ECE 430 Exam #2, Fall 2009 Name: Solution

90 Minutes

Section (Check One) MWF 10am ______ MWF 2pm ______

1. ________ / 35 2. ________ / 35

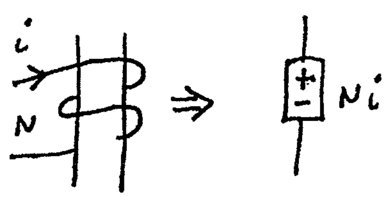
3. ________ / 30 Total ________ / 100

Useful information

$\sin(x) = \cos(x - 90^\circ)$ $\quad \bar{V} = \bar{Z}\bar{I}$ $\quad \bar{S} = \bar{V}\bar{I}^*$ $\quad \mu_0 = 4\pi \cdot 10^{-7}$ H/m

$\int_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot \mathbf{n} da$ $\quad \int_C \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot \mathbf{n} da$ $\quad \Re = \frac{l}{\mu A}$ $\quad MMF = Ni = \phi \Re$

$\Re = \frac{l}{\mu A}$ $\quad B = \mu H$ $\quad \phi = BA$ $\quad \lambda = N\phi$ $\quad \lambda = Li$ (if linear)



$W_m = \int_0^\lambda i \, d\hat{\lambda}$ $\quad W_m' = \int_0^i \lambda \, d\hat{i}$ $\quad W_m + W_m' = \lambda i$ $\quad f^e = \frac{\partial W_m'}{\partial x} = -\frac{\partial W_m}{\partial x}$ $\quad x \to \theta$

$f^e \to T^e$

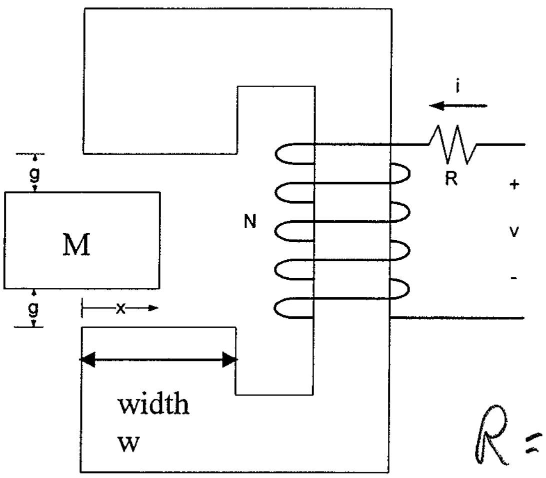
$\underset{a \to b}{EFE} = \int_a^b i \, d\lambda$ $\quad \underset{a \to b}{EFM} = -\int_a^b f^e dx$ $\quad \underset{a \to b}{EFE} + \underset{a \to b}{EFM} = W_{mb} - W_{ma}$ $\quad \lambda = \frac{\partial W_m'}{\partial i}$ $\quad i = \frac{\partial W_m}{\partial \lambda}$

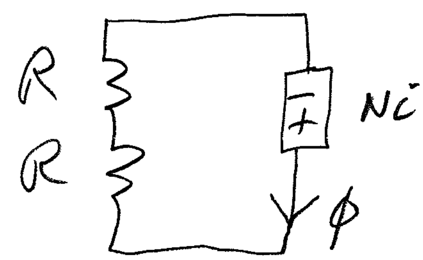
1. **(35 points)**

For the structure drawn in the figure below, the movable mass M is constrained to move left and right only as indicated in the figure where "x" is the distance from the left edge of the stationary member to the right edge of the movable mass. The depth into the page for both members is 2cm. The width shown is 4 cm. The gap g is 1mm, and the number of turns N =100. Find:

a) Flux linkage λ (defined for the voltage polarity shown) in terms of current i and distance x.
b) An expression for the voltage v in terms of the current i, distance x, resistance R, the velocity (v) of the mass in the positive x direction, and time t.
c) An expression for the force of electric origin on the movable mass M in the positive x direction.
d) A numerical value for the energy stored in the coupling field (in Joules) when the current is 10 Amps and the distance x is 3cm

You may neglect fringing in the gap, and you may assume the iron is infinitely permeable.





