ECE 330 Exam 1: Fall 2019   NAME___Solution_____

90 minutes   J. Schuh and R. Zhang

Section (Check one)   MWF 10am_____   MWF 2pm_____

1. _____/25   2. _____/25

3. _____/25   4. _____/25   TOTAL _____/100

USEFUL INFORMATION
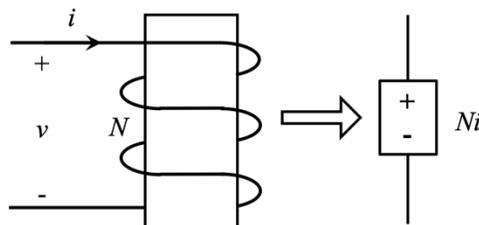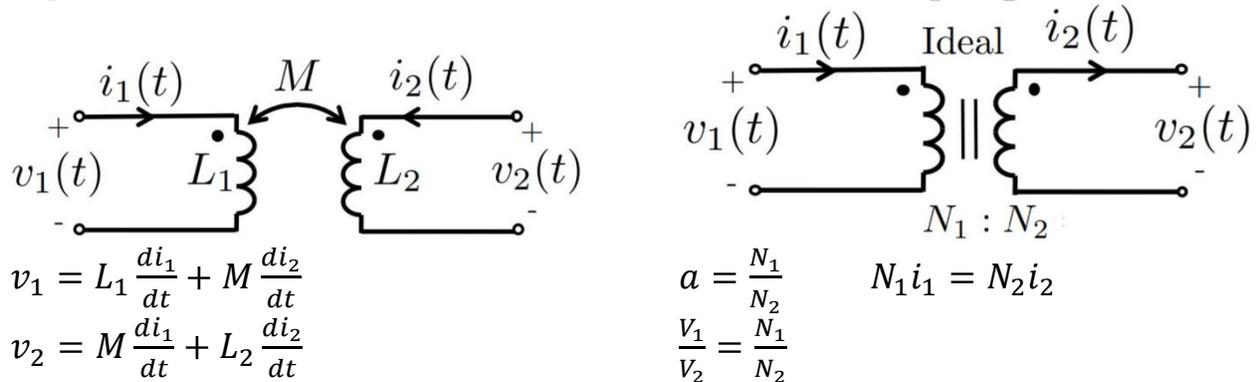
$\sin(x)=\cos(x-90°)$ $\bar{V} = \bar{Z}\bar{I}$ $\bar{S} = \bar{V}\bar{I}^* = P + jQ$ $\bar{S}_{3\varphi} = \sqrt{3}V_L I_L \angle\theta$

$0<\theta<180°$ (lag) $I_L = \sqrt{3}I_\varphi$ (delta) $\bar{Z}_Y = \bar{Z}_\Delta/3$

$-180°<\theta<0$ (lead) $V_L = \sqrt{3}V_\varphi$ (wye) $\mu_0 = 4\pi \times 10^{-7}$H/m

ABC phase sequence has A at 0, B at -120°, and C at +120°

$\oint \underline{H} \cdot \underline{dl} = \int \underline{J_f} \cdot \hat{n}dA$ $\oint \underline{E} \cdot \underline{dl} = -\frac{d}{dt}\left(\int \underline{B} \cdot \hat{n}dA\right)$ $\mathcal{R} = \frac{l}{\mu A}$ $Ni = \mathcal{R}\varphi$

$\varphi = BA$ $\lambda = N\varphi = Li$ (if linear) $v = \frac{d\lambda}{dt}$ $k = \frac{M}{\sqrt{L_1 L_2}}$

1hp=746 W

$v_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$

$v_2 = M\frac{di_1}{dt} + L_2\frac{di_2}{dt}$

$a = \frac{N_1}{N_2}$ $N_1 i_1 = N_2 i_2$

$\frac{V_1}{V_2} = \frac{N_1}{N_2}$

Problem 1 (25 Points)

A single phase source is serving three loads connected in parallel, through a feeder with impedance $\bar{Z}_{feeder} = 1 + j2 \; \Omega$.

- Load 1 is an impedance of $30 + j40 \; \Omega$.
- Load 2 draws 200 W at a lagging PF of 0.8.
- Load 3 draws 1 A at a leading PF of 0.6.

