\omega/2)}{\sin(\omega/2)}\Big|_{\omega=0} \to 100$$

$$= \begin{cases} 0 & k \neq 0 \\ 100 & k = 0 \end{cases}$$

$$\frac{\sin(\omega N/2)}{\sin(\omega/2)}$$



$X[k]$ MODIFIED ONLY AT

$k \in \{15, 16, 17, 85, 84, 83\}$

12)

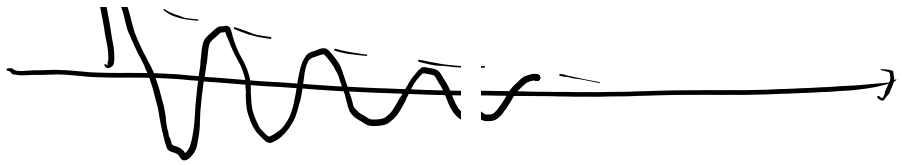
$$X(z) = \frac{1}{1 - 0.9e^{0.1\pi j} z^{-1}}$$

$$\longleftrightarrow \quad x[n] = \left(0.9e^{0.1\pi j}\right)^n u[n]$$

$\hat{h}$ Re($x[n]$)

$\omega_{REF} = 0.1\pi$

$\sigma = -\ln(0.9)$



$$0.9^n = e^{-\sigma n} \quad \Rightarrow \quad \sigma = -\ln(0.9)$$

$$\frac{|x[n]|}{|x[0]|} \leq \frac{1}{e} = e^{-1} \qquad \frac{|x[n]|}{|x[0]|} = 0.9^n$$

$$|x[n]| = \left|\left(0.9\, e^{j0.1\pi}\right)^n u[n]\right| \to 1$$

$$= |0.9|^n \cdot \left|e^{j0.1\pi}\right| \cdot |u[n]|$$

$$= (0.9)^n u[n]$$

$$\left|e^{j0.1\pi}\right| = \left|\cos(0.1\pi) + j\sin(0.1\pi)\right|$$

$$= \sqrt{\cos^2(0.1\pi) + \sin^2(0.1\pi)}$$

FIND n S.T.

$$(0.9)^n \leq e^{-1}$$

11) $y[n] = x[n] - 0.6x[n-$

$Y(z) = X(z)\left(1 - ($

$= \frac{X(z)}{z^2}\left(z^2 - 0.\right.$

2 Eqns

$z = \frac{0.6 \pm \sqrt{0.36 -}}{2}$

$= \frac{0.6 \pm \sqrt{-0.44}}{2}$

$= 0.3 \pm j\frac{\sqrt{0.44}}{2}$

$\angle r_1 = \theta$

$\tan\theta = \frac{O}{A} = \frac{\frac{\sqrt{.44}}{2}}{0.3}$

$\angle r_1 = \text{atan}\left(\frac{\frac{\sqrt{.44}}{2}}{0.6}\right) = a$

$\angle r_2 = -\angle r_1 = -\text{atan}($

$1) + 0.2x[n-2]$

$0.6(z^{-1} + 0.2z^{-2})$

$6z + 0.2)$

0.8

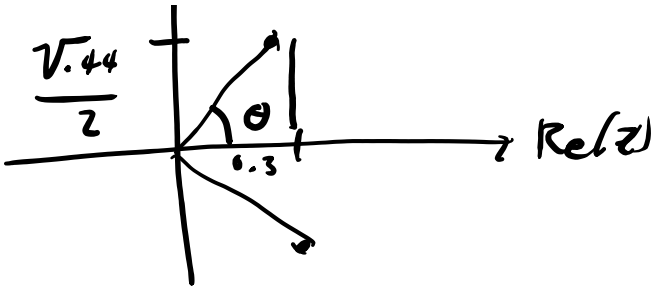
0.80

$.36$

$.44$

$\text{Im}(z)$

$\uparrow$



$$\tan\left(\frac{\sqrt{0.8-0.3^2}}{0.6}\right)$$

$$\frac{\sqrt{0.8-0.3^2}}{0.6}$$

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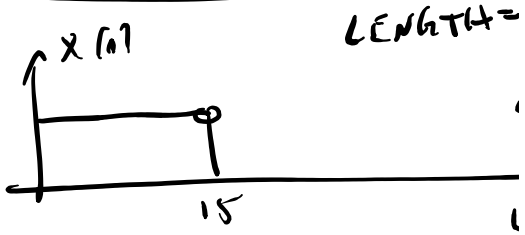


$$Y[k] = e^{-j\pi k} X[k]$$ WI

$$= H[k] X[k]$$ FOR

$$y[n] = h[n] \circledast x[n]$$

$$H[k] = e^{-j\pi k} = e^{-j\omega}$$

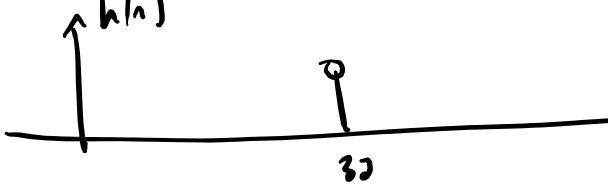
64

ERE $N = 64$

$$H[k] = e^{-j\pi k}$$

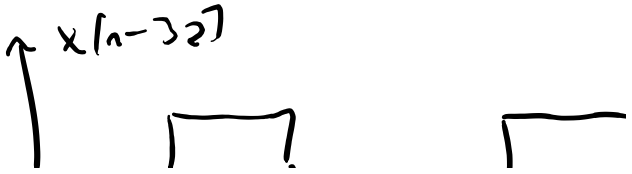
$$n_0 = \frac{N}{2}$$

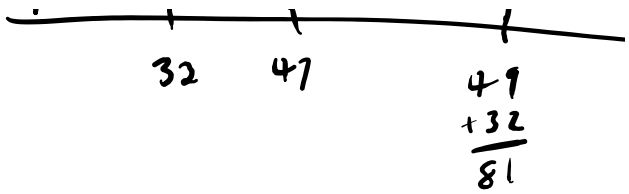
$$H[k] = e^{-j\left(\frac{2\pi k}{N}\right)\frac{N}{2}}$$ ←



$$h[n] * x[n] = x[n-32]$$

$n_0 \Big|_{\omega = \frac{2\pi k}{N}}$ IF





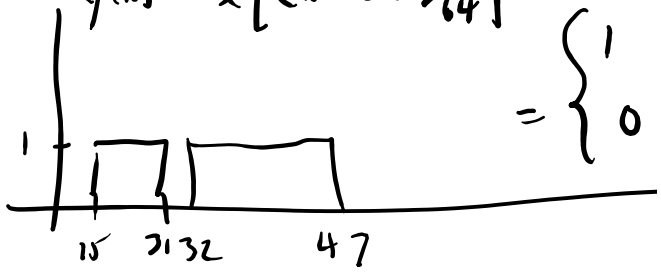
$\rightarrow h[n] = \delta\left[n - \frac{N}{2}\right]$

$= \delta[n-32]$

$h[n] \circledast x[n] = x[\langle n-32 \rangle_{64}]$

$y[n] = x[\langle n-32 \rangle_{64}]$

$= \begin{cases} 1 \\ 0 \end{cases}$



$\longrightarrow$

$= v[n]$

]

$\frac{63}{32}$
$\frac{95}{}$

$4] = \sum_{l=-\infty}^{\infty} v[n-64l]$

$81 - 64 = 15$

$15 \le n \le 47$ $\quad 95 - 64 = 31$

else
