ECE 330 Exam 1: Spring 2021

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USEFUL INFORMATION

sin(x)=cos(x-90°) $\quad \bar{V} = \bar{Z}\bar{I} \quad \bar{S} = \bar{V}\bar{I}^* = P + jQ \quad \bar{S}_{3\varphi} = \sqrt{3}V_L I_L \angle\theta$

0<θ<180° (lag) $\quad I_L = \sqrt{3}I_\varphi$ (delta) $\quad \bar{Z}_Y = \bar{Z}_\Delta/3$

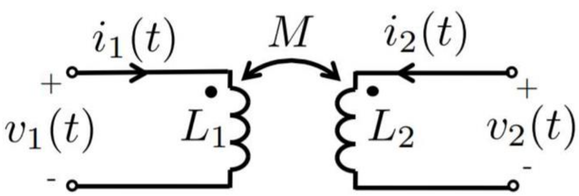
-180°<θ<0 (lead) $\quad V_L = \sqrt{3}V_\varphi$ (wye) $\quad \mu_0 = 4\pi \times 10^{-7}$H/m

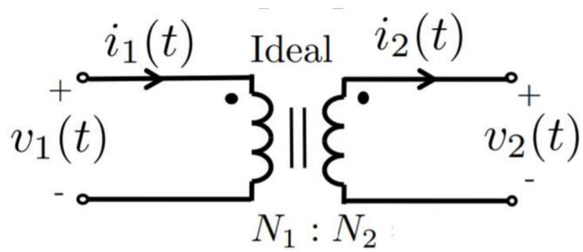
ABC phase sequence has A at 0, B at -120°, and C at +120°

$\int \underline{H} \cdot \underline{dl} = \int \underline{J_f} \cdot \hat{n} dA \quad \int \underline{E} \cdot \underline{dl} = -\frac{d}{dt}\left(\int \underline{B} \cdot \hat{n} dA\right) \quad \mathcal{R} = \frac{l}{\mu A} \quad Ni = \mathcal{R}\varphi$

$\varphi = BA \quad \lambda = N\varphi = Li$ (if linear) $\quad v = \frac{d\lambda}{dt} \quad k = \frac{M}{\sqrt{L_1 L_2}}$

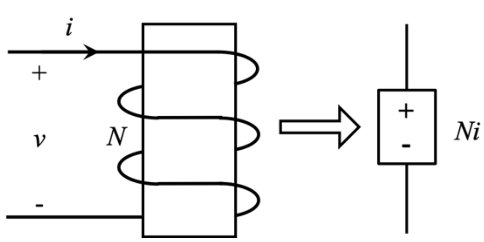
1hp=746 W





$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \qquad a = \frac{N_1}{N_2} \qquad N_1 i_1 = N_2 i_2$

$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt} \qquad \frac{V_1}{V_2} = \frac{N_1}{N_2}$



Problem 1 (25 Points)

a) A single-phase load has a voltage of 157 cos (377t + 15°) Volts with a current into the positive terminal of 12 sin (377t + 70°) Amps. Find the average power absorbed by this load.

b) Two single-phase loads in parallel, 16 kW at 0.8 PF lag and a load with impedance Z=20+j20 Ω, are supplied by a source V=100∠0° V. Find the total current supplied by the source and the combined PF (specify lag or lead).

Problem 2 (25 Points)

The following three-phase balanced loads are connected in parallel across a three - phase wye-connected, 60 Hz source of 4160 V (line to line).

Load 1: 120 kVA at 0.8 PF lag (Wye connected)
Load 2: 180 kW at 0.7 PF lag (Wye connected)
Load 3: 13 Amps phase current, unity power factor (Delta connected)

a) Find the complex power P + jQ consumed by each load.
b) Find the total source line current (magnitude)

① a) $v(t) = 157\cos(377t + 15°)$ V

$i(t) = 12\sin(377t + 70°)$ A $= 12\cos(377t + 70° - 80°) =$

$= 12\cos(377t - 20°)$ A

P – ?

