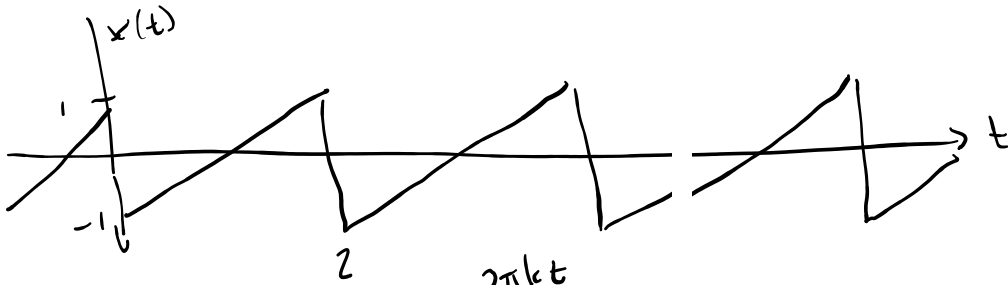


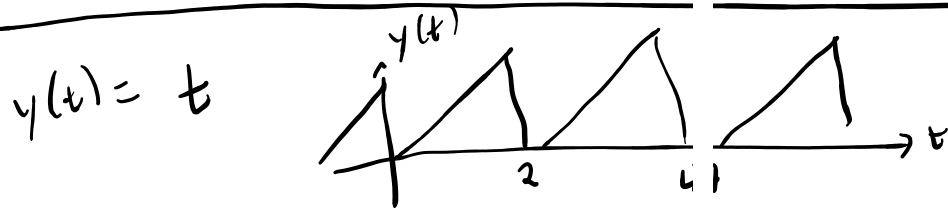
$$x(t) = \begin{cases} -1 + t & 0 \leq t \leq 2 \\ x(t-2) & \text{otherwise} \end{cases}$$



$$X_k = \frac{1}{T_0} \int_0^{T_0} x(t) e^{-j \frac{2\pi k t}{T_0}} dt$$

$$= \frac{1}{2} \int_0^2 (-1 + t) e^{-j \frac{2\pi k t}{2}} dt$$

$$= -\frac{1}{2} \int_0^2 e^{-j \frac{2\pi k t}{2}} dt + \frac{1}{2} \int_0^2 t e^{-j \frac{2\pi k t}{2}} dt$$



$x(t) = -1 + y(t)$

CONSTANT OFFSET PROPERTY:

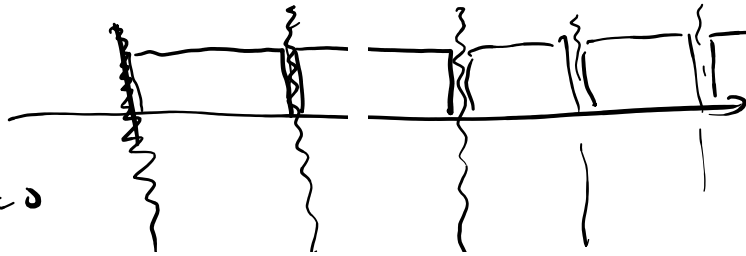
$\therefore Y_0 - 1 \quad \text{if} \quad k=0$

$$X_k = \begin{cases} Y_k & ; k \neq 0 \end{cases}$$

$$Y_k = \frac{1}{T_0} \int_0^{T_0} y(t) e^{-j \frac{2\pi k t}{T_0}} dt$$

$$= \frac{1}{2} \int_0^2 t e^{-j \frac{2\pi k t}{2}} dt$$

$$z(t) = \begin{cases} 1 & t > 0 \\ \text{undef} & \text{at } t=0 \end{cases}$$



$$z(t) = \frac{dy}{dt}$$

BUT UNDEFINED AT

$$t=0,$$

SO CAN'T USE IT



$$Y_k = \frac{1}{2} \int_0^2 t e^{-j \frac{2\pi k t}{2}} dt$$

INT BY PARTS:  $\int u dv = uv - \int v du$

$$t = u$$

$$du = dt$$

$$v = \frac{1}{-j2\pi k} e^{-j\pi k t}$$

$$dv = \frac{1}{2} e^{-j\pi k t} dt$$

$$\int u dv = \left( \frac{1}{-j\pi k} \right) \left( \frac{1}{2} \right) t e^{-j\pi k t} \Big|_0^2$$

