Problem 1  (25 points)

Each of the following is sampled at $F_s = 10000$ samples/second, producing either $x[n] =$constant, or $x[n] = \cos \omega n$ for some value of $\omega$. Specify the constant if possible; otherwise, specify $\omega$ such that $-\pi \leq \omega < \pi$.

(a) $x(t) = \cos (2\pi 900t)$
   **Solution:** $\omega = \frac{9\pi}{50}$

(b) $x(t) = \cos (2\pi 10000t)$
   **Solution:** $x[n] = 1$

(c) $x(t) = \cos (2\pi 11000t)$
   **Solution:** $\omega = \frac{\pi}{5}$

Problem 2  (25 points)

Consider the signal $x(t) = 2 \cos (2\pi 440t) - 3 \sin (2\pi 440t)$

This signal can also be written as $x(t) = A \cos (\omega t + \theta)$ for some $A = \sqrt{M}$, $\omega$, and $\theta = \text{atan}(R)$. Find $M$, $\omega$, and $R$.

**Solution:**

\[
A = \sqrt{13} \quad (M = 13) \\
\omega = 2\pi 440 \\
\theta = \text{atan} \left( \frac{3}{2} \right) \quad (R = \frac{3}{2})
\]

Problem 3  (25 points)

A signal $x(t)$ is periodic with $T_0 = 0.02$ seconds, and its values are specified by

\[
x(t) = \begin{cases} 
-1 & 0 \leq t \leq 0.01 \\
0 & 0.01 < t < 0.02 
\end{cases}
\]

Its CTFS representation is defined by

\[
x(t) = \sum_{k=-\infty}^{\infty} X_k e^{jk\omega_0 t}
\]

(a) Sketch $x(t)$ as a function of $t$ for $0 \leq t \leq 0.02$ seconds. Label at least one important tic mark, each, on the horizontal and vertical axes.

**Solution:** Useful tic marks include $t = 0.01$ or $t = 0.02$, and $x(t) = -1$ between 0 and 0.01.

(b) What is $\omega_0$?

**Solution:** $\omega_0 = 100\pi$
(c) Find \( X_0 \) without doing any integral.

**Solution:** \( X_0 = -\frac{1}{2} \)

(d) Find \( X_k \) for all the other values of \( k \), i.e., for \( k \neq 0 \). Simplify; your answer should have no exponentials in it.

**Solution:** \( X_k = 0 \) for even \( k \), \( X_k = \frac{j}{k\pi} \) for odd \( k \).

**Problem 4  (25 points)**

Consider the signal

\[ x[n] = \begin{cases} 
    \left(\frac{1}{2}\right)^n & n \geq 0 \\
    0 & n < 0 
\end{cases} \]

(a) Find the DTFT, \( X(\omega) \).

**Solution:** \( X(\omega) = \frac{1}{1 - \frac{1}{2}e^{-j\omega}} \)

(b) Find the power spectrum \( |X(\omega)|^2 \), and sketch it for \(-\pi \leq \omega \leq \pi \). Specify its values at \( \omega = 0, \omega = \frac{\pi}{2}, \) and \( \omega = \pi \).

**Solution:** \( |X(\omega)|^2 = \frac{1}{4 - \cos \omega} \). \( |X(0)|^2 = 4, \; |X(\frac{\pi}{2})|^2 = \frac{4}{5}, \; |X(\pi)|^2 = \frac{4}{9} \).