$P = Re\{\bar{V}\bar{I}^*\}$

$\bar{V} = \frac{157}{\sqrt{2}}\angle 15° = 111\angle 15°$ V

$\bar{I} = \frac{12}{\sqrt{2}}\angle -20°$ A $= 8.5\angle -20°$ A

$P = Re\{111\angle 15° \cdot 8.5\angle 20°\} = Re\{943.5\angle 35°\} =$

$= 773$ W

b) $\bar{S}_t = \bar{V}\cdot\bar{I}^* \Rightarrow \bar{I} = \left(\frac{\bar{S}_t}{\bar{V}}\right)^*$

$\bar{S}_t = \bar{S}_1 + \bar{S}_2$

$S_1 = \frac{P}{PF} = \frac{16\,k}{0.8} = 20$ kVA

$\theta_1 = \cos^{-1}(0.8) = 36.9°$

$\bar{S}_1 = 20\angle 36.9°$ kVA

$Z = 20 + j20 = 28\angle 45°\,\Omega$

$\bar{S}_2 = \bar{V}\cdot\bar{I}^* = \frac{V^2}{Z^*} = \frac{100^2}{28\angle -45°} =$

$\bar{I} = \frac{\bar{V}}{Z}$

$= 0.357\angle 45°$ kVA

$\bar{S}_t = 16 + j12 + 0.252 + j0.25 = 16.25 + j12.25 =$

$= 20.35\angle 37°$ kVA

$\bar{I} = \left(\frac{20.35k\angle 37°}{100\angle 0°}\right)^* = 203.5\angle -37°$ A

$PF = \cos(37°) = 0.8$ lag.

Q2: a) $\bar{S}_{3\phi} = \bar{S}_{13\phi} + \bar{S}_{23\phi} + \bar{S}_{33\phi}$

$\bar{S}_{13\phi} = 120\angle 36.9^\circ \text{ KVA} = 96 + j72 \text{ KVA}$ $\quad \theta_1 = \cos^{-1}(0.8) = 36.9^\circ$

$S_{33\phi} = \frac{P}{PF} = \frac{180}{0.7} = 257 \text{ KVA}$

$\theta_2 = \cos^{-1}(0.7) = 45.5^\circ$

$\bar{S}_{23\phi} = 257\angle 45.5^\circ \text{ KVA} = 180 + j183 \text{ KVA}$

$\bar{S}_{33\phi} = 3 V_L I_\phi \angle\theta =$ $\quad \theta_3 = 0^\circ$

$= 3 \cdot 4160 \cdot 13 \angle 0^\circ = 162 \angle 0^\circ \text{ KVA} = 162 \text{ KVA}$

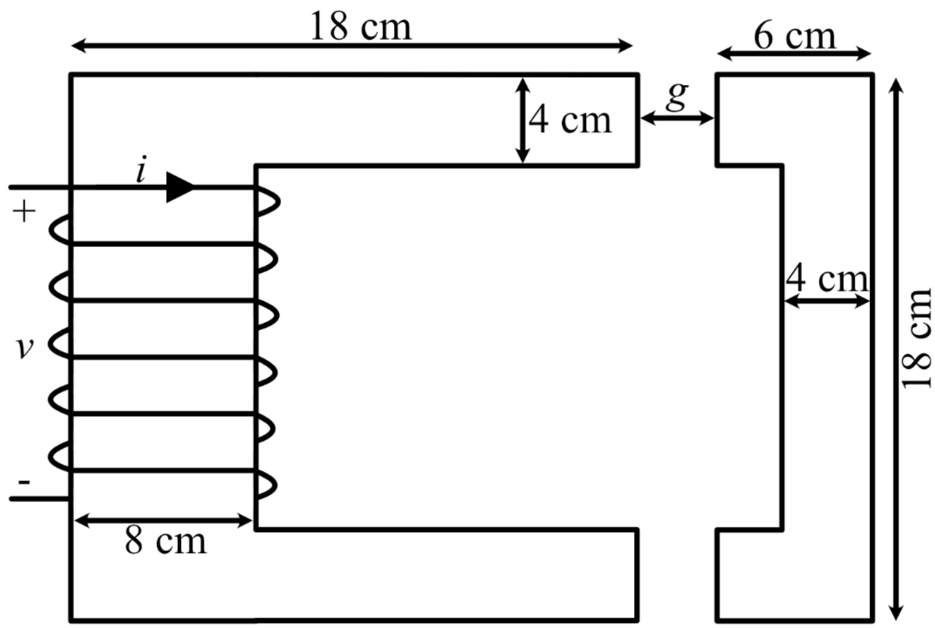
$\bar{S}_{3\phi} = 96 + j72 + 180 + j183 + 162 =$

