Name Solution
(Print Name)

Section: (circle one) 10 AM (Pai) 12PM (Sauer)

ECE330 C&N

Final Exam

SP 2003

Monday, May 12, 2003, 1:30 – 4:30 PM

Three sheets provided

Problem 1 ______

Problem 2 ______

Problem 3 ______

Problem 4 ______

Problem 5 ______

Problem 6 ______

Problem 7 ______

Problem 8 ______

Problem 9 ______

Problem 10 ______

TOTAL: ______

Problem 1 (10 pts. – no partial credit)

a) If $v(t)=200\cos(377t-10°)$ and $i(t)=10\sin(377t+125°)$, find the complex $P+jQ$, PF (specifically lead or lag)

$P+jQ =$ 707 − j707 P.F.= 0.707 Lead or Lag (circle one)

$\bar{V} = \frac{200}{\sqrt{2}}\angle{-10°}$ $\bar{I} = \frac{10}{\sqrt{2}}\angle{35°}$

$\bar{S} = 1000\angle{-45°}$

b) Two single phase loads in parallel, 10 kVA at 0.8 PF lag and 16 kW at 0.8 PF lead are supplied by a source $\bar{V} = 240\angle 0°$. Find the total current (magnitude) supplied by the source and the combined PF (specify lead/lag).

I = 103 P.F.= 0.97 Lead or Lag (circle one)

$\bar{S} = 10k\angle{37°} + \frac{16k}{.8}\angle{-37°} = 7986 + j6018$
$+ \; 15973 - j12036$
$= 23{,}959 - j6{,}018$

$\bar{S} = 24703\angle{-14°} = 240\angle{0}\;\bar{I}^*$ $I = \frac{24703}{240} = 103$

$PF = \cos(-14°) = .97$

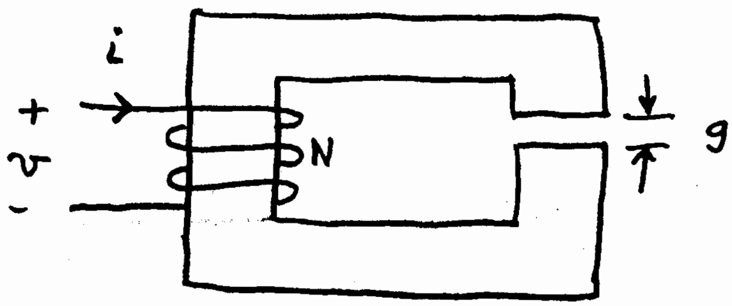
c) A 3 phase, delta connected load has a line to line voltage of 480 V. The complex power per phase is 1000+j500 VA. The magnitude of the line current is 4.03 A.

$3 \times 1118\angle = \sqrt{3} \times 480\, I$

$I = 4.03$

Problem 2 (10 pts. – no partial credit)

Given the following device



$Hg = Ni$

$\mu_{iron} = \infty$ cross section area A

Find the following in terms of μ_0, N, A, g, i

a) H_{gap} (directed down) = Ni/g

b) B_{gap} (directed down) = $\mu_0 Ni/g$

c) B_{iron} (directed clockwise) = $\mu_0 Ni/g$

d) ϕ (directed clockwise) = $\mu_0 ANi/g$

e) λ (where $v = d\lambda/dt$) = $\mu_0 AN^2 i/g$

Problem 3 (10 pts. – no partial credit)

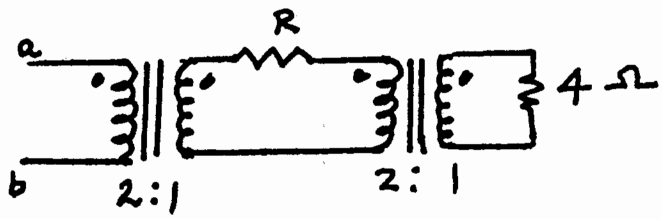
a) A coil of 500 turns is wound on an iron core whose reluctance $\Re = 4.6 \times 10^6$ At/W. The inductance of the coil is $\underline{.054}$ H.

$$R\phi = Ni \qquad \lambda = \frac{N^2}{R} i = Li$$

b) Two coils which are coupled have self-inductances of 10 and 20 mH respectively and a coupling coefficient of 0.9. The mutual inductance is $\underline{12.7}$ mH.

$$.9 = \frac{m}{\sqrt{.01 \times .02}} \qquad m = 12.7\,mH$$

c) Input resistance at "ab" is $100\Omega\,(R > 0)$. The value of R is $\underline{9}$.