The voltage across the loads is $\bar{V}_{load} = 100 \angle 0° \; V$.

a) Compute the current phasors $\bar{I}_1$, $\bar{I}_2$, and $\bar{I}_3$ for each of the three loads (20 points total).

$$\bar{V}_L = 100 \angle 0° \; V$$

$$\bar{I}_1 = \frac{\bar{V}_L}{\bar{Z}_1}$$

$$\bar{Z}_1 = 30 + j40 \Omega$$
$$= 50 \angle 53.13° \Omega$$

$$\bar{I}_1 = \frac{100 \angle 0°}{50 \angle 53.13°} \Rightarrow \boxed{\bar{I}_1 = 2 \angle -53.13° A}$$

$$I_3 = 1 \; A$$
$$\theta_{i3} = + \cos^{-1}(0.6)$$
$$\theta_{i3} = 53.13°$$

$$\boxed{\bar{I}_3 = 1 \angle 53.13° A}$$

$$P_2 = V I_2 (PF)$$

$$I_2 = \frac{P_2}{V(PF)} \Rightarrow I_2 = \frac{200}{100(0.8)} \Rightarrow I_2 = 2.5 A$$

$$\theta_{i2} = -\cos^{-1}(0.8) = -36.87°$$

$$\boxed{\bar{I}_2 = 2.5 \angle -36.87° A}$$

c) What is the source voltage $\bar{V}_{source}$? (3 points)

$$\bar{I}_{tot} = \bar{I}_1 + \bar{I}_2 + \bar{I}_3$$

$$= (1.2 - j1.6) + (2 - j1.5) + (0.6 + j0.8)$$

$$= 3.8 - j2.3 \, A$$

$$= 4.44 \angle -31.18° \, A$$

$$\bar{V}_s = \bar{V}_{load} + \bar{Z}_{line} \bar{I}_{tot}$$

$$= 100 \angle 0° + (2.24 \angle 63.43°)(4.44 \angle -31.18°)$$

$$= 100 \angle 0° + 9.9456 \angle 32.25°$$

$$\boxed{\begin{array}{l} \bar{V}_s = 108.41 + j 5.307 \, V \\ = 108.54 \angle 2.80° V \end{array}}$$

d) What is the power factor at the source? State whether it is leading or lagging. (1 point)

$$\theta_s = (2.80 - [-31.18]) = 33.98$$

$$PF = 0.829 \, lagging$$

e) What is the power factor at the load? State whether it is leading or lagging. (1 point)

$$\theta = 31.18°$$

$$PF = 0.856 \, lagging$$

Problem 2 (25 Points)

A Wye-connected three-phase generator delivers 1200 kVA at 0.6 PF lagging and 4160 volt (line-to-line) to the following loads in parallel.
- Load 1 is connected in Wye, and draws a total of 300 kW at unity power factor.
- Load 2 is an impedance load connected in Delta, with unknown per-phase impedance $\bar{Z}$.

a) Draw the three-phase circuit diagram. Label all relevant instances of the following:
- Phase-to-neutral voltage phasor (e.g. $10\angle 50°$ V).
- Per-phase impedance for load 2 as symbolic expressions (e.g. $\bar{Z}/\sqrt{5}\ \Omega$).
Include polarity markers (+/-) in all of your voltage labels. (11 points)
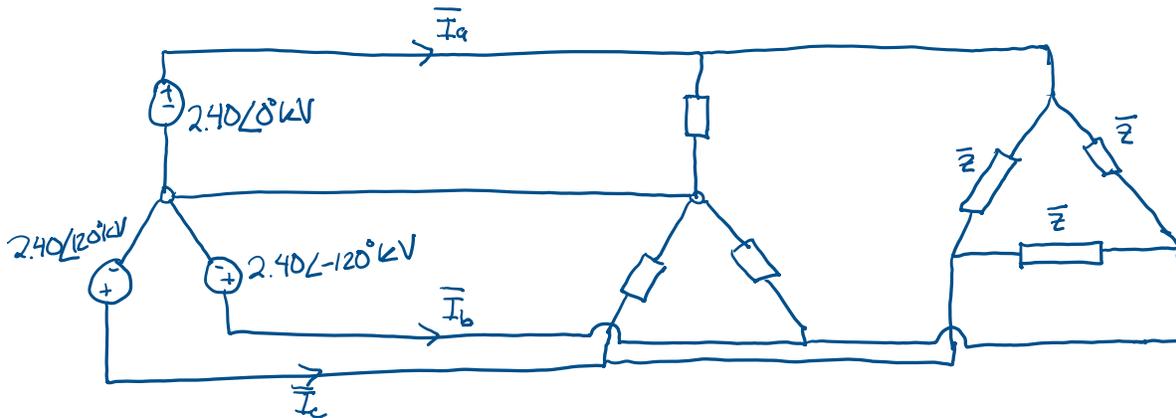
b) Draw the single-phase equivalent circuit for phase a. Label one instance of the following:
- Voltage phasor (e.g. $10\angle 50°$ V).
- Power for the generator in terms of kVA and PF (e.g. 10 kVA at 0.1 PF leading).
- Power for load 1 in terms of kVA and PF (e.g. 10 kVA at 0.1 PF leading).
- Impedance for load 2 as a symbolic expression (e.g. $\bar{Z}/\sqrt{5}\ \Omega$).
All quantities should be stated in per-phase values. (6 points)