$= 438 + j255 \text{ KVA}$

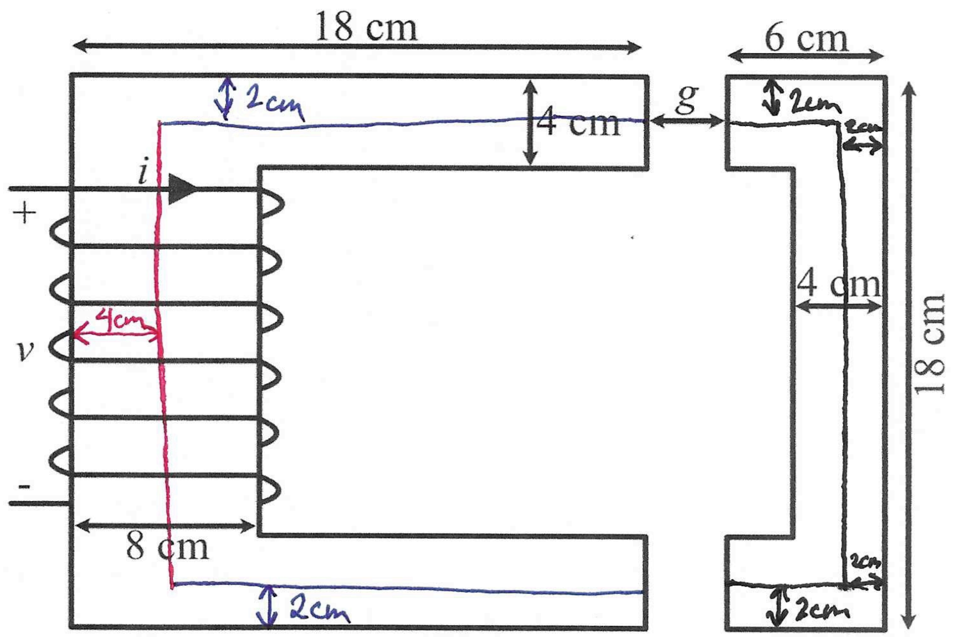
b) $S_{3\phi} = \sqrt{3} V_L \cdot I_L$

$I_L = \frac{S_{3\phi}}{\sqrt{3} V_L} = \frac{507k}{\sqrt{3}\, 4160} = 70 \text{ A}$

Problem 3 (25 Points)



An iron core with a depth into the page of 5 cm and permeability μ=2500μ_0 has a coil with 250 turns wrapped around it as shown. If the air gap g=10 mm, what is the inductance of the coil? Neglect fringing effects.



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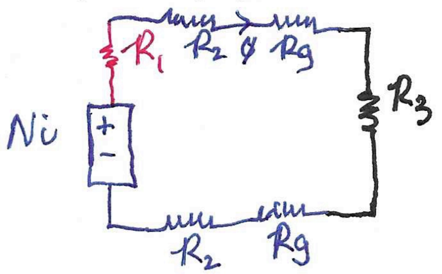


$$\mathcal{R}_1 = \frac{14\text{cm}}{2500\mu_0(40\text{cm}^2)} = 1.114\times10^4 \text{ At/Wb}$$

$$\mathcal{R}_2 = \frac{14\text{cm}}{2500\mu_0(20\text{ cm}^2)} = 2.228\times10^4 \text{ At/Wb}$$

$$\mathcal{R}_3 = \frac{22\text{ cm}}{2500\mu_0(20\text{cm}^2)} = 3.501\times10^4 \text{ At/Wb}$$

$$\mathcal{R}_g = \frac{10\text{ mm}}{\mu_0(20\text{cm}^2)} = 3.979\times10^6 \text{ At/Wb}$$

$$\mathcal{R}_{TOT} = \mathcal{R}_1 + \mathcal{R}_3 + 2\mathcal{R}_2 + 2\mathcal{R}_g$$