$$\mathcal{R} = \frac{10^{-3}}{4\pi\times10^{-7}(2\times10^{-2}X)}$$

$$\lambda = N\phi$$

$$\phi = \frac{100i}{\frac{2\times10^{-3}}{8\pi\times10^{-9}X}} = .0126\,X\,i$$

$$\boxed{\lambda = .126\,X\,i}$$

$$w_m' = .063\,X\,i^2$$

$$\boxed{f^e = .063\,i^2}$$

$$v = iR + .126\,x\frac{di}{dt} + .126\,v\,i$$

$$v = \frac{dx}{dt}$$

$$w_m = w_m' = .063(.03)100 = \boxed{0.189\text{ J}}$$

2. (35 points)

An electric machine (1 = stator, 2 = rotor) has the following linear flux linkage vs current characteristic:

$$\lambda_1 = 0.2 i_1 + 0.1 \sin\theta \, i_2$$

$$\lambda_2 = 0.1 \sin\theta \, i_1 + 0.3 \, i_2$$

a) What is the energy stored in the coupling field when $\theta = 90$ degrees, $i_1 = 3$ Amps, and $i_2 = 5$ Amps?
b) How much energy is given to the coupling field by the mechanical system if θ is changed from 90 degrees to 60 degrees while the two currents remain constant?
c) How much energy is given to the coupling field by the electrical system during that same path from θ equals 90 degrees to 60 degrees while the two currents remain constant?

a) $W_m = W_m' = \frac{1}{2} \cdot .2 \, i_1^2 + 0.1 \sin\theta \, i_1 i_2 + \frac{1}{2} \cdot .3 i_2^2$

at $\theta = 90$, $i_1 = 3$, $i_2 = 5$: $= \frac{1}{2} 0.2 \times 9 + .1 \times 3 \times 5 + \frac{1}{2} 0.3 \times 25 = \boxed{6.15 \text{ J}}$

b) $T^e = 0.1 \cos\theta \, i_1 i_2$ $\quad EFm_{a-b} = -\int_{90^\circ}^{60^\circ} 1.5 \cos\theta \, d\theta = \boxed{0.2 \text{ J}}$

c) $EFE_{a-b} = \int_{.6+.5}^{.6+.433} 3 \, d\lambda_1 + \int_{.3+1.5}^{.2598+1.5} 5 \, d\lambda_2 = \boxed{-0.4 \text{ J}}$

Check $\quad W_m\Big|_{60^\circ} = W_m'\Big|_{60^\circ} = \frac{1}{2} 0.2 \times 9 + .0866 \times 3 \times 5 + \frac{1}{2} .3 \times 25$

$= 5.949 \text{ J}$

$W_{mb} - W_{ma} = -0.2 \text{ J} = EFE_{a-b} + EFm_{a-b}$ ✓

3. (30 points)

Consider the following nonlinear equations

$$\dot{x}_1 = x_1 - x_1 x_2$$
$$\dot{x}_2 = 0.5\, x_1 x_2 - 2\, x_2$$

Assume the initial conditions for this system are

$$\mathbf{x}(0) = \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 2 \\ 0.5 \end{bmatrix}$$

a) Using Euler's method with a time step $\Delta t = 0.1$, determine the value of $\mathbf{x}(0.2)$.

b) Determine two separate equilibrium points for this system.

a) $$x(0.1) = x(0) + \left.\frac{dx}{dt}\right|_{0} 0.1 = \begin{bmatrix} 2 \\ 0.5 \end{bmatrix} + \begin{bmatrix} 1 \\ -0.5 \end{bmatrix} \cdot 1 = \begin{bmatrix} 2.1 \\ 0.45 \end{bmatrix}$$

$$x(0.2) = x(0.1) + \left.\frac{dx}{dt}\right|_{.1} 0.1 = \begin{bmatrix} 2.1 \\ 0.45 \end{bmatrix} + \begin{bmatrix} 1.155 \\ -.4275 \end{bmatrix} \cdot 1 = \begin{bmatrix} 2.2155 \\ .40725 \end{bmatrix}$$

$$= \begin{bmatrix} 2.2155 \\ .40725 \end{bmatrix}$$

b) $$0 = x_1 - x_1 x_2$$
$$0 = .5 x_1 x_2 - 2 x_2$$

$$\boxed{x_{1e} = 0 \quad x_{2e} = 0}$$

for $x_{2e} \neq 0$, $x_{1e} \neq 0$

$$\boxed{\begin{aligned} x_{2e} &= 1 \\ x_{1e} &= 4 \end{aligned}}$$