MIDTERM EXAM

Friday, October 3, 2014

- This is a CLOSED BOOK exam.
- There are a total of 100 points in the exam. Each problem specifies its point total. Plan your work accordingly.
- You must SHOW YOUR WORK to get full credit.

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Name: ____________________________________________
Problem 1  (25 points)

Each of the following is sampled at $F_s = 10000$ samples/second, producing either $x[n] =$ constant, or $x[n] = \cos \omega n$ for some value of $\omega$. Specify the constant if possible; otherwise, specify $\omega$ such that $-\pi \leq \omega < \pi$.

(a) $x(t) = \cos (2\pi 900t)$

(b) $x(t) = \cos (2\pi 10000t)$

(c) $x(t) = \cos (2\pi 11000t)$
Problem 2  (25 points)

Consider the signal
\[ x(t) = 2 \cos(2\pi 440t) - 3 \sin(2\pi 440t) \]

This signal can also be written as \( x(t) = A \cos(\omega t + \theta) \) for some \( A = \sqrt{M}, \omega, \) and \( \theta = \text{atan}(R) \).
Find \( M, \omega, \) and \( R. \)
Problem 3  (25 points)

A signal \( x(t) \) is periodic with \( T_0 = 0.02 \) seconds, and its values are specified by

\[
x(t) = \begin{cases} 
-1 & 0 \leq t \leq 0.01 \\
0 & 0.01 < t < 0.02
\end{cases}
\]

Its CTFS representation is defined by

\[
x(t) = \sum_{k=-\infty}^{\infty} X_k e^{jk\omega_0 t}
\]

(a) Sketch \( x(t) \) as a function of \( t \) for \( 0 \leq t \leq 0.02 \) seconds. Label at least one important tic mark, each, on the horizontal and vertical axes.

(b) What is \( \omega_0 \)?

(c) Find \( X_0 \) without doing any integral.
PROBLEM 3 CONTINUED

(d) Find $X_k$ for all the other values of $k$, i.e., for $k \neq 0$. Simplify; your answer should have no exponentials in it.
Problem 4  (25 points)

Consider the signal

\[ x[n] = \begin{cases} 
\left( \frac{1}{2} \right)^n & n \geq 0 \\
0 & n < 0
\end{cases} \]

(a) Find the DTFT, \( X(\omega) \).
PROBLEM 4 CONTINUED

(b) Find the power spectrum $|X(\omega)|^2$, and sketch it for $-\pi \leq \omega \leq \pi$. Specify its values at $\omega = 0$, $\omega = \frac{\pi}{2}$, and $\omega = \pi$. 