$$\mathcal{R}_{TOT} = 8.048\times10^6 \text{ At/Wb}$$

$$\phi = \frac{Ni}{\mathcal{R}_{TOT}}$$

$$\lambda = N\phi = \frac{N^2}{\mathcal{R}_{TOT}} i = Li$$

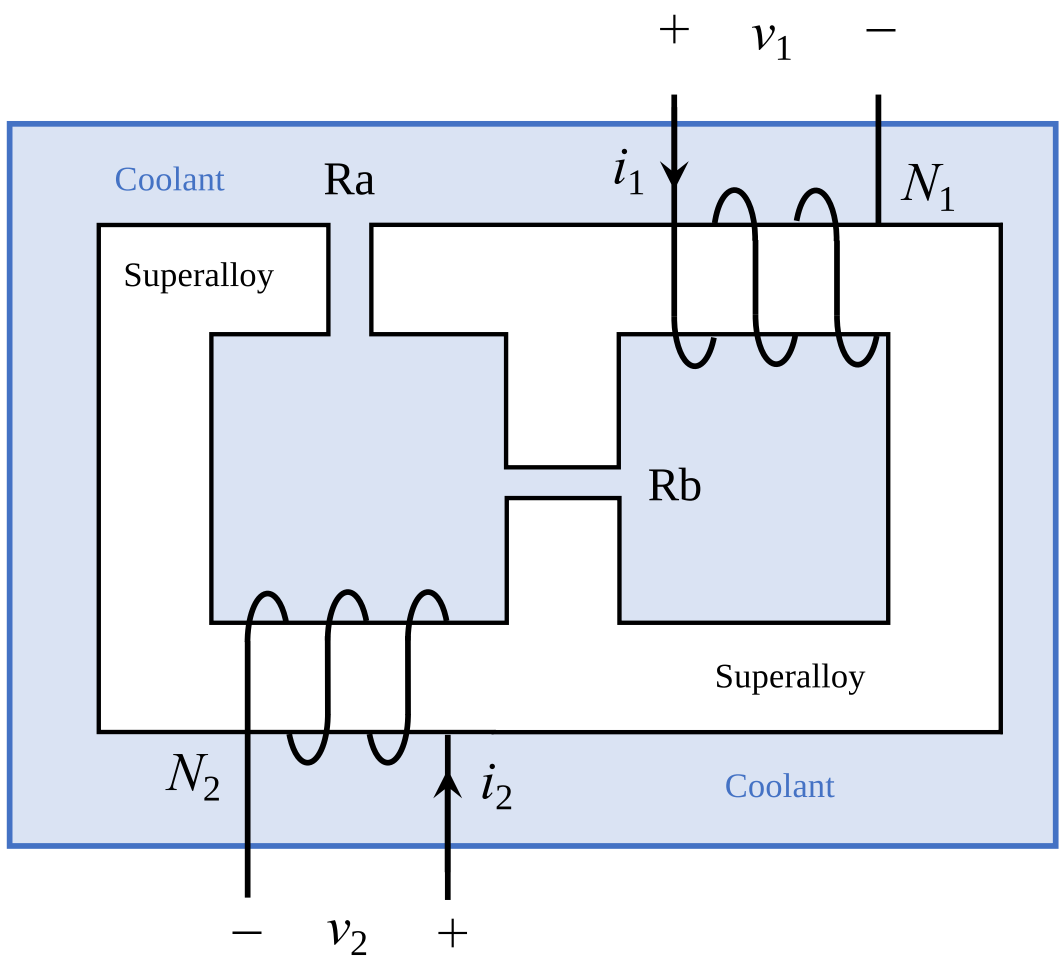
$$L = \frac{N^2}{\mathcal{R}_{TOT}}$$

$$\boxed{L = 7.765 \text{ mH}}$$

Problem 4 (25 Points)

You have been hired by a stealth-mode startup to develop a superconducting transformer for a possible mission to Mars.

