www. ecs.umass.edu/ece/ece344/ECE344_2.pdf

- **Diffusion capacitance.**
The concentration of excess carriers diffusing in the quasi-neutral regions can be obtained from Eq. (148):

$$\begin{cases} \delta p_n(x) = p_n(x) - p_{n0} = p_{n0}\left[e^{eV_a/(k_BT)} - 1\right] e^{-(x-x_{n0})/L_p} & (x > x_{n0}) \\ \delta n_p(x) = n_p(x) - n_{p0} = n_{p0}\left[e^{eV_a/(k_BT)} - 1\right] e^{(x+x_{p0})/L_n} & (x < -x_{p0}) . \end{cases} \quad (185)$$

The charge per unit area will be (considering only holes, a similar expression will hold for electrons):

$$Q_{diff,p} = e \int_{x_{n0}}^{\infty} \delta p_n(x)\, dx = eL_p p_{n0}\left[e^{eV_a/(k_BT)} - 1\right] . \quad (186)$$

Under strong forward bias, $e^{eV_a/(k_BT)} >> 1$, so:

$$C_{diff,p} \approx \frac{dQ_{diff,p}}{dV_a} = \frac{e^2 L_p p_{n0}}{k_BT} e^{eV_a/(k_BT)} \approx \frac{e}{k_BT}\frac{L_p^2}{D_p} J_p(x_{n0}) = \frac{e}{k_BT}\tau_p J_p(x_{n0}) , \quad (187)$$

where we have used Eq. (150) in the last step. Accounting now for the charge of the minority electrons diffusing into the p quasi-neutral region:

$$C_{diff} \approx \frac{dQ_{diff,p}}{dV_a} + \frac{dQ_{diff,n}}{dV_a} = \frac{e}{k_BT}[\tau_p J_p(x_{n0}) + \tau_n J_n(-x_{p0})] . \quad (188)$$

Note: This is the equation found in most textbooks. However, in a text by Karl Hess (and in a brief comment on the lastest edition of the Streetman-Banerjee's book) one finds instead:

$$C_{diff} \approx \frac{e}{2k_BT}[\tau_p J_p(x_{n0}) + \tau_n J_n(-x_{p0})] , \quad (189)$$

where the additional factor of 1/2 is explicitly commented and the claim is made that Eq. (188) is in error. This factor can be justified in hand-waving fashion by noting that the charges $Q_{diff,p}$ and $Q_{diff,n}$ are like the charges in the two opposite plates of a capacitor, so that the capacitance should be given by the change w.r.t. the applied bias of the average of the electron and hole charges. A more sophisticated and rigorous explanation is given by S. E. Laux and K. Hess, IEEE Trans. Electron. Device vol. 46, no. 2 (February 1999), p. 396. Their argument is based on the observation that – rigorously speaking – the diffusion charge extends also inside the depletion region, so that the integration in Eq. (186) should extend from 0 to ∞, not from x_{n0} (and similarly for the expression for $Q_{diff,n}$). Since as V_a changes charges will leave the depletion region, we will obtain a lower estimate for the charge, and so for the capacitance. In a way, this argument is equivalent to our 'hand-waving' argument since both reduce to accounting for the charges throughout the entire junction, not just in the quasi-neutral regions.

The junction capacitance dominates the reactance of a p-n junction under reverse bias; for forward bias, however, the charge storage capacitance C_s becomes dominant. To calculate the capacitance due to charge storage effects,

let us assume that a p⁺-n junction is forward biased with a steady current I. The stored charge in the injected hole distribution is

$$Q_p = I\tau_p = qA\,\Delta p_n L_p = qAL_p p_n e^{qV/kT} \qquad \text{for } V \gg 0.0259 \text{ V} \quad (5\text{-}64)$$

The capacitance due to small changes in this stored charge is

$$C_s = \frac{dQ_p}{dV} = \frac{q^2}{kT}AL_p p_n e^{qV/kT} = \frac{q}{kT}I\tau_p \quad (5\text{-}65)$$

Similarly, we can determine the *a-c conductance* by allowing small changes in the current:

$$G_s = \frac{dI}{dV} = \frac{qAL_p p_n}{\tau_p}\frac{d}{dV}(e^{qV/kT}) = \frac{q}{kT}I \quad (5\text{-}66)$$

Thus the a-c component of current is

$$i(\text{a-c}) = G_s v(\text{a-c}) + C_s\frac{dv(\text{a-c})}{dt} \quad (5\text{-}67)$$

where

$$G_s = \frac{q}{kT}I(\text{d-c}) \qquad \text{and} \qquad C_s = G_s\tau_p$$

The charge storage capacitance can be a serious limitation for forward-biased p-n junctions in high-frequency circuits. As in the case of the switching performance discussed in the two preceding sections, the high-frequency a-c response of a junction can be improved by reducing the carrier lifetime. Since C_s is proportional to τ_p, a short hole lifetime can make the forward-biased capacitance of a p⁺-n junction acceptably small for many applications.

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