ECE 333 Green Electric Energy

Homework 2 - Solution

P492-7.1:

a. 

\[ P = \eta \cdot \frac{1}{2} \rho A v^3 = \frac{1}{2} \rho \cdot \frac{\pi d^2}{4} \cdot v^3 = 30\% \cdot \frac{1}{2} \cdot 1.225 \cdot \frac{\pi 20^2}{4} \cdot 10^3 = 57.7 \, kW \]

b. use equation 7.17

\[ \rho_{2500,10^\circ C} = \frac{353.1}{T} \exp(-0.0342 \frac{T}{T}) = \frac{353.1}{273.15 + 10} \exp(-0.0342 \frac{2500}{273.15 + 10}) = 0.922 \, \text{kg} / \text{m}^3 \]

c. The wind speed is the same, so we just correct the air density (using the same efficiency)

\[ P_{2500,10^\circ C} = \eta \cdot \frac{1}{2} \rho_{2500,10^\circ C} A v^3 = \rho_{2500,10^\circ C} \eta \cdot \frac{1}{2} \rho \cdot \frac{\pi d^2}{4} \cdot v^3 = \rho_{2500,10^\circ C} P = \frac{0.922}{1.225} \cdot 57.7 = 43.43 \, kW \]

P492-7.2:

a. Here \( \alpha = 0.2 \), based on equation 7.18:

\[ \frac{v_{120}}{v_{10}} = \left( \frac{H_{120}}{H_{10}} \right)^\alpha \]

\[ v_{120} = v_{10} \cdot \left( \frac{H_{120}}{H_{10}} \right)^\alpha = 5 \cdot \left( \frac{120}{10} \right)^{0.2} = 8.218 \, \text{m} / \text{s} \]

\[ P_{120} = \frac{1}{2} \rho v_{120}^3 = \frac{1}{2} \cdot 1.225 \cdot 8.218^3 = 339.9 \, \text{W} / \text{m}^2 \]

b. 

\[ v_{40} = v_{10} \cdot \left( \frac{H_{40}}{H_{10}} \right)^\alpha = 5 \cdot \left( \frac{40}{10} \right)^{0.2} = 6.598 \, \text{m} / \text{s} \]

\[ P_{40} = \frac{1}{2} \rho v_{40}^3 = \frac{1}{2} \cdot 1.225 \cdot 6.598^3 = 175.9 \, \text{W} / \text{m}^2 \]

c. 

\[ \frac{P_{120}}{P_{40}} = \frac{339.9}{175.9} = 1.93 \]
based on equation 7.20

\[
p_{120} \over p_{40} = \left( \frac{H_{120}}{H_{40}} \right)^3 = 1.93
\]

d.

\[
\rho_{120m,15^\circ C} = \frac{353.1}{T} \exp(-0.0342 \frac{z}{T}) = 1.225 \exp(-0.0342 \frac{120}{273.15+15}) = 1.208 \text{ kg / m}^3
\]

\[
p_{120m,15^\circ C} = \frac{1}{2} \rho_{120m,15^\circ C} v_{120}^3 = \frac{1}{2} \cdot 1.208 \cdot 8.218^3 = 335.2 \text{ W / m}^2
\]

Because

\[
\frac{p_{120m,15^\circ C}}{p_{120m}} = \frac{335.2}{339.9} = 0.986
\]

there is no need to consider the density difference in this problem

Problem a. Compare the total wind energy at 0 °C, 1 atm of pressure, contained in 1-m² surface area under the following wind patterns:

\[
\rho = \frac{353.1}{T} \exp(-0.0342 \frac{z}{T}) = \frac{353.1}{273.15} = 1.293 \text{ kg / m}^3
\]

(i) 100 hours of 10 m/s winds

\[
\text{energy}_1 = \frac{1}{2} \rho A v_{10}^3 \cdot t_{10} = \frac{1}{2} \cdot 1.293 \cdot 1 \cdot 10^3 \cdot 100 = 64.65 \text{ kWh}
\]

(ii) 50 hours of 8 m/s winds plus 50 hours of 12 m/s winds

\[
\text{energy}_2 = \frac{1}{2} \rho A v_{8}^3 \cdot t_{8} + \frac{1}{2} \rho A v_{12}^3 \cdot t_{12} = \frac{1}{2} \cdot 1.293 \cdot 1 \cdot 8^3 \cdot 50 + \frac{1}{2} \cdot 1.293 \cdot 1 \cdot 12^3 \cdot 50 = 72.41 \text{ kWh}
\]

Although the average wind speeds of two cases above are the same, the total energy produced in the second case is higher than that of the first one. It tells us the average wind
speed is not an appropriate index for us to judge which wind pattern is better because the wind power is proportional to the $v_{wind}^3$.

Problem b. (iv), using the rotor area and the upwind velocity. We use the rotor area to compute the mass flow rate that determines the difference in kinetic energy between the upwind and downwind air flows, see Eq. 7.21. We use the upwind velocity since it determines the power in wind before its extraction and that is how Betz’s efficiency limit is computed.