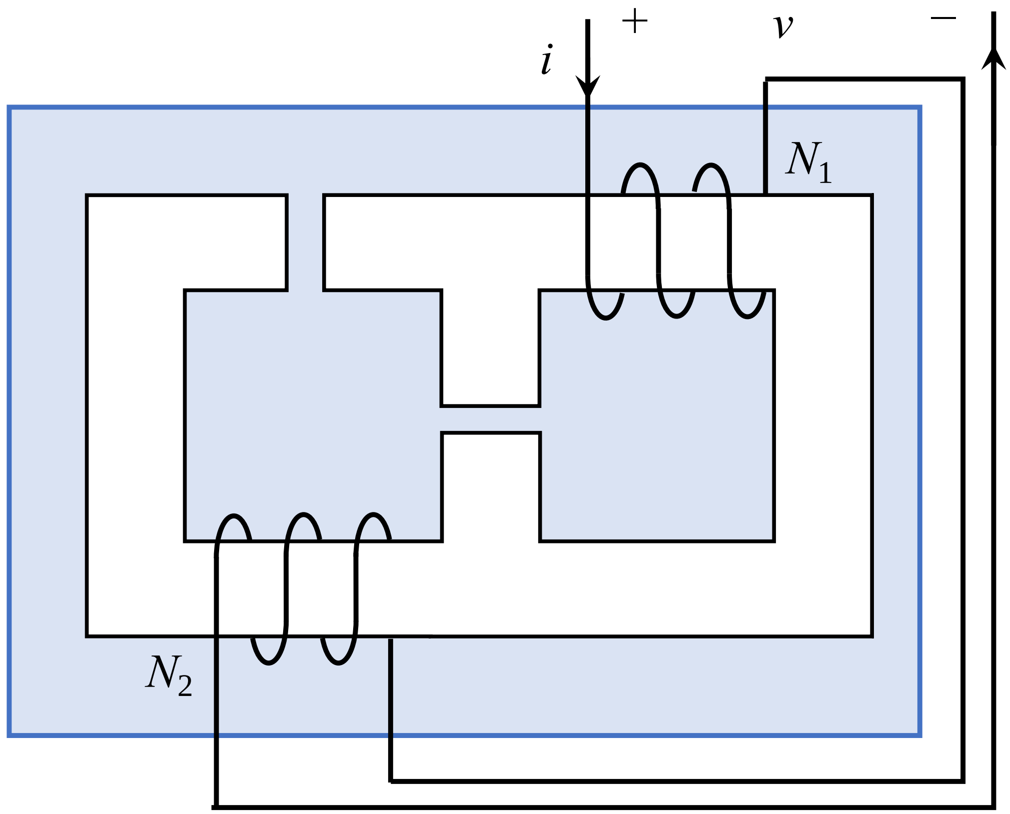
The transformer is built out of magnetic super alloy, and immersed in a liquid coolant. The super alloy is so magnetically permeable that its permeability is essentially infinite ($\mu_r \rightarrow \infty$) while the coolant is a factor of five times more permeable than vacuum ($\mu_r = 5$). The two coils are connected to a resonant power converter circuit with the voltage and current labels as shown. The first coil has $N_1 = 20$ turns while the second coil has $N_2 = 10$ turns.



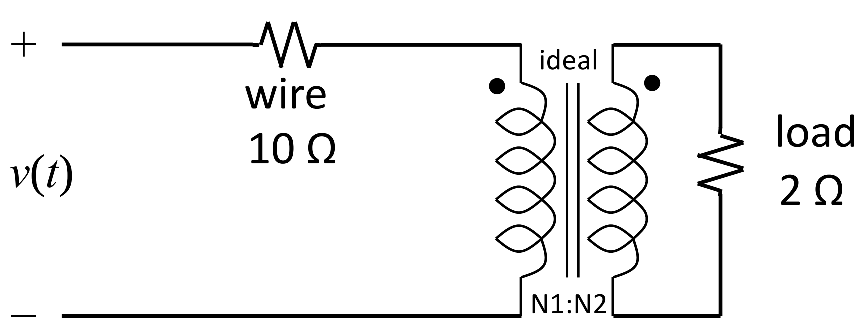
a) We computed Ra = 200 At/Wb and Rb = 800 At/Wb by neglecting the effects of fringing. Use these figures to compute the mutual inductance between the two coils, and state the unit.

b) If we were to account for the effects of fringing in the calculation above, would the mutual inductance increase or decrease? Please explain your reasoning. (Your answer should take into consideration the effects of fringing in <u>both</u> gaps.)