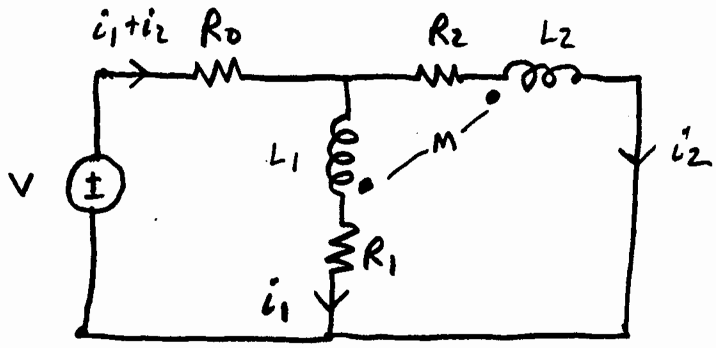
Ideal transformers

$$100 = 4(R + 16) \qquad R = 9$$

d) Write the two loop equations for the circuits shown

Left $V = (i_1 + i_2)R_0 + L_1 \frac{di_1}{dT} + i_1 R_1 - m\frac{di_2}{dT}$

Right $0 = R_2 i_2 + L_2 \frac{di_2}{dT} - m\frac{di_1}{dT} - R_1 i_1 - L_1 \frac{di_1}{dt} + m\frac{di_2}{dT}$



Check $W_{m_a} = \int_0^{10} \frac{\lambda}{2} d\lambda = \frac{\lambda^2}{4}\Big|_0^{10} = 25\text{J}$

$12.5 - 25 \overset{?}{=} -25 + 12.5$ ✓

Problem 4 (10 pts. – no partial credit)

a) An electromechanical device has the following flux-linkage versus current relationship:

$$\lambda = \frac{.04}{.02 + x} i$$

$W_m' = \frac{.02}{.02+x} i^2$ $f^e = \frac{-.02 i^2}{(.02+x)^2}$

The system starts at point a $(i = 5A, x = 0)$ and moves to point b $(i = 5A, x = .02m)$ along a constant current path. Find the following:

$W_{m_b} = \int_0^{\lambda_b} i \big|_{x=.02} d\lambda = \int_0^5 \lambda d\lambda = \frac{25}{2}$

$W_{m_b} =$ 12.5 J

$EFE_{a-b} = \int_{\lambda_a}^{\lambda_b} i \, d\lambda = \int_{10}^{5} 5 \, d\lambda = 5(5-10) =$

$EFE_{a-b} =$ −25 J

$EFM_{a-b} = -\int_0^{.02} \frac{-.02 \times 25}{(.02+x)^2} = \frac{-.5}{.02+x}\Big|_0^{.02} = -12.5 + 25 = 12.5$

$EFM_{a-b} =$ 12.5 J

b) An R-L circuit has the following differential equation:

$$\frac{di}{dt} = -2i + 12 \qquad i(0) = 0$$

Use Euler's method with a step size of 0.01 sec to estimate the current at 0.01 and 0.02 seconds.

$i(0.01) =$ $0 + .01(0 + 12) = 0.12$ A

$i(0.02) =$ $0.12 + .01(-.24 + 12) = 0.2376$ A

Problem 5 (10 pts. – no partial credit)

A nonlinear dynamic model of a system is:

$$\frac{dx}{dt} + x^2 - 16 = 0$$

a) The two equilibrium points for this system are:

x_{e_1} = 4

x_{e_2} = -4

b) The linearized model valid for either x_e is

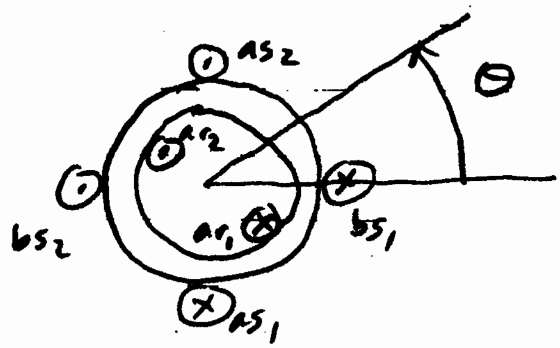
$$\frac{d\Delta x}{dt} = -2x_e \Delta x$$

c) Is x_{e_1} a stable or unstable (circle one) equilibrium point?

d) Is x_{e_2} a stable or unstable (circle one) equilibrium point?