$S_{1,1\phi} = 100\ kVA,\ PF = 1$

$S_{gen,1\phi} = 400\ kVA,\ PF = 0.6\ lagging$

d) How much three-phase kVARs do we need to bring the combined power factor to unity? How would you connect the capacitors (in Wye or Delta) and why? (3 points)

$$\bar{S}_{3\phi} = 1200 \angle 53.13° \text{ kVA}$$
$$= 720 + j960 \text{ kVA}$$

$$\boxed{\text{Add } 960 \text{ kVAR of capacitance}}$$

Delta: lower capacitance needed to get given power, lower phase current

Wye: lower phase voltage

Either: complex power equation is the same for Wye and Delta

e) Compute the numerical value for the unknown impedance $\bar{Z}$. (5 points)

$$\bar{S}_2 = \bar{S}_{tot} - \bar{S}_1$$
$$= 420 + j960 \text{ kVA}$$
$$= 1047.85 \angle 66.37°$$

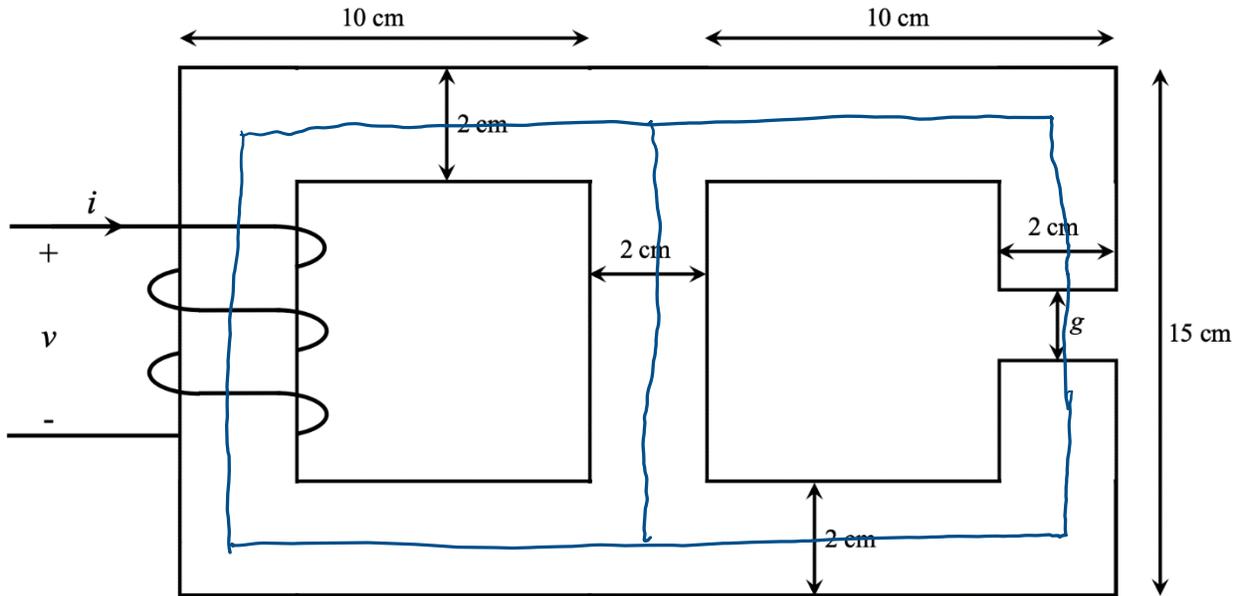$$\bar{S}_2 = 3\bar{V}_L \bar{I}_\phi^* \implies \bar{S}_2 = \frac{3V_L^2}{\bar{Z}^*} \implies \bar{Z} = \left(\frac{3V_L^2}{\bar{S}_2}\right)^*$$

$$\bar{I}_\phi = \frac{V_L}{\bar{Z}}$$

$$\boxed{\bar{Z} = 49.54 \angle 66.37° \ \Omega}$$

Problem 3 (25 Points)

