6. Limits on Conversion of Wind Into Electricity

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We analytically characterize the power in wind as a cubic function of the wind speed $v$.

The wind energy is in the form of kinetic energy, whose extraction from the wind is used to rotate the generator shaft mounted in the nacelle.

We examine the constraint – the so-called Betz limit – that limits the ability of a wind turbine to convert the wind kinetic energy into mechanical energy to rotate the turbine generator shaft.
THE BETZ LIMIT

- The limit was derived in 1919 by Albert Betz, a German physicist.
- We consider the wind as it passes through a wind turbine rotor and we examine the wind stream.
- We explain the conceptual basis on the limit of the conversion of wind into electricity by means of the diagram below.
THE BETZ LIMIT

Stream tube is formed by the air mass passing through the turbine rotor.

Area a swept by rotor.

Upwind

Downwind

$$v$$

$$v_r$$

$$v_d$$
THE BETZ LIMIT

- Clearly, the turbine cannot extract all the kinetic energy in the wind because that implies that the air would have to stop completely after passing the turbine – an impossible situation since it would prevent all the continuing wind to pass through the rotor.

- Furthermore, the downwind velocity $v_d$ cannot equal $v$ since that would imply that no energy is extracted by the turbine.
THE BETZ LIMIT

- Betz formulated the relationship to determine the maximum mechanical power obtainable from wind.
- We focus on what happens with the wind as it passes through the plane of the rotor blades at velocity $v_r$, with

$$v_d < v_r < v$$

where, we explicitly take into account that, as the wind mass of air goes through the stream tube and kinetic energy is extracted, the downwind speed must be lower than the upwind speed.
THE BETZ LIMIT

❑ The conservation of energy implies that

\[
\text{kinetic energy upwind} = \text{kinetic energy downwind} + \text{energy extracted by blade rotor}
\]

❑ Therefore, as the mass flow rate \( \frac{dm}{dt} \) throughout the stream tube remains unchanged, the power extracted by the rotor blades is

\[
p_r = \frac{d}{dt} \left( \text{kinetic energy upwind} - \text{kinetic energy downwind} \right)
\]

\[
= \frac{1}{2} \frac{dm}{dt} \left( v^2 - v_d^2 \right)
\]
Now, we can determine \( \frac{dm}{dt} \) anywhere in the stream tube and at the rotor blade plane since

\[
\frac{dm}{dt} = \rho a v_r,
\]

We assume that

\[
v_r = \frac{v + v_d}{2}
\]

and therefore

\[
p_r = \frac{1}{2} \rho a \left( \frac{v + v_d}{2} \right) \left( v^2 - v_d^2 \right)
\]
We introduce the ratio $\lambda$ defined by

\[ v_d = \lambda v \]

so that the expression for $p_r$ becomes

\[ p_r = \frac{1}{2} \rho a v \left( \frac{1 + \lambda}{2} \right) v^2 \left( 1 - \lambda^2 \right) \]

\[ = \frac{1}{2} \rho a v^3 \frac{1}{2} \left( 1 + \lambda \right) \left( 1 - \lambda^2 \right) \]

\[ \text{power in the wind} \quad \text{fraction extracted} \]
We can think of the fraction of power extracted by the rotor as the rotor efficiency $\eta_r$.

$$\eta_r = \frac{1}{2} (1 + \lambda) (1 - \lambda^2)$$

so that we may write

$$p_r = \frac{1}{2} \rho \, \text{av}^3 \, \eta_r$$
To determine the maximum rotor efficiency, we evaluate the derivative of $p_r$ w.r.t. $\lambda$

$$\frac{dp_r}{d\lambda} = \frac{1}{2} \rho \ av^3 \ \frac{1}{2} \left[ (1 + \lambda)(-2\lambda) + (1)(1 - \lambda^2) \right]$$

$$= \frac{1}{4} \rho \ av^3 \left[ -2\lambda^2 - 2\lambda + 1 - \lambda^2 \right]$$

$$= \frac{1}{4} \rho \ av^3 \left[ 1 - 2\lambda - 3\lambda^2 \right]$$

$$= \frac{1}{4} \rho \ av^3 \left[ (1 + \lambda)(1 - 3\lambda) \right]$$
THE BETZ LIMIT

- We set \( \frac{dp_r}{d \lambda} \) to be 0 and we solve for \( \lambda \)

- The only physically meaningful solution is

\[
\lambda = \frac{1}{3},
\]

i.e., the efficiency is maximized when the ratio of \( v_d \) to \( v \) is 1/3 so that
THE BETZ LIMIT

\[ \eta_r = \frac{1}{2} \left( 1 + \frac{1}{3} \right) \left( 1 - \frac{1}{3^2} \right) = \frac{16}{27} = 59.3 \% \]

