ECE 330 Exam #2, Spring 2014 Name: Solution
90 Minutes

Section (Check One) MWF 10am _______ MWF 2pm _______

1. 25 / 25 2. 25 / 25

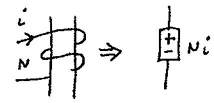
3. 25 / 25 4. 25 / 25 Total 100 / 100

Useful information

$\sin(x) = \cos(x - 90^\circ)$ $\quad \overline{V} = \overline{Z}\overline{I}$ $\quad \overline{S} = \overline{V}\overline{I}^*$ $\quad \mu_0 = 4\pi \cdot 10^{-7}$ H/m

$\int_C \mathbf{H} \cdot \mathbf{dl} = \int_S \mathbf{J} \cdot \mathbf{n} da$ $\quad \int_C \mathbf{E} \cdot \mathbf{dl} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot \mathbf{n} da$ $\quad MMF = Ni = \phi \Re$

$\Re = \frac{l}{\mu A}$ $\quad B = \mu H$ $\quad \phi = BA$ $\quad \lambda = N\phi$ $\quad \lambda = Li$ (if linear)



$W_m = \int_0^\lambda i d\hat{\lambda}$ $\quad W_m' = \int_0^i \lambda d\hat{i}$ $\quad W_m + W_m' = \lambda i$ $\quad f^e = \frac{\partial W_m'}{\partial x} = -\frac{\partial W_m}{\partial x}$ $\quad x \to \theta$

$f^e \to T^e$

$\underset{a \to b}{EFE} = \int_a^b i d\lambda$ $\quad \underset{a \to b}{EFM} = -\int_a^b f^e dx$ $\quad \underset{a \to b}{EFE} + \underset{a \to b}{EFM} = W_{mb} - W_{ma}$ $\quad \lambda = \frac{\partial W_m'}{\partial i}$ $\quad i = \frac{\partial W_m}{\partial \lambda}$

$M\frac{dv}{dt} = \sum forces\ in + x\ direction$ $\quad \underline{\dot{x}} = \underline{f}(\underline{x}, \underline{u})$ $\quad$ Equilibrium: $\underline{f}(\underline{x}_{eq}, \underline{u}_{eq}) = 0$

$\underline{x}(t_{n+1}) = \underline{x}(t_n) + \Delta t \cdot \underline{f}\left(\underline{x}(t_n), \underline{u}(t_n)\right)$

Linearization:
$$\Delta\dot{x}_1 = \frac{\partial f_1}{\partial x_1}\Delta x_1 + \frac{\partial f_1}{\partial x_2}\Delta x_2 + \cdots + \frac{\partial f_1}{\partial u}\Delta u$$
$$\Delta\dot{x}_2 = \frac{\partial f_2}{\partial x_1}\Delta x_1 + \frac{\partial f_2}{\partial x_2}\Delta x_2 + \cdots + \frac{\partial f_2}{\partial u}\Delta u$$

Stability: $A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}$ $\quad |\lambda I - A| = 0$ $\quad$ stable if $\mathrm{Re}\{\lambda\} < 0$

Problem 1. (25 points)

In class, generally we treated mutual inductances to be sinusoidal in rotational devices. In the system below, we have added a third harmonic (the 3θ terms). This gives the mutual inductance a more trapezoidal shape, which is common in some actual machines.

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_r \end{bmatrix} = \begin{bmatrix} L_s & 0 & M\left(\cos\theta - 0.1\cos(3\theta)\right) \\ 0 & L_s & M\left(\sin\theta + 0.1\sin(3\theta)\right) \\ M\left(\cos\theta - 0.1\cos(3\theta)\right) & M\left(\sin\theta + 0.1\sin(3\theta)\right) & L_r \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_r \end{bmatrix}$$

a) Find the co-energy in terms of the currents.

b) Find an expression for the energy stored in the coupling field in terms of currents.

c) Find the torque of electric origin T^e in terms of the currents.

d) Suppose $i_a = i_b = i_r = 1$ A, $L_s = L_r = 1$ H, $M = 0.9$ H, and $\theta = 60°$. What is the contribution to the torque due to including the third harmonic (3θ) mutual inductance terms?

a) $W_m' = \left(\frac{1}{2} L_s i_a^2\right) + \left(0 + \frac{1}{2} L_s i_b^2\right) + \Big(m(\cos\theta - 0.1\cos3\theta) i_a i_r$