10 cm   10 cm

2 cm

i

+

v

-

2 cm

2 cm

2 cm

15 cm

g

2 cm

A coil is wrapped 300 times around an iron core with the given dimensions and a depth of 2 cm into the page. The iron has a permeability $\mu=2000\mu_0$ and contains an air gap of $g=2$ mm. Fringing effects can be neglected.

a) Draw the magnetic path through the iron core on the figure above and draw the corresponding magnetic equivalent circuit. (15 points).
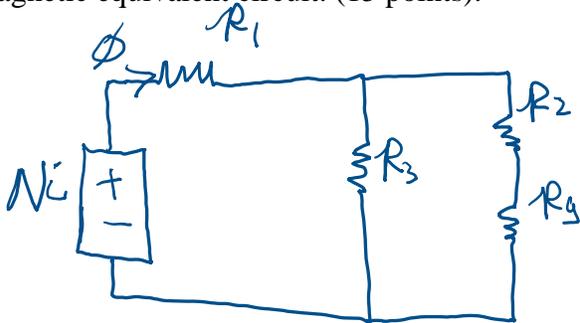
$$R_1 = \frac{33cm}{2000\mu_0(4cm^2)} \Rightarrow R_1 = 3.283\times10^5 \, At/wb$$

$$R_2 = \frac{33cm - 2mm}{2000\mu_0(4cm^2)} \Rightarrow R_2 = 3.263\times10^5 \, At/wb$$

$$R_3 = \frac{13cm}{2000\mu_0(4cm^2)} \Rightarrow R_3 = 1.293\times10^5 \, At/wb$$

$$R_g = \frac{2mm}{\mu_0(4cm^2)} \Rightarrow R_g = 3.979\times10^6 \, At/wb$$

b) What is the total flux ϕ through the circuit? (7 points)

