UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 401 Signal and Image Analyais
Spring 2021

EXAM 3

Tuesday, December 14, 2021, 8:00-11:00am

- This is a CLOSED BOOK exam.
- You are permitted two sheets of handwritten notes, 8.5x11.
- Calculators and computers are not permitted.
- Do not simplify explicit numerical expressions. The expression "$e^{-5}\cos(3)$" is a MUCH better answer than "-0.00667".
- If you're taking the exam online, you will need to have your webcam turned on. Your exam will appear on Gradescope at exactly 1:00pm; you will need to photograph and upload your answers by exactly 2:00pm.
- There will be a total of 200 points in the exam. Each problem specifies its point total. Plan your work accordingly.
- You must SHOW YOUR WORK to get full credit.

Name: _____

netid: _____

Phasors

$$A\cos(2\pi ft+\theta) = \Re\left\{Ae^{j\theta}e^{j2\pi ft}\right\} = \frac{1}{2}e^{-j\theta}e^{-j2\pi ft} + \frac{1}{2}e^{j\theta}e^{j2\pi ft}$$

Fourier Series

$$\textbf{Analysis:}\quad X_k = \frac{1}{T_0}\int_0^{T_0} x(t)e^{-j2\pi kt/T_0}dt$$

$$\textbf{Synthesis:}\quad x(t) = \sum_{k=-\infty}^{\infty} X_k e^{j2\pi kt/T_0}$$

Sampling and Interpolation:

$$x[n] = x\left(t = \frac{n}{F_s}\right)$$

$$f_a = \min\left(f \bmod F_s, -f \bmod F_s\right)$$

$$z_a = \begin{cases} z & f \bmod F_s < -f \bmod F_s \\ z^* & f \bmod F_s > -f \bmod F_s \end{cases}$$

$$y(t) = \sum_{n=-\infty}^{\infty} y[n]p(t-nT_s)$$

Convolution

$$h[n] * x[n] = \sum_{m=-\infty}^{\infty} h[m]x[n-m] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]$$

Frequency Response and DTFT

$$H(\omega) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n}$$

$$h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(\omega)e^{j\omega n}d\omega$$

$$h[n] * \cos(\omega n) = |H(\omega)|\cos\left(\omega n + \angle H(\omega)\right)$$

Rectangular & Hamming Windows; Ideal LPF

$$w_R[n] = \begin{cases} 1 & 0 \le n \le N-1 \\ 0 & \text{otherwise} \end{cases} \leftrightarrow W_R(\omega) = e^{-\frac{j\omega(N-1)}{2}}\frac{\sin(\omega N/2)}{\sin(\omega/2)}$$

$$w_H[n] = 0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right)w_R[n] \leftrightarrow W_H(\omega) = 0.54W_R(\omega) - 0.23W_R\left(\omega - \frac{2\pi}{N-1}\right) - 0.23W_R\left(\omega + \frac{2\pi}{N-1}\right)$$

$$h_{\text{ideal}}[n] = \frac{\omega_c}{\pi}\text{sinc}(\omega_c n) \leftrightarrow H_{\text{ideal}}(\omega) = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & \text{otherwise} \end{cases}$$

Discrete Fourier Transform

$$\textbf{Analysis:}\quad X[k] = \sum_{n=0}^{N-1} x[n]e^{-j2\pi kn/N}$$

$$\textbf{Synthesis:}\quad x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j2\pi kn/N}$$

Z Transform Pairs

$$b_k z^{-k} \leftrightarrow b_k \delta[n-k]$$

$$\frac{1}{1-az^{-1}} \leftrightarrow a^n u[n]$$

$$\frac{1}{(1-e^{-\sigma_1 - \jmath\omega_1} z^{-1})(1-e^{-\sigma_1 + \jmath\omega_1} z^{-1})} \leftrightarrow \frac{1}{\sin(\omega_1)} e^{-\sigma_1 n} \sin\left(\omega_1 (n+1)\right) u[n]$$