$+ m(\sin\theta + 0.1\sin3\theta) i_b i_r + \frac{1}{2} L_r i_r^2\Big)$

b) $W_m = W_m'$

c) $T^e = -m\sin\theta\, i_a i_r + 0.3m\sin3\theta\, i_a i_r$

$+ m\cos\theta\, i_b i_r + 0.3m\cos3\theta\, i_b i_r$

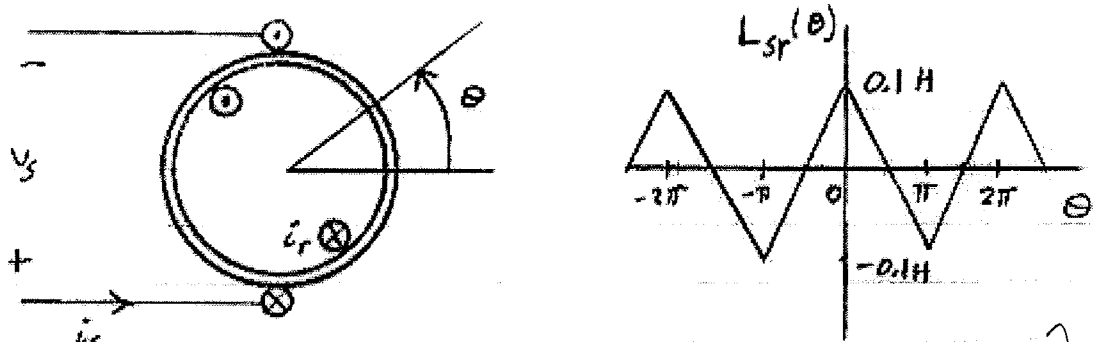
d) $T^e_{3\theta} = 0.3 \times 0.9 \sin 180° \times 1 \times 1 + 0.3 \times 0.9 \cos 180° \times 1 \times 1$

$\underline{= -0.27 \text{ NM}}$

Problem 2. (25 points.)

A single-phase generator consists of a coil on the stator and a coil on the rotor with a mutual inductance variation with θ as shown in the figure below. The rotor is being driven at a constant speed of 377 radians per second and the rotor coil has a constant dc current $i_r = 5$ A. The self inductances are constants and you may assume a linear magnetic core.

$v_s = \frac{d\lambda_s}{dt}$



$\lambda_s = L_s i_s + L_{sr}(\theta) i_r$

a) Plot the open-circuit stator voltage ($i_s = 0$) as a function of θ (label all points)

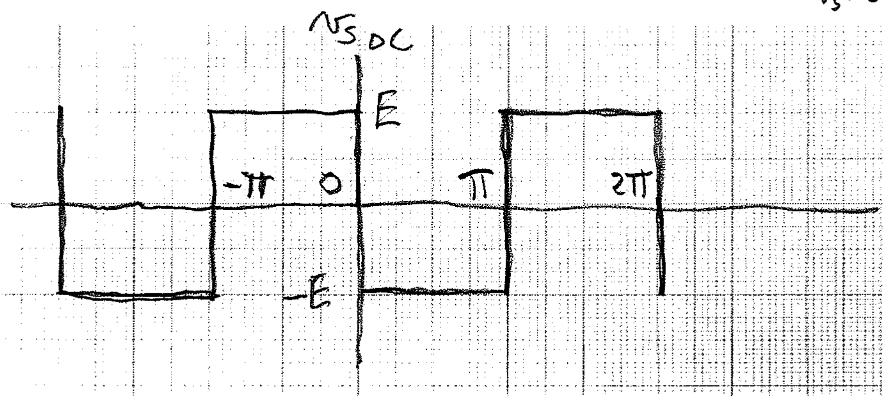
$$E\Big|_{i_s=0} = \frac{dL_{sr}(\theta)\, i_r}{dt}$$

$$= \frac{\partial L_{sr}}{\partial \theta} \times 377 \times 5$$

$$= \frac{0.2}{\pi} 377 \times 5$$

$$= 24 \times 5$$

$$= 120\,V$$



b) What is the torque of electrical origin when $i_s = 10$ Amps and θ = 45°?

$$W_m' = \frac{1}{2} L_s i_s^2 + L_{sr}(\theta) i_s i_r$$

$$T^e = \frac{\partial W_m'}{\partial \theta} = \frac{\partial L_{sr}(\theta)}{\partial \theta} i_s i_r = -\frac{0.2}{\pi} \times 10 \times 5 \text{ Nm}$$

$$= -3.18 \text{ Nm}$$

Problem 3. (25 points.)