- This optimal theoretical efficiency – better known as the Betz efficiency – cannot be higher than 59.3 %; this value is the essence of the Betz limit.
The Betz limit implies that even under ideal conditions less than 60% of the power in wind can be extracted; indeed, in actual systems, the best that is attainable is, typically, below 50% – in other words, at most half of the energy in wind can be converted into mechanical energy to rotate the generator shaft.
The tip speed of the rotor is a function of the rate of rotation of the rotor specified by its r.p.m.: in each revolution of the rotor, the tip traverses a distance $\pi d$ and so the tip speed is $(\pi d) (r.p.m.)$.

A convenient way to express rotor efficiency is in terms of the tip speed ratio $\tau$, where

$$
\tau = \frac{\text{rotor tip speed}}{v} = \frac{(r.p.m.) \cdot \text{min}}{60 \text{ sec}} \cdot \frac{\pi d}{v}
$$
TIP SPEED RATIO

- Studies indicate that modern turbines attain maximum efficiency for $4 \leq \tau \leq 6$: the tip of the blade moves 4 – 6 times faster than the wind speed.

- It follows that for maximum efficiency it is desirable that turbine blades change their speed as wind speed changes – as is the case in the so-called variable speed generators.
Betz’s law states that the maximum power that we can extract from wind is

\[ p_r = \left( \frac{1}{2} \rho a v^3 \right) 0.593 \]

Engineers define the efficiency as the ratio of the output to the input quantity

\[ \eta_r = \frac{p_{\text{out}}}{p_{\text{in}}} \]

and so the natural question is what is \( p_{\text{in}} \)
A convenient way to think about $p_{in}$ is that $p_{in}$ is the power in the wind prior to the installation of a turbine: absent the turbine, $v_r = v$ and so

$$p_{in} = \frac{1}{2} \rho a v^3$$

The Betz efficiency determines the limit on the conversion of $p_{in}$ into mechanical power to rotate the generator shaft.
The wind turbine generators or wind energy conversion systems may be classified into two principal categories:

- variable–speed rotors
- fixed–speed rotors

The variable–speed turbines are able advantageously use the fact that wind speed varies to...
adjust the rotor speed in order to *optimally match* wind speed

- The fixed–speed rotor generators are simpler but do not operate at *optimal efficiency*; moreover, the stresses from the rapidly varying wind speeds, typically, require sturdier design of such turbines.
The turbine design uses an induction generator connected to a fixed—speed wind turbine.

This design needs two additional components for grid connection:

WECS TYPE A
WECS TYPE A

- a soft-starter to decrease current transients during startup phase
- a capacitor bank to supply reactive power to the generator

- As a result of the capacitor bank, the generator can operate essentially as a zero-value generation source or consumption sink of reactive power
- However, such capacitive compensation unable to provide flexible reactive power control by the wind turbine
The *Vestas*-developed type B WECS generator is designed to work with a **limited variable speed**.
WECS TYPE B

wind turbine

- The turbine uses the variable resistor in the rotor to control the real power output

- The capacitor bank and soft-starter device roles are analogous to those in the type A design
This design uses two AC/DC converters rated at 25% of total generator power with a capacitor between them to control the WECS.

The wound rotor induction generator topology is also known as a doubly fed induction generator (DFIG).
The term “doubly” comes from the fact that the generator has two electrical ports – one stator and one rotor; unlike the squirrel cage rotor, the DFIG has windings in the rotor, which are accessible via the use of brushes and slip rings.

DFIGs can be controlled to provide active and reactive power to the grid.

The WECS type C is the most widespread of all wind turbines on the market.
The type D design uses a full-scale frequency converter with different types of generators.

The most common design in use is the permanent magnet synchronous generator (PMSG).
This design allows full control over the active and the reactive power production that results in a high-wind-energy extraction value.

Full power control improves power and frequency stability in the grid and reduces the short circuit power.

Most type D designs do not require a gearbox – a distinct advantage of type D WECS.
WIND TURBINE CLASSIFICATION

Wind energy conversion systems

Variable-speed turbines

Indirect drive with gearbox

- Wound-rotor synchronous generator
- Permanent-magnet synchronous generator
- Squirrel-cage induction generator
- Doubly-fed induction generator
- Wound-rotor induction generator with variable R

Direct drive without gearbox

- Wound-rotor synchronous generator
- Permanent-magnet synchronous generator

Fixed-speed turbines

- Squirrel-cage induction generator