Problem 6 (10 pts. – no partial credit)

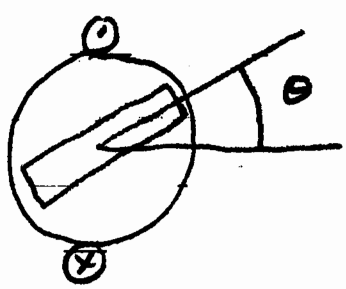
a) A two phase synchronous machine (round rotor) is shown below



Write the 3×3 inductance matrix for this machine in terms of L_S, L_γ, M and θ

$$\begin{array}{cc} & \begin{array}{ccc} as & bs & ar \end{array} \\ \begin{array}{c} as \\ bs \\ ar \end{array} & \begin{bmatrix} L_s & 0 & M\cos\theta \\ 0 & L_s & M\sin\theta \\ M\cos\theta & M\sin\theta & L_r \end{bmatrix} \end{array}$$

b) A single coil is located on the stator of a salient-pole machine as shown below



When $\theta = 0$, the self inductance is a maximum value L_A

When $\theta = \pi/2$ the self inductance is a minimum value L_B

Write a general form of the self inductance of the coil in terms L_A, L_B and θ

$$L = L_0 + L_1\cos 2\theta \qquad \left.\begin{array}{l} L_A = L_0 + L_1 \\ L_B = L_0 - L_1 \end{array}\right\} \quad \begin{array}{l} L_0 = \frac{L_A + L_B}{2} \\ L_1 = \frac{L_A - L_B}{2} \end{array}$$

$$L = \frac{L_A + L_B}{2} + \left(\frac{L_A - L_B}{2}\right)\cos 2\theta$$

Problem 7 (10 pts. – no partial credit)

A 3-phase, 4-pole, 60 Hz induction motor is running at a slip of 0.05

a) The speed of the motor is 1710 RPM

b) The speed of the motor is 179 mechanical radians per second

c) The frequency of the rotor currents is 3 Hz

d) The rotor copper losses are 5.26 % of the output power (Pm)

e) The air gap power is 105.26 % of the output power (Pm)

I^2R vs $I^2R\left(\frac{.95}{.05}\right)$

$\frac{I^2R}{.05}$ vs $I^2R\left(\frac{.95}{.05}\right)$

Problem 8 (30 pts.)

A 100 kVA, 2300/230 V, single phase 60 Hz transformer has the following parameters.
(Side 1 is HV and side 2 is LV)

$R_1 = 0.3\Omega$, $R_2 = .003\Omega$, $R_{c1} = \infty$

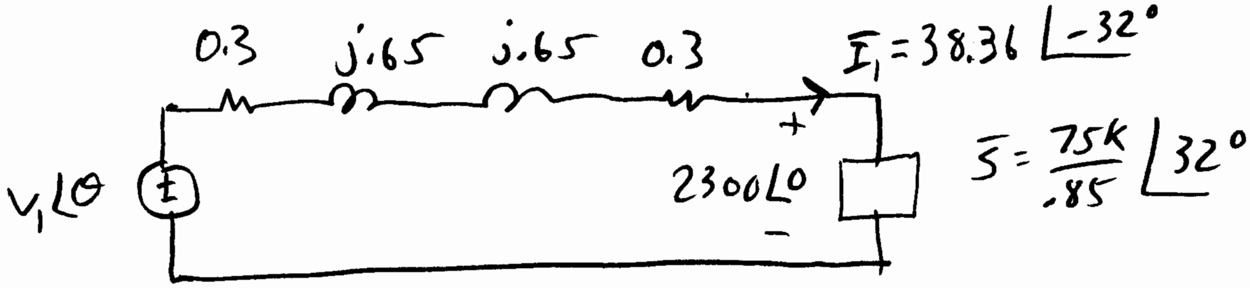
$X_{\ell 1} = 0.65\Omega$ $X_{\ell 2} = 0.0065\Omega$ $X_{m1} = \infty$

Transformer delivers 75 kW at 230 V at 0.85 PF logging

1) Find

   (a) input voltage
   (b) input current
   (c) input power
   (d) efficiency
   (e) voltage regulation

2) Suppose the transformer is accidentally short circuited at the load terminals, what is the input current assuming input voltage does not change?