An electric machine (1 = stator, 2 = rotor) has the following linear flux linkage vs current characteristic:

$$\lambda_1 = 0.2 i_1 + 0.1 \sin\theta \, i_2$$

$$\lambda_2 = 0.1 \sin\theta \, i_1 + 0.3 \, i_2$$

a) What is the energy stored in the coupling field when $\theta = 90$ degrees, $i_1 = 3$ Amps, and $i_2 = 5$ Amps?
b) How much energy is given to the coupling field by the mechanical system if θ is changed from 90 degrees to 60 degrees while the two currents remain constant?
c) How much energy is given to the coupling field by the electrical system during that same path from θ equals 90 degrees to 60 degrees while the two currents remain constant?

a) $W_m = W_m' = 0.1 i_1^2 + 0.1 \sin\theta \, i_1 i_2 + \frac{0.3}{2} i_2^2$

$$W_m \Big|_{\substack{\theta=90 \\ i_1=3 \\ i_2=5}} = 0.9 + 1.5 + 3.75 = 6.15 \text{ J}$$

c) $$EFE_{90^\circ-60^\circ} = \int_{.2\times3 + .1\times1\times5}^{.2\times3 + .1\sin60\times5} 3\,d\lambda_1 + \int_{.1\times1\times3 + .3\times5}^{.1\sin60\times3 + .3\times5} 5\,d\lambda_2 = \int_{1.1}^{1.033} 3\,d\lambda_1 + \int_{1.8}^{1.76} 5\,d\lambda_2 = -.2 - .2$$

$$= -.4 \text{ J}$$

b) $$W_m \Big|_{\substack{\theta=60 \\ i_1=3 \\ i_2=5}} = 0.9 + 1.3 + 3.75 = 5.95 \text{ J}$$

$$EFM_{90^\circ-60^\circ} = W_{m60} - W_{m90} - EFE_{90-60} = 5.95 - 6.15 + .4 \text{ J}$$

$$= 0.2 \text{ J}$$

OR $T^e = \frac{\partial W_m'}{\partial \theta} = 0.1 \cos\theta \, i_1 i_2$

$$EFM_{90-60} = -\int_{90}^{60} 0.1\cos\theta \times 3 \times 5 \, d\theta = -1.5 \sin\theta \Big|_{90}^{60} = -1.3 - (-1.5)$$

$$= 0.2 \text{ J}$$

Problem 4. (25 points.)

Consider the following nonlinear equations

$$\dot{x}_1 = x_1 - x_1 x_2$$
$$\dot{x}_2 = 0.5\, x_1 x_2 - 2\, x_2$$

Assume the initial conditions for this system are

$$\mathbf{x}(0) = \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 2 \\ 0.5 \end{bmatrix}$$

a) Using Euler's method with a time step $\Delta t = 0.1$sec, determine the value of $\mathbf{x}(0.1)$ and $\mathbf{x}(0.2)$.
b) Determine all of the equilibrium points for this system.
c) Find the eigenvalues for each of these equilibrium points
d) Are these points stable or unstable or neither? Explain why.

a) $x_1(0.1) = 2 + (2 - 2\times.5).1 = 2.1$

$x_2(0.1) = 0.5 + (0.5\times2\times0.5 - 2\times0.5).1 = 0.45$

$x_1(0.2) = 2.1 + (2.1 - 2.1\times0.45).1 = 2.22$

$x_2(0.2) = 0.45 + (0.5\times2.1\times.45 - 2\times.45).1 = 0.407$

b) $x_1 = 0,\ x_2 = 0$

$x_1 = 4,\ x_2 = 1$

c) + d)

$J = \begin{bmatrix} 1-x_2 & -x_1 \\ 0.5x_2 & 0.5x_1-2 \end{bmatrix}$ $\quad J_1 = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$

$\begin{vmatrix} \lambda-1 & 0 \\ 0 & \lambda+2 \end{vmatrix} = (\lambda-1)(\lambda+2) = 0$ $\quad \lambda = 1, -2$

unstable

$J_2 = \begin{bmatrix} 0 & -4 \\ 0.5 & 0 \end{bmatrix}$ $\quad \begin{vmatrix} \lambda & 4 \\ -0.5 & \lambda \end{vmatrix} = \lambda^2 + 2 = 0$ $\quad \lambda = \pm j\sqrt{2}$

Neither