ECE 401 Signal and Image Analysis
Homework 4

UNIVERSITY OF ILLINOIS
Department of Electrical and Computer Engineering

Assigned: 10/11/2022; Due: 10/19/2022
Reading: *DSP First* Chapter 6

Problem 4.1

Consider this filter:

$$y[n] = x[n] + x[n-1]$$

Show that the magnitude response of this filter is $|H(\omega)| = 2\cos(\omega/2)$.

Solution:

$$h[n] = \begin{cases} 1 & n = 0,1 \\ 0 & \text{otherwise} \end{cases}$$

$$H(\omega) = \sum_n h[n]e^{-j\omega n} = 1 + e^{-j\omega} = e^{-j\omega/2}(e^{j\omega/2} + e^{-j\omega/2}) = e^{-j\omega/2} 2\cos(\omega/2)$$

Problem 4.2

Suppose you have a filter whose frequency response is

$$H(\omega) = 14e^{-j6\omega}$$

Show that, if $x[n] = \cos(\omega n)$, the effect of convolving $y[n] = x[n] * h[n]$ is to

(a) scale $x[n]$ by a factor of 14, and

(b) delay it by 6 samples.

Solution: Using the frequency response formula,

$$\begin{aligned} y[n] &= |H(\omega)| \cos(\omega n + \angle H(\omega)) \\ &= 14\cos(\omega n - 6\omega) \\ &= 14\cos(\omega(n-6)) \end{aligned}$$

Problem 4.3

The signals $x_1(t)$ and $x_2(t)$ are cosines an octave apart (roughly C6 and C7):

$$x_1(t) = \cos(2\pi 1000 t)$$
$$x_2(t) = \cos(2\pi 2000 t)$$

The signals are sampled (at $F_s = 16000$ samples/second), then the resulting signals $x_1[n]$ and $x_2[n]$ are passed through a first-difference operator:

$$y_1[n] = x_1[n] - x_1[n-1]$$
$$y_2[n] = x_2[n] - x_2[n-1]$$

What are the amplitudes of the signals $y_1[n]$ and $y_2[n]$?

Solution: In radians/second, the frequencies are

$$\omega_1 = \frac{2\pi 1000}{16000} = \frac{\pi}{8}, \quad \omega_2 = \frac{2\pi 2000}{16000} = \frac{\pi}{4}$$

The frequency response of the first-difference operator is

$$H(\omega) = 1 - e^{-j\omega} = 2je^{-j\omega/2}\sin(\omega/2)$$

Which has these magnitude responses:

$$\left|H\left(\frac{\pi}{8}\right)\right| = 2\sin(\pi/16) \approx 0.39, \quad \left|H\left(\frac{\pi}{4}\right)\right| = 2\sin(\pi/8) \approx 0.77$$

Problem 4.4

The signals $x_1(t)$ and $x_2(t)$ are cosines an octave apart (roughly C6 and C7):

$$x_1(t) = \cos(2\pi 1000 t)$$
$$x_2(t) = \cos(2\pi 2000 t)$$

The signals are sampled (at $F_s = 16000$ samples/second), then the resulting signals $x_1[n]$ and $x_2[n]$ are passed through a seven-sample local average:

$$y_1[n] = \frac{1}{7}\sum_{m=-3}^{3} x_1[n-m]$$
$$y_2[n] = \frac{1}{7}\sum_{m=-3}^{3} x_2[n-m]$$

What are the amplitudes of the signals $y_1[n]$ and $y_2[n]$?

Solution: In radians/second, the frequencies are

$$\omega_1 = \frac{2\pi 1000}{16000} = \frac{\pi}{8}, \quad \omega_2 = \frac{2\pi 2000}{16000} = \frac{\pi}{4}$$

The frequency response of the seven-sample central local average filter is

$$H(\omega) = \frac{\sin(7\omega/2)}{7\sin(\omega/2)}$$

Which has these magnitude responses:

$$\left|H\left(\frac{\pi}{8}\right)\right| = \frac{\sin(7\pi/16)}{7\sin(\pi/16)} \approx 0.72, \quad \left|H\left(\frac{\pi}{4}\right)\right| = \frac{\sin(7\pi/8)}{7\sin(\pi/8)} \approx 0.14$$