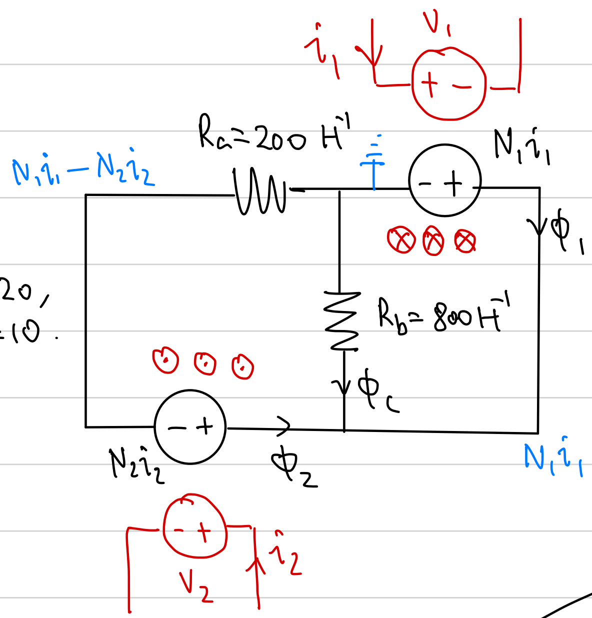
c) Now, we connect the two coils in series, by connecting the negative terminal of the first coil onto the positive terminal of the second coil as shown. What is the combined inductance of this structure? (Neglect fringing and use Ra = 200 At/Wb and Rb = 800 At/Wb.)



d) The purpose of our transformer is to drive a 2 Ω load from a source that is connected through a 10 Ω wire, as shown below. If v(t) = cos(t) V and we model the transformer as ideal, what is the average power delivered to the load? What could we do to deliver more power to the load?



a)



1) Draw electrical diagram.

2) Magnetic circuit:

→ RHR for polarities of mmf

→ ϕ_1, ϕ_2 pointing outwards from + terminal

3) Pick a "ground" and label all "nodal voltages"

4) Write KCL eqns

$$\underbrace{\frac{0 - (N_1 i_1 - N_2 i_2)}{R_a}}_{\phi_2} + \underbrace{\frac{0 - N_1 i_1}{R_b}}_{\phi_c} + \phi_1 = 0$$

$$\phi_2 = \frac{N_2}{R_a} i_2 - \frac{N_1}{R_a} i_1 \qquad \phi_1 = \frac{N_1 i_1}{R_b} + \frac{N_1 i_1}{R_a} - \frac{N_2 i_2}{R_a}$$

$$\lambda_2 = \underbrace{\frac{N_2^2}{R_a}}_{L_2} i_2 - \underbrace{\frac{N_1 N_2}{R_a}}_{M} i_1 \qquad \lambda_1 = \underbrace{\frac{N_1^2}{R_a \| R_b}}_{L_1} i_1 - \underbrace{\frac{N_1 N_2}{R_a}}_{M} i_2$$

$$L_1 = N_1^2 / (R_a \| R_b) = 400 / (200 \| 800) = 400 / 160 = 5/2 = \underline{2.5\ H}$$

$$L_2 = N_2^2 / R_a = 100 / 200 = \underline{0.5\ H}$$

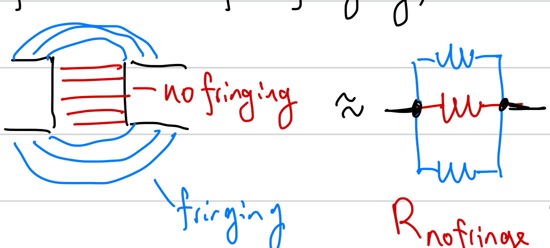
$$M = N_1 N_2 / R_a = 200 / 200 = \underline{1\ H}$$

$$\left(200 \| 800 = \frac{(200)(800)}{200 + 800} = \frac{160000}{1000} = 160 \right)$$

$$\boxed{M = 1H}$$

b) If account for fringing, then R_a, R_b decrease. (More path to take)

