The solar declination angle is given by the formula:

\[ \delta|_d = 0.41\sin\left(\frac{2\pi}{365}(d - 81)\right) \]

which implies that: 
\[ -0.41 \leq \delta|_d \leq 0.41 \]

At solar noon the altitude angle is given by:

\[ \beta(0)|_d = \frac{\pi}{2} - \ell - \delta|_d = \frac{\pi}{2} - \frac{\pi}{6} - \delta|_d = \frac{\pi}{3} + \delta|_d \]

Consequently, 
\[ -\frac{\pi}{3} - 0.41 \leq \beta(0)|_d \leq -\frac{\pi}{3} + 0.41 \]

and therefore:

\[ x_1 \geq \frac{2}{\tan\left(\beta(0)|_d\right)} = \frac{2}{\tan\left(\frac{\pi}{3} - 0.41\right)} \approx 2.7 \text{ ft} \]

\[ x_2 \geq \frac{4}{\tan\left(\beta(0)|_d\right)} = \frac{4}{\tan\left(\frac{\pi}{3} - 0.41\right)} \approx 5.4 \text{ ft} \]

4.3

\[ d = 172 \Rightarrow \delta|_{d=172} = 0.41\sin\left(\frac{2\pi}{365}(172 - 81)\right) = 0.41 \text{ rads} \]

a) For June solstice:

\[ \beta(0)|_{d=172} = \frac{\pi}{2} - 0.698 + 0.41 = 1.28 \text{ rads} \]

Therefore,

\[ P \geq \frac{8}{\tan(1.28)} = 2.37 \text{ ft} \]
b) For winter solstice: \( \delta|_d = -0.41 \text{ rads} \)

Similarly with question a) we get:

\[
\beta\left(0\right)|_d = \frac{\pi}{2} - 0.697 - 0.41 = 0.463 \Rightarrow \\
Y = P \tan\left(0.463\right) = 2.37 \tan\left(0.463\right) = 1.183 \text{ ft}
\]

c) Skip this part

4.5

a)

For June 21, \( d = 172 \Rightarrow \delta|_{d=172} = 0.41 \text{ rads} \)

\[
\beta\left(h = 1\right)|_{d=172} = \sin^{-1}\left(\cos\left(\frac{32\pi}{180}\right)\cos\left(0.41\right)\cos\frac{\pi}{12} + \sin\left(\frac{32\pi}{180}\right)\sin\left(0.41\right)\right) = 1.286 \text{ rads}
\]

Therefore the height of the tree is:

\( H = 30 \tan\left(1.286\right) = 106.56 \text{ ft} \)

b) \( \phi = \arcsin\left(\frac{\cos\left(\delta\right)\sin\left(H\right)}{\cos\left(\beta\right)}\right) = \arcsin\left(\frac{\cos\left(0.41\right)\sin\left(\pi / 12\right)}{\cos\left(1.286\right)}\right) = 1.06 = 60.9^\circ \)

c) The altitude angle of the tree relative to the site is \( \arctan\left(106 / 100\right) = 46.7^\circ \)

and the azimuth was computed in part b.

With the sun path diagram for a latitude of 32 degrees north, we can see that the site will be shaded briefly during each day from early September to early April.
4.7

a) At sunrise, the altitude angle is zero. Moreover, at summer solstice the declination angle is 0.41 radians. With the given latitude of 47.63 degrees (or 0.83 radians), we can compute the solar hour angle using equation 4.16 or 4.8. We can then compute the azimuth angle using equation 4.9 and the azimuth angle checking procedure.

\[ \theta(h) = \cos^{-1}(-\tan(L)\tan(\delta)) = \cos^{-1}(-\tan(0.83)\tan(0.41)) = 2.067 \]

\[ \sin(\phi) = \left( \frac{\cos(\delta)\sin(\theta(h))}{\cos(\beta)} \right) = \left( \frac{\cos(0.41)\sin(2.067)}{\cos(0)} \right) = 0.807 \]

So

\[ \phi \in [0.939, \pi - 0.939] \]

Azimuth check:

\[ \cos(\theta(h)) = -0.475 < 0.396 = \frac{\tan(\delta)}{\tan(L)} \Rightarrow |\phi| > \pi / 2 \Rightarrow \phi = \pi - 0.939 = 2.203 \]

b) We use the solar hour angle to compute the solar time

\[ 12 : 00 - \frac{2.067}{\pi} = 4 : 06 \text{ a.m. solar time} \]

c) June 21st is day 172 (see Table 4.1). Here we will need equations 4.12 and 4.13.
\[ b = \frac{2\pi}{364} (172 - 81) = \frac{\pi}{2} \]
\[ e = 9.87 \sin(2b) - 7.53 \cos(b) - 1.5 \sin(b) = -1.5 \]

So we have that the sunrise in solar time is 4:06 am and the solar day deviation is -1.5 minutes. The longitude is 2.135 radians and the local time meridian is \( \frac{2\pi}{3} \). So we use the relationship in equation 4.14.

\[
solar\ time = clock\ time + e + 4\left(\frac{180}{\pi}\right) (local\ time\ meridian - longitude)\]

\[
4:06\ am = clock\ time - (1.5 + 9.3\ min)\]

Sunrise time: 4:17 am

The website will give a different time, since they use the top rim of the sun instead of its center and they take refraction into account.

4.8

a)  
\[
\kappa^+_{d=172} = \left(12:00 - 4:11\right) \frac{\pi}{12} = \left(12 - \left(4 + \frac{11}{60}\right)\right) \frac{\pi}{12} = 2.0453 \text{ rads} \]

\[
\kappa^-_{d=172} = \left(12:00 - 20:11\right) \frac{\pi}{12} = -2.141 \text{ rads} \]

Therefore, clock time for solar noon is \((4:11 + 20:11)/2 = 12:11\) pm.

b)  
\[
\beta_{d=172} = \frac{2\pi}{364} (172 - 81) = 1.57 \text{ rads} \]

\[
\Rightarrow e_{d=172} = -1.5 \text{ minutes} \]

Therefore,  
\[
solar\ noon - noon = 12:00 - 12:11 = -11\ minutes \]

\[
\Rightarrow -11 = e_{d=172} + 4\left(local\ time\ meridian - local\ longitude\right) \]

where local time meridian and local longitude are given in degrees.

Solving for the local longitude we get: local longitude = 122.375 deg
Since the sunrise is 8 hours before noon solar noon, the sunrise hour angle is $8\pi/12$ or simply $2\pi/3$. We can then use equation 4.17 to find a latitude of 49 degrees or 0.855 radians

4.9

a)

$$a = 1160 + 75\sin\left(\frac{2\pi}{365}(1 - 275)\right) = 1235 \text{ W} / \text{m}^2$$

$$k = 0.174 + 0.035\sin\left(\frac{2\pi}{365}(1 - 100)\right) = 0.1393$$

$$\delta = 0.41\sin\left(\frac{2\pi}{365}(1 - 81)\right) = -0.40 \text{ rad}$$

$$\beta = \frac{\pi}{2} - \text{latitude} + \delta = \frac{\pi}{2} - 0.7 - 0.4 = 0.47 \text{ rad}$$

$$r = \sqrt{(708\sin(\beta))^2 + 1417 - 708\sin(\beta)} = 2.197$$

$$i_v = ae^{-kr} = 909.4 \text{ W} / \text{m}^2$$