$$Ni = \phi R_{tot}$$

$$R_{tot} = \left(\frac{1}{R_3} + \frac{1}{R_2 + R_3}\right)^{-1} + R_1$$

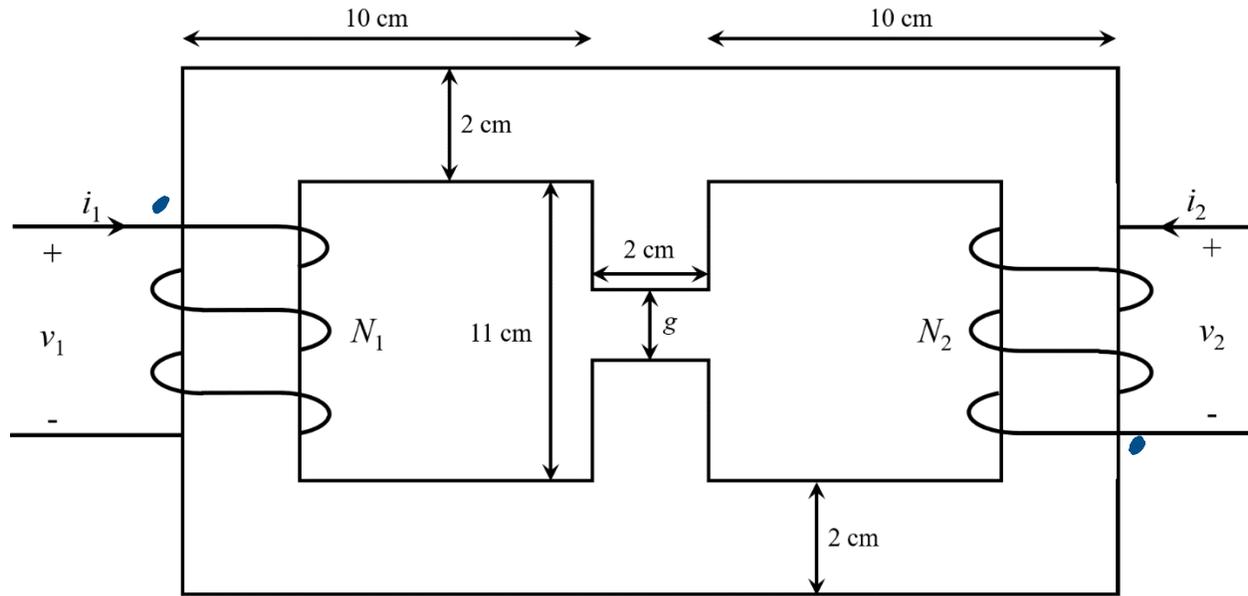$$= 4.538 \times 10^5 \; At/wb$$

$$300i = 4.538 \times 10^5 \phi$$

$$\boxed{\phi = 6.610 \times 10^{-4} i}$$

c) What is the inductance of the coil? (3 points)

$$\lambda = N\phi = Li$$

$$\lambda = 300 \left(6.610 \times 10^{-4}\right) i$$
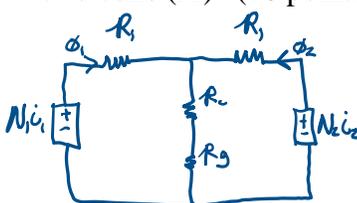
$$\lambda = 0.198 i$$

$$\boxed{L = 0.198 \; H}$$

Problem 4 (25 Points)

Two coils are wrapped around an iron core with the given dimensions and depth of 3 cm into the page. Coil 1 contains 400 turns and Coil 2 contains 200 turns. The iron has a permeability $\mu=1500\mu_0$ and contains an air gap of $g=4$ mm. Fringing effects can be neglected.

a) Draw on the figure above where the dot marks should go for each coil. (2 points)

b) What is the self-inductance for each coil (L_1 and L_2) and the mutual inductance between the two coils (M)? (18 points)

$$R_1 = \frac{33cm}{1500\mu_0(6cm^2)} \Rightarrow R_1 = 2.918\times10^5 \ At/Wb$$

$$R_c = \frac{13cm - 4mm}{1500\mu_0(6cm^2)} \Rightarrow R_c = 1.114\times10^5 \ At/Wb$$

$$R_g = \frac{4mm}{\mu_0(6cm^2)} = R_g = 5.305\times10^6 \ At/Wb$$

$$N_1 i_1 = R_1\phi_1 + (R_c + R_g)(\phi_1 + \phi_2)$$
$$-N_2 i_2 = R_1\phi_2 + (R_c + R_g)(\phi_1 + \phi_2)$$
$$N_1 i_1 - N_2 i_2 = R_1(\phi_1 - \phi_2)$$
$$\phi_1 = \frac{N_1 i_1 - N_2 i_2}{R_1} + \phi_2$$

$$N_2 i_2 = (R_1 + R_c + R_g)\phi_2 + (R_c + R_g)\phi_1$$
$$N_2 i_2 = (R_1 + R_c + R_g)\phi_2 + (R_c + R_g)\left[\frac{N_1 i_1 - N_2 i_2}{R_1} + \phi_2\right]$$
$$= (R_1 + 2R_c + 2R_g)\phi_2 + \left(\frac{R_c + R_g}{R_1}\right)[N_1 i_1 - N_2 i_2]$$
$$-\left(\frac{R_c + R_g}{R_1}\right)N_1 i_1 + \left(1 + \frac{R_c + R_g}{R_1}\right)N_2 i_2 = (R_1 + 2R_c + 2R_g)\phi_2$$

$$\lambda_1 = N_1 \phi_1 \qquad\qquad \lambda_2 = N_2\phi_2$$

$$\boxed{\begin{array}{l} L_1 = 0.284 \ H \\ L_2 = 0.0703 \ H \\ M = 0.1335 \ H \end{array}}$$

c) What is the open circuit voltage measured across Coil 2 if a current of $i_1=\sqrt{2}(10)\cos(377t)$ is applied across Coil 1? Write your answer as a cosine function. (5 points)

$$V_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$V_2 = -M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

Open circuit: $i_2 = 0$

$$V_2 = -M \frac{di_1}{dt}$$

$$V_2 = -0.1335 \left(-\sqrt{2}(10)(377) \sin(377t) \right)$$

$$V_2 = \sqrt{2}(503.30) \sin(377t)$$

$$\boxed{V_2 = \sqrt{2}(503.30) \cos(377t - 90°)}$$

(Blank page for extra work)