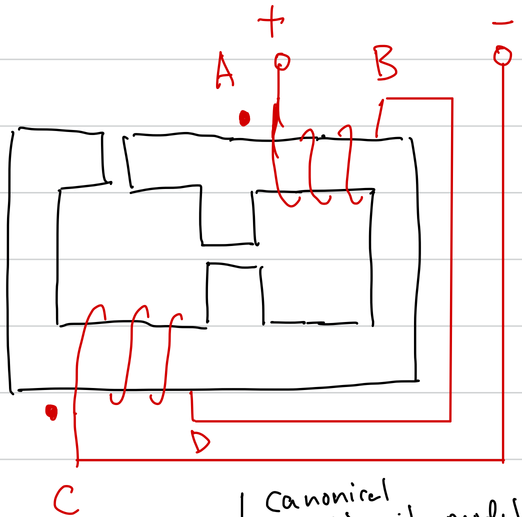
M is inversely proportional to R_a.



$$R_{nofringe} \| R_{fringe} < R_{nofringe}$$

$$\boxed{M \text{ must } \underline{\text{increase}}}$$

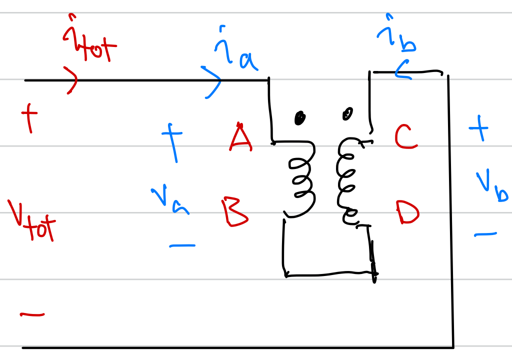
c)



(See also HW4, Prob 3.17)

1) Label the dots, terminals of two mutual inductors.
2) Draw equiv circuit diagram, label terminals.
3) Connect the terminals correctly.
4) Solve circuit model.

↓ canonical circuit model



$$V_a = L_1 \frac{di_a}{dt} + M \frac{di_b}{dt}$$

$$V_b = M \frac{di_a}{dt} + L_2 \frac{di_b}{dt}$$

$$i_a = -i_b = i_{tot}$$

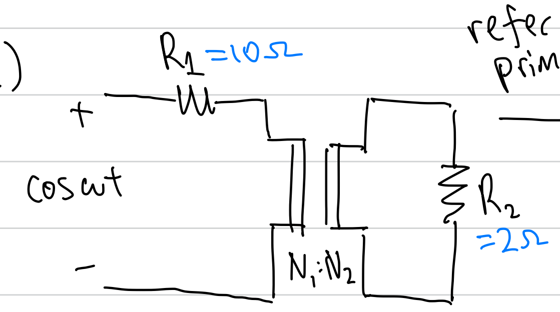
$$\Rightarrow \quad V_a = L_1 \frac{di_{tot}}{dt} - M \frac{di_{tot}}{dt}$$

$$V_b = M \frac{di_{tot}}{dt} - L_2 \frac{di_{tot}}{dt}$$

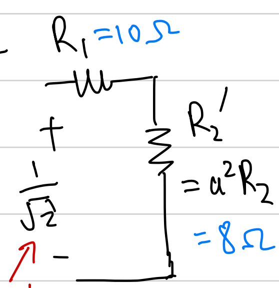
$$V_{tot} = V_a - V_b = (L_1 + L_2 - 2M) \frac{di_{tot}}{dt}$$

$\boxed{\text{Inductance of combined structure is } L_1 + L_2 - 2M = 1\text{ H}}$

d)



refer to primary →



not 1 V!!

$N_1 = 20$, $N_2 = 10$, $a = N_1/N_2 = 2$

$$\bar{I} = \frac{\bar{V}}{R_1 + R_2'}$$

$$P_{load} = I^2 R_2' = \frac{V^2 R_2'}{(R_1 + R_2')^2}$$

$$= \frac{V^2 a^2 R_2}{(R_1 + a^2 R_2)^2}$$

$$P_{out} = \frac{1}{2} \frac{(4)(2)}{(10 + (4)(2))^2} = \frac{4}{(18)^2} = \frac{1}{81}\text{ W}$$

$\boxed{P_{out} = 0.0123\text{ W}}$

To increase power: 1) Decrease R_1; 2) Increase R_2 (up to 2.5Ω); 3) Increase a (up to $\sqrt{5}$); 4) Increase voltage.