2022dec07

Wednesday, December 7, 2022 13:03

Sample Exam q5

$x(t) = -2 + \sin(40\pi t)$

$F_s = 100$ Hz

$N = 20$ - sample DFT

$$x[n] = x(t)\Big|_{t=\frac{n}{F_s}} = -2 + \sin\left(40\pi \frac{n}{100}\right)$$

$$= -2 + \sin(0.4\pi n)$$ MULTIPLE OF 2π

$$X[k] = \sum_{n=0}^{19} x[n] e^{-j\frac{2\pi k n}{20}}$$ $0.4\pi = \frac{8\pi}{20}$ N

$$\boxed{x[n] = -2e^{0} + \frac{1}{2j}\left[e^{j8\pi n/20} - e^{-j8\pi n/20}\right]}$$

$$X[k] = X(\omega_k)\Big|_{\omega_k = \frac{2\pi k}{N}}$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$e^{j8\pi n/20}$ $\left(e^{-j8\pi n/20}\right)$ $[n]$

$$x[n] = \left(-2 + \frac{1}{2j} e \qquad - \frac{1}{2j} e \right. \qquad jw_R[n]$$

$$W_R(\omega) = e^{-j\omega\left(\frac{N-1}{2}\right)} \frac{\sin(\omega N/2)}{\sin(\omega/2)}$$

$$\delta[n-n_0] \longleftrightarrow z^{-n_0} = -e^{-j\omega n_0}$$

$$X(\omega) = \frac{1}{2\pi} \mathcal{F}\left(-2 + \frac{1}{2j} e^{j\frac{8\pi n}{20}} - \frac{1}{2j} e^{-j\frac{8\pi n}{20}}\right) * W_R(\omega)$$

$$\mathcal{F}(1) = 2\pi\delta(\omega)$$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega) e^{j\omega n} d\omega$$

$$= \frac{1}{\cancel{2\pi}}\int_{-\pi}^{\pi} \cancel{2\pi}\, \underline{\delta(\omega)}\, \underline{e^{j\omega n}}\, d\omega$$

$$= e^{j\omega n}\Big|_{\omega=0} = 1$$

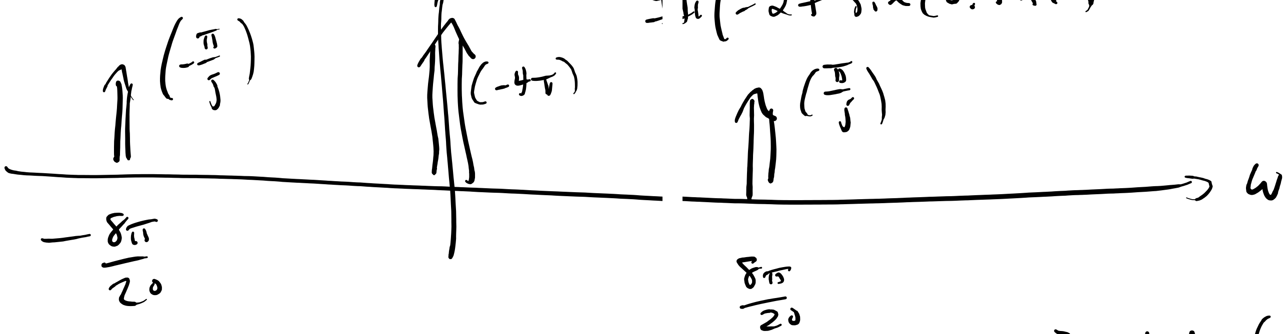
$$\mathcal{F}\left\{-2 + \frac{1}{2j} e^{j\frac{8\pi n}{20}} - \frac{1}{2j} e^{-j\frac{8\pi n}{20}}\right\}$$

$$= -2 \cdot 2\pi\delta(\omega) + \frac{1}{2j} \cdot 2\pi\delta\left(\omega - \frac{8\pi}{20}\right)$$