$$- \left( \frac{1}{-j2\pi k} \right) \int_0^2 e^{-j\pi k t} dt$$

$$= \frac{1}{-j2\pi k} t e^{-j\pi k t} \Big|_0^2$$

$$+ \frac{1}{j2\pi k} \left( \frac{1}{-j\pi k} \right) e^{-j\pi k t} \Big|_0^2$$

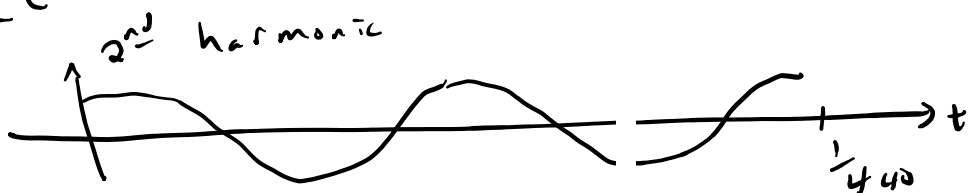
SLIDES EXAMPLE

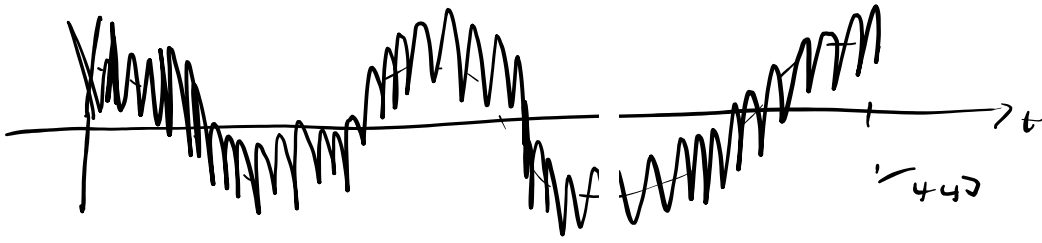
$$F_0 = 440 \text{ Hz}$$

$$k \in \{2, 220\}$$

$$x(t) = \cos(2\pi \cdot 2 \cdot 440t) + \cos(2\pi \cdot 220 \cdot 440t)$$

$$= \frac{1}{2} e^{j880\pi 2t} + \frac{1}{2} e^{-j880\pi 2t} + \frac{1}{2} e^{j880\pi 220t} + \frac{1}{2} e^{-j880\pi 220t}$$





$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{j 880\pi k t}$$

$$X_k = \begin{cases} \frac{1}{2} & k \in \{2, -2, 220, -220\} \\ 0 & \text{otherwise} \end{cases}$$

$$y(t) = \frac{dx}{dt}$$

$$Y_k = j 2\pi k F_0 X_k = j 880\pi k X_k$$

$$\begin{aligned} \text{MIT} & \left\{ \begin{array}{ll} \frac{1}{2} \cdot (j 880\pi 2) & k = 2 \\ \frac{1}{2} (-j 880\pi 2) & k = -2 \end{array} \right. \\ \text{HUGLE} & \left\{ \begin{array}{ll} \frac{1}{2} (j 880\pi 220) & k = 220 \\ \frac{1}{2} (-j 880\pi 220) & k = -220 \\ 0 & \text{otherwise} \end{array} \right. \end{aligned}$$

$$j 880\pi 2 t$$

$$y(t) = -\frac{1}{2j} 880\pi 2 e^{-j880\pi 2t} + \frac{1}{2j} 880\pi 2 e^{-j880\pi 2t} + \dots$$

$$\sin(x) = \frac{1}{2j} (e^{jx} - e^{-jx})$$

$$y(t) = -880\pi 2 \sin(880\pi 2t)$$

$$-880\pi 220 \sin(880\pi 22 \cdot 0t)$$

BIGGER

GIGANTIC!