ECE 401 Signal and Image Analysis
Homework 6

UNIVERSITY OF ILLINOIS
Department of Electrical and Computer Engineering

Assigned: 11/16/2022; Due: 11/30/2022
Reading: *DSP First* Chapters 9 and 10

Problem 6.1

Consider the difference equation:

$$y[n] = x[n] - \frac{1}{2}x[n-1] + \frac{1}{4}x[n-2]$$

Find the frequencies, $\omega = \angle z_1$ and $\omega = \angle z_2$, of the two zeros.

Solution: The transfer function is

$$H(z) = 1 - \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2}$$

which has zeros at

$$z = \frac{1}{4} \pm j\frac{\sqrt{3}}{4}.$$

The frequencies of these two zeros are

$$\omega = \pm\frac{\pi}{3}\ \frac{\text{radians}}{\text{sample}}$$

Problem 6.2

A particular filter has the difference equation

$$y[n] = x[n] - 1.2e^{j3\pi/5}x[n-1] + 0.8e^{j2\pi/5}y[n-1]$$

Express the frequency response of this filter as

$$H(\omega) = \frac{e^{j\omega} - z_1}{e^{j\omega} - p_1}$$

for some zero z_1 and pole p_1.

Solution:

$$H(\omega) = \frac{e^{j\omega} - 1.2e^{j3\pi/5}}{e^{j\omega} - 0.8e^{j2\pi/5}}$$

Problem 6.3

Remember that

$$G(z) = \frac{1}{1 - 0.8z^{-1}} \leftrightarrow g[n] = (0.8)^n u[n]$$

Use the linearity and time-shift properties of the Z-transform to find $h[n]$, where

$$H(z) = \frac{1 - 0.3z^{-1}}{1 - 0.8z^{-1}} = \frac{1}{1 - 0.8z^{-1}} - 0.3z^{-1}\frac{1}{1 - 0.8z^{-1}}$$

Solution: The time-shift property is

$$z^{-n_0}G(z) \leftrightarrow g[n - n_0],$$

so

$$G(z) - 0.3z^{-1}G(z) \leftrightarrow g[n] - 0.3g[n - 1],$$

therefore

$$h[n] = (0.8)^n u[n] - 0.3(0.8)^{n-1}u[n - 1]$$

Problem 6.4

What is $h[n]$ if

$$H(z) = \frac{1}{(1 - e^{j0.1\pi}z^{-1})(1 - e^{-j0.1\pi}z^{-1})}$$

Solution: Using PFE, we get

$$H(z) = \frac{C_1}{1 - e^{j0.1\pi}z^{-1}} + \frac{C_1^*}{1 - e^{-j0.1\pi}z^{-1}}$$

Solving for C_1, we can find that $C_1 = p_1/(p_1 - p_1^*) = e^{j0.1\pi}/(2j\sin(0.1\pi))$, so

$$h[n] = \left(\frac{e^{j0.1\pi(n+1)}}{2j\sin(0.1\pi)} - \frac{e^{-j0.1\pi(n+1)}}{2j\sin(0.1\pi)}\right)u[n] = \frac{\sin(0.1\pi(n+1))}{\sin(0.1\pi)}u[n]$$