ECE 401 Signal and Image Analysis
Homework 3

UNIVERSITY OF ILLINOIS
Department of Electrical and Computer Engineering

Assigned: Tuesday, 9/22/2020; Due: Monday, 10/5/2020
Reading: *DSP First* Chapter 6

Problem 3.1

Consider this filter:
$$y[n] = x[n] + x[n-1]$$
Show that the magnitude response of this filter is $|H(\omega)| = 2\cos(\omega/2)$.

Problem 3.2

Suppose you have a filter whose frequency response is
$$H(\omega) = 14e^{-j6\omega}$$
Show that, if $x[n] = \cos(\omega n)$, the effect of convolving $y[n] = x[n] * h[n]$ is to

(a) scale $x[n]$ by a factor of 14, and

(b) delay it by 6 samples.

Problem 3.3

The signals $x_1(t)$ and $x_2(t)$ are cosines an octave apart (roughly C6 and C7):
$$x_1(t) = \cos(2\pi 1000t)$$
$$x_2(t) = \cos(2\pi 2000t)$$
The signals are sampled (at $F_s = 16000$ samples/second), then the resulting signals $x_1[n]$ and $x_2[n]$ are passed through a first-difference operator:
$$y_1[n] = x_1[n] - x_1[n-1]$$
$$y_2[n] = x_2[n] - x_2[n-1]$$
What are the amplitudes of the signals $y_1[n]$ and $y_2[n]$?

Problem 3.4

The signals $x_1(t)$ and $x_2(t)$ are cosines an octave apart (roughly C6 and C7):
$$x_1(t) = \cos(2\pi 1000t)$$
$$x_2(t) = \cos(2\pi 2000t)$$

The signals are sampled (at $F_s = 16000$ samples/second), then the resulting signals $x_1[n]$ and $x_2[n]$ are passed through a seven-sample local average:

$$y_1[n] = \frac{1}{7} \sum_{m=-3}^{3} x_1[n-m]$$

$$y_2[n] = \frac{1}{7} \sum_{m=-3}^{3} x_2[n-m]$$

What are the amplitudes of the signals $y_1[n]$ and $y_2[n]$?