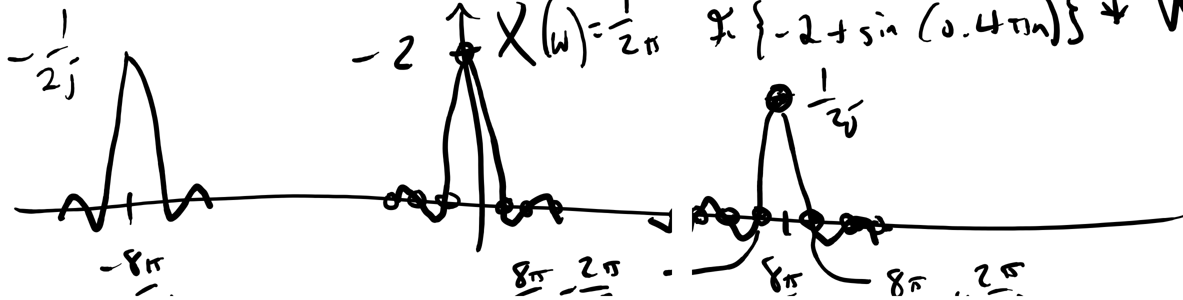
$$- \frac{1}{2j} \cdot 2\pi\delta\left(\omega + \frac{8\pi}{20}\right)$$

$$= -4\pi\,\delta(\omega) + \frac{\pi}{j}\,\delta\left(\omega - \frac{8\pi}{20}\right) - \frac{\pi}{j}\,\delta\left(\omega + \frac{8\pi}{20}\right)$$

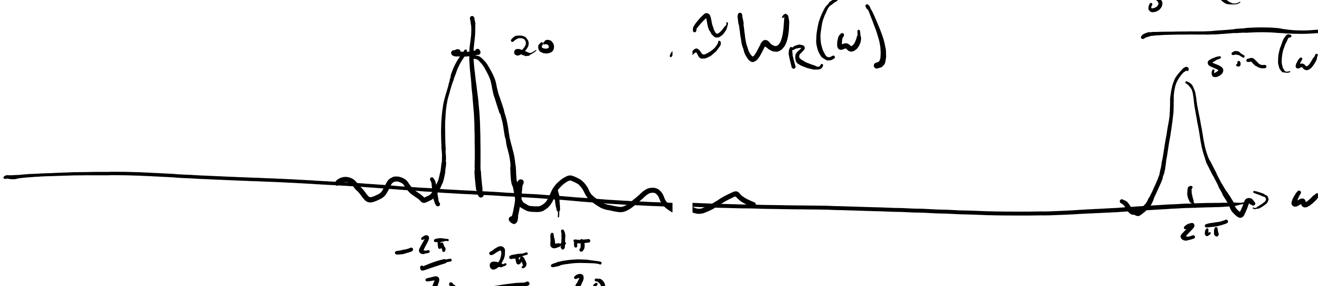
$= \mathcal{F}(-2 + \sin(0.4\pi n))$



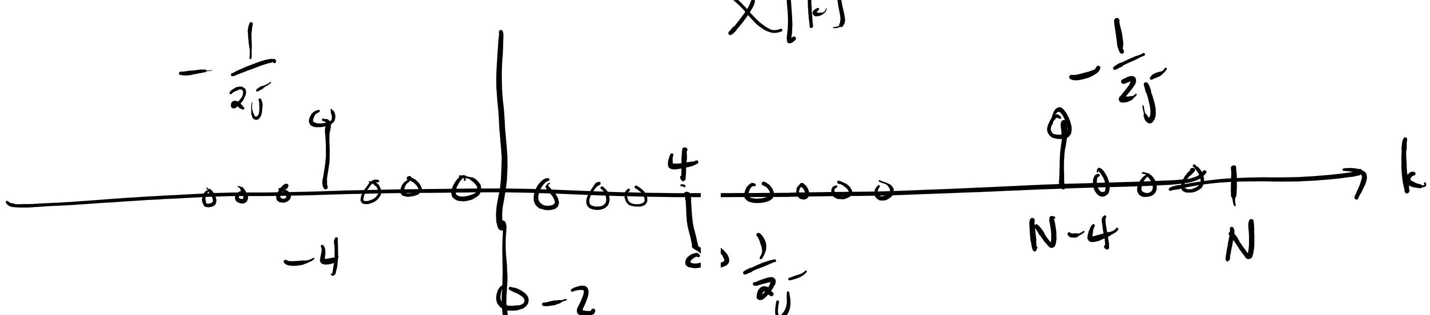
$X(\omega) = \frac{1}{2\pi}\,\mathcal{F}\{-2 + \sin(0.4\pi n)\} * W_R(\omega)$



$\tilde{W}_R(\omega)$ $\quad \frac{\sin(\omega 20/2)}{\sin(\omega/2)}$



$X[k]$



$k=0$

$$X[k] = \begin{cases} \frac{1}{2j} & k = 4 \\ -\frac{1}{2j} & k = N - 4 = 16 \\ 0 & \text{otherwise} \end{cases}$$

SINUSOIDAL COMPONENTS WERE AT $\omega = 0$ AND $\omega = 0.4\pi$, BOTH OF WHICH ARE MULTIPLES OF $\frac{2\pi}{N}$

$\Rightarrow$ WINDOWING DOES NOT CAUSE ANY ENERGY TO LEAK FROM THE BINS $k = \{-4, 0, 4\}$ INTO ANY OTHER BINS.

