Problem 4.1
Consider this filter:
\[ y[n] = x[n] + x[n-1] \]
Show that the magnitude response of this filter is \( |H(\omega)| = 2\cos(\omega/2) \).

Solution:
\[
H(\omega) = \sum_n h[n]e^{-j\omega n} = 1 + e^{-j\omega} = e^{-j\omega/2}(e^{j\omega/2} + e^{-j\omega/2}) = e^{-j\omega/2}2\cos(\omega/2)
\]

Problem 4.2
Suppose you have a filter whose frequency response is
\[ H(\omega) = 14e^{-j6\omega} \]
Show that, if \( x[n] = \cos(\omega n) \), the effect of convolving \( y[n] = x[n] * h[n] \) is to
(a) scale \( x[n] \) by a factor of 14, and
(b) delay it by 6 samples.

Solution: Using the frequency response formula,
\[
y[n] = |H(\omega)|\cos(\omega n + \angle H(\omega)) \\
= 14\cos(\omega n - 6\omega) \\
= 14\cos(\omega(n - 6))
\]

Problem 4.3
The signals $x_1(t)$ and $x_2(t)$ are cosines an octave apart (roughly C6 and C7):

\[
\begin{align*}
  x_1(t) &= \cos(2\pi 1000 t) \\
  x_2(t) &= \cos(2\pi 2000 t)
\end{align*}
\]

The signals are sampled (at $F_s = 16000$ samples/second), then the resulting signals $x_1[n]$ and $x_2[n]$ are passed through a first-difference operator:

\[
\begin{align*}
  y_1[n] &= x_1[n] - x_1[n-1] \\
  y_2[n] &= x_2[n] - x_2[n-1]
\end{align*}
\]

What are the amplitudes of the signals $y_1[n]$ and $y_2[n]$?

**Solution:** In radians/second, the frequencies are

\[
\omega_1 = \frac{2\pi 1000}{16000} = \frac{\pi}{8}, \quad \omega_2 = \frac{2\pi 2000}{16000} = \frac{\pi}{4}
\]

The frequency response of the first-difference operator is

\[
H(\omega) = 1 - e^{-j\omega} = 2je^{-j\omega/2}\sin(\omega/2)
\]

Which has these magnitude responses:

\[
\left|H\left(\frac{\pi}{8}\right)\right| = 2\sin(\pi/16) \approx 0.39, \quad \left|H\left(\frac{\pi}{4}\right)\right| = 2\sin(\pi/8) \approx 0.77
\]

**Problem 4.4**

The signals $x_1(t)$ and $x_2(t)$ are cosines an octave apart (roughly C6 and C7):

\[
\begin{align*}
  x_1(t) &= \cos(2\pi 1000 t) \\
  x_2(t) &= \cos(2\pi 2000 t)
\end{align*}
\]

The signals are sampled (at $F_s = 16000$ samples/second), then the resulting signals $x_1[n]$ and $x_2[n]$ are passed through a seven-sample local average:

\[
\begin{align*}
  y_1[n] &= \frac{1}{7} \sum_{m=-3}^{3} x_1[n - m] \\
  y_2[n] &= \frac{1}{7} \sum_{m=-3}^{3} x_2[n - m]
\end{align*}
\]

What are the amplitudes of the signals $y_1[n]$ and $y_2[n]$?

**Solution:** In radians/second, the frequencies are

\[
\omega_1 = \frac{2\pi 1000}{16000} = \frac{\pi}{8}, \quad \omega_2 = \frac{2\pi 2000}{16000} = \frac{\pi}{4}
\]

The frequency response of the seven-sample central local average filter is

\[
H(\omega) = \frac{\sin(7\omega/2)}{7\sin(\omega/2)}
\]

Which has these magnitude responses:

\[
\left|H\left(\frac{\pi}{8}\right)\right| = \frac{\sin(7\pi/16)}{7\sin(\pi/16)} \approx 0.72, \quad \left|H\left(\frac{\pi}{4}\right)\right| = \frac{\sin(7\pi/8)}{7\sin(\pi/8)} \approx 0.14
\]