1)

(a) $V_1\angle\theta = (0.6 + j1.3)(38.36\angle -32°) + 2300\angle 0$
$= 54.9\angle 33° + 2300\angle 0 = 2346 + j30 = \underline{2346\angle 1°}$

(b) $\underline{I_1 = 38.36\angle -32°}$

(c) $S = 2346\angle 1° \times 38.36\angle +32°$
$= 89{,}992\angle 33°$ $P = 75474$

(d) $\eta = \frac{75000}{75474} \times 100 = 99\%$

(e) $VR = \frac{2346 - 2300}{2300} \times 100 = 2\%$

2) $I_{sc} = \frac{2346\angle 1°}{0.6 + j1.3}$

$I_{sc} = 1638$ Amps

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Problem 9 (30 pts)

A 230 Volt (line to line), 2-pole, 3-phase, 60Hz, balanced, symmetrical, round-rotor synchronous machine has negligible armature (stator) resistance. Two tests were performed on the machine as follows:

Open-circuit test:
$I_a = 0$ (no line current)
$V_a = 120V$ (rated open circuit voltage line to neutral)
$I_r = 4$ Amps (field current)

Short-circuit test:
$I_a = 5$ Amps (rated line current)
$V_a = 0$ (shorted stator terminals)
$I_r = 2$ Amps (field current)

a) Compute M, where the magnitude of the internal voltage E_{ar} is equal to $\omega_s M I_r/\sqrt{2}$.

$$\frac{2\pi 60 M \times 4}{\sqrt{2}} = 120 \qquad M = 0.113$$

b) Compute the synchronous reactance X_s.

$$\frac{377 \times .113 \times 2}{\sqrt{2}} = 5 X_s \qquad X_s = 12\,\Omega$$

If you cannot solve a) for M and X_s, then use M=0.1 and X_s = 10 for the remainder of this problem.

c) Compute the line current I_a and field current I_r for the following three cases where the machine is loaded as a generator to 2,300 Watts (3-phase) with the terminal voltage fixed at 120 Volts (line to neutral):

- minimum field current (δ = 90 degrees)
- unity power factor (Q = 0)
- delivers 1,000 VARS (3-phase) to the load

(USE BLANK PAGE PROVIDED AFTER THIS PAGE) c1) $\delta = 90°$ $\quad 2300 = \dfrac{3 \times 120 \times 377 \times .113 \times I_r/\sqrt{2}}{12}$

$I_r = 2.5A$

$$\frac{377 \times .113 \times 2.5}{\sqrt{2}} \angle 90° = j12\bar{I} + 120\angle 0$$

$$\bar{I} = \frac{120 - j75}{j12}$$ $\quad I_a = 11.8A$

c2) $3 \times 120\, I_a = 2300$ $\quad I_a = 6.4A$

$$\frac{377 \times .113 \times I_r}{\sqrt{2}} \angle \delta = 6.4\angle 0\, j12 + 120 = 143 \angle 33°$$ $\quad I_r = 4.73$

c3) $2300 + j1000 = 3 \times 120 \angle 0\, \bar{I}_a^*$ $\quad \bar{I}_a = 7 \angle -23.5°$

$$\frac{377 \times .113 \times I_r}{\sqrt{2}} \angle \delta = 7 \angle -23.5 \times j12 + 120\angle 0$$

$$= 84 \angle 66.5 + 120\angle 0 = 153 + j77$$

$$= 173 \angle$$

$I_r = 5.7A$

$$S = \frac{1200 - 1050}{1200} = .125$$

Problem 10 (30 pts)

A 230 Volt (line to line), 6-pole, 3-phase, 60Hz, balanced, symmetrical, round-rotor induction machine has negligible stator copper loss, negligible core loss, and negligible stator leakage reactance. The magnetizing reactance as seen on the stator side is 40 Ohms and the rotor leakage reactance as seen on the stator side is 2 Ohms. The full-load (Pm = 3,700Watts 3-phase) speed is 1050 RPM.

a) What are the rotor copper losses?

$$P_m = 3{,}700 = 3 I^2 R_r' \left(\frac{1-.125}{.125}\right) = 7 P_{RCL}$$

$$P_{RCL} = 3 I^2 R_r'$$

$$P_{RCL} = \frac{3{,}700}{7} = 529\,W$$

b) What is the full-load torque in Newton-Meters?

$$T_{fL} = \frac{3{,}700}{1050 \times \frac{2\pi}{60}} = 33.6\ Nm$$

c) What is the maximum torque that this motor can deliver?

$$T_{max} = \frac{6}{2} \frac{3}{2} \frac{\left(\frac{230}{\sqrt{3}}\right)^2}{2\pi 60 (2)} = 105\ Nm$$