$\delta(\omega)$ INTRODUCED IN LECTURE 26.

$\boxed{\text{DEFINITION}}$

- $\delta(0) = \infty$
- $\delta(\omega) = 0$ FOR $\omega \neq 0$
- $\int_{-\epsilon}^{\epsilon} \delta(\omega) f(\omega)\, d\omega = f(0)$

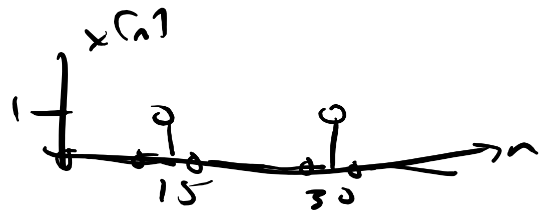
$$\int_{-\epsilon}^{\epsilon} \delta(\omega)\, d\omega = 1$$

$\boxed{\text{RESULTING PROPERTIES}}$

- $\delta(\omega-\omega_0) * W(\omega) = W(\omega-\omega_0)$
- $\mathcal{F}\{e^{j\omega_0 n}\} = 2\pi \underline{\delta(\omega-\omega_0)}$

PROBLEM 1

1 $x[n] = \delta[n-15] + \delta[n-30]$

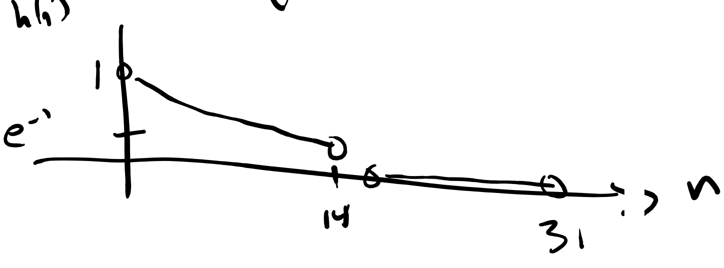


a $X[k] = \sum_{n=0}^{31} x[n]\, e^{-j2\pi k n/N}$

$= e^{-j2\pi 15k/N} + e^{-j2\pi 30k/N}$

$= e^{-j2\pi 15k/32} + e^{-j2\pi 30k/32}$

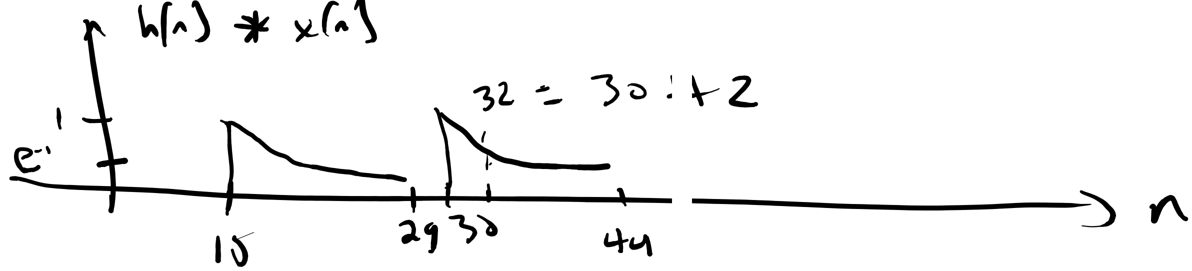
b $h[n] = \begin{cases} e^{-n/14} & 0 \le n \le 14 \\ 0 & \text{else} \end{cases}$



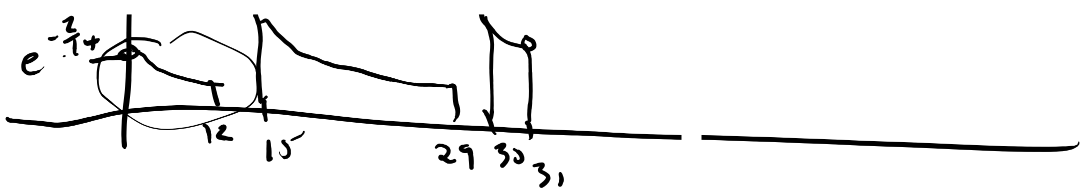
$Y[k] = H[k] X[k]$

$y[n] = h[n] \circledast x[n]$

$= h[n-15] + h[n-30] \Big|_{\text{MODULO } 32}$



$h[n] \circledast x[n] = h[n] * x[n]$ MODULO 32



$$y[n] = \begin{cases} e^{-(n-15)/14} & 15 \le n \le 29 \\ e^{-(n-30)/14} & 30 \le n \le 31 \\ e^{-2/14} e^{-n/14} & 0 \le n \le 12 \\ 0 & 13 \le n \le 14 \end{cases}$$

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$$v[n] = x[n] + 0.9\,v[n-1]$$

$$y[n] = v[n] - 0.7\,y[n-1]$$

$$V(z) = X(z) + 0.9\,z^{-1}V(z)$$

$$Y(z) = V(z) - 0.7\,z^{-1}Y(z)$$

$$V(z)\left(1 - 0.9z^{-1}\right) = X(z)$$

$$Y(z)\left(1 + 0.7z^{-1}\right) = V(z)$$

$$\frac{V(z)}{X(z)} = \frac{1}{1-0.9z^{-1}}$$

$$\frac{Y(z)}{V(z)} = \frac{1}{1+0.7z^{-1}}$$

$$\boxed{H(z) = \frac{Y(z)}{X(z)} = \frac{1}{(1-0.9z^{-1})(1+0.7z^{-1})}}$$

(b)

$$\frac{1}{(1-0.9z^{-1})(1+0.7z^{-1})} = \frac{C_1}{1-0.9z^{-1}} + \frac{C_2}{1+0.7z^{-1}}$$

$$\frac{1}{1+0.7(0.9)^{-1}} = C_1 = \frac{1}{1+7/9}$$

$$\frac{1}{1-0.9(0.7)^{-1}} = C_2 = \frac{1}{1-9/7}$$

$$h[n] = \left(\frac{1}{1+\frac{7}{9}}\right)(0.9)^n u[n] + \left(\frac{1}{1-\frac{9}{7}}\right)(-0.7)^n u[n]$$

$\boxed{3}$ -

$$\omega = \frac{2\pi 440}{F_s} = \frac{2\pi 440}{10000} \qquad F_s = 10{,}000$$

$$\sigma = \frac{\pi 20}{F_s} = \frac{\pi 20}{10000}$$

$$y[n] = x[n] + a_1 y[n-1] + a_2 y[n-2]$$

$$Y(z) = X(z) + a_1 z^{-1} Y(z) + a_2 z^{-2} Y(z)$$

$$Y(z)(1 - a_1 z^{-1} - a_2 z^{-2}) = X(z)$$

$$H(z) = \frac{1}{1 - a_1 z^{-1} - a_2 z^{-2}} = \frac{1}{(1 - p_1 z^{-1})(1 - p_1^* z^{-1})}$$

$$= \frac{1}{1 - (p_1 + p_1^*) z^{-1} + |p_1|^2 z^{-2}}$$

$$p_1 = e^{-\sigma + j\omega_0} = e^{-\frac{\pi 20}{10000} + j\frac{2\pi 440}{10000}}$$

$\pi 20$ $\left(2\pi 440\right.$

$$a_1 = p_1 + p_1^* = 2e^{-\frac{}{10000}} \cos\left(\frac{}{10000}\right)$$

$$a_2 = -|p_1|^2 = -e^{-\frac{2\pi 20}{10000}}$$

$\boxed{D}$

PURE DAMPED RESONATOR,

$$h[n] = \frac{1}{\sin(\omega_1)} e^{-\sigma_1 n} \sin(\omega_1(n+1))\, u[n]$$

$$= \frac{1}{\sin\left(\frac{2\pi 440}{10000}\right)} e^{-\frac{\pi 20 n}{10000}} \sin\left(\frac{2\pi 440}{1000}(n+1)\right) u[n]$$

BY PFE YOU'D GET

$$h[n] = C_1 e^{-\left(\left(\frac{\pi 20}{1000}\right) + j\left(\frac{2\pi 440}{10000}\right)\right)}$$

WHICH IS ALSO VALID A,

FIND OUT WHAT C_1

$$)n \quad + C_2 e^{-\left(\left(\frac{\pi 20}{1000}\right) - j\left(\frac{2\pi 440}{10000}\right)\right)n}$$

NSWER IF YOU CAN

AND C_2 ARE