ECE 330 Exam 2: Fall 2021 NAME________________________________
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Section (Check one) MWF 10am__________ MWF 12pm________ MWF 2pm__________

USEFUL INFORMATION

sin(x)=cos(x-90°) $\bar{V} = \bar{Z}\bar{I}$ $\bar{S} = \bar{V}\bar{I}^* = P + jQ$ $\bar{S}_{3\varphi} = \sqrt{3}V_L I_L \angle\theta$

$0<\theta<180°$ (lag) $I_L = \sqrt{3}I_\varphi$ (delta) $\bar{Z}_Y = \bar{Z}_\Delta/3$

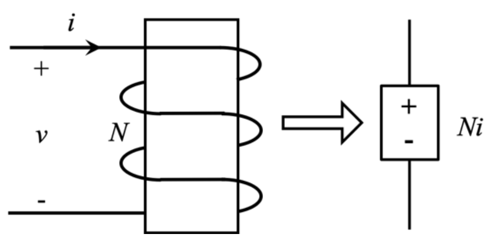
$-180°<\theta<0$ (lead) $V_L = \sqrt{3}V_\varphi$ (wye) $\mu_0 = 4\pi \times 10^{-7}$ H/m

ABC phase sequence has A at 0, B at -120°, and C at +120°

$\int \underline{H} \cdot \underline{dl} = \int \underline{J_f} \cdot \hat{n} dA$ $\int \underline{E} \cdot \underline{dl} = -\frac{d}{dt}\left(\int \underline{B} \cdot \hat{n} dA\right)$ $\mathcal{R} = \frac{l}{\mu A}$ $Ni = \mathcal{R}\varphi$

$\varphi = BA$ $\lambda = N\varphi = Li$ (if linear) $v = \frac{d\lambda}{dt}$ $k = \frac{M}{\sqrt{L_1 L_2}}$

1hp=746 W



$W_m = \int_0^\lambda i d\hat{\lambda}$ $W_m' = \int_0^i \lambda d\hat{i}$ $W_m + W_m' = i\lambda$ $f^e = -\frac{\partial W_m}{\partial x} = \frac{\partial W_m'}{\partial x}$

$x \to \theta, f^e \to T^e$

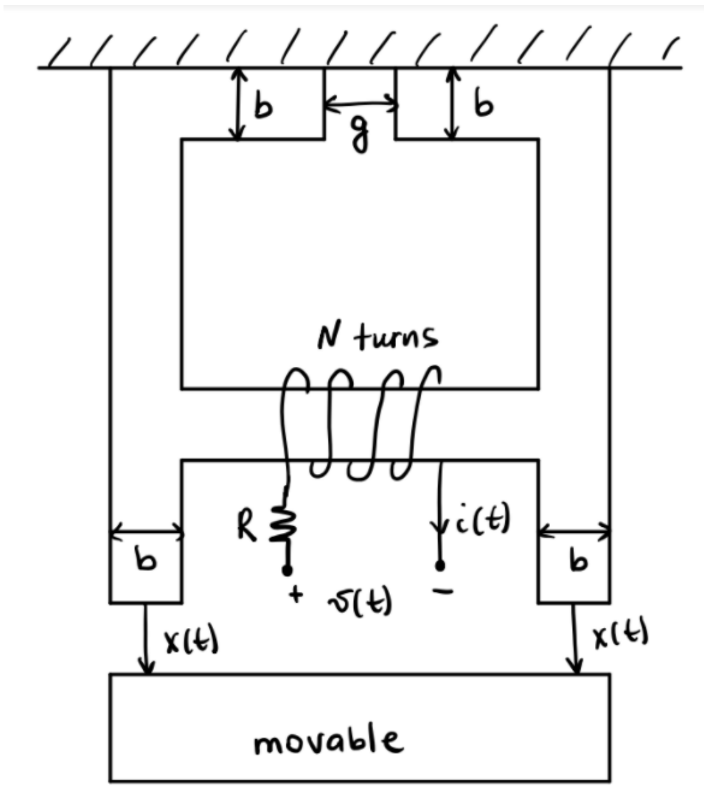
$EFE_{a\to b} = \int_a^b i d\lambda$ $EFM_{a\to b} = \int_a^b -f^e dx$

$EFE_{a\to b} + EFM_{a\to b} = W_{mb} - W_{ma}$

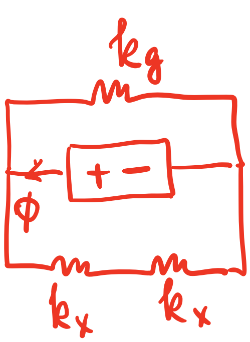
$i = \frac{\partial W_m}{\partial \lambda}$ $\lambda = \frac{\partial W_m'}{\partial i}$

Problem 1 (25 Points)

For the structure shown below, the movable member is constrained to move up and down only as indicated, and the air gap between the fixed and movable member is x(t). The structure has a width b. The coil has number of turns N, current *i(t)* and resistance R. The airgap g is fixed. Depth into the page for all members is D. Neglect fringing and assume that the permeability of the structure to be infinite. Permeability of free space is μ_0.



a) Draw the magnetic circuit and find the total reluctance in terms of the given variables.



$$R_{total} = R_g \parallel (R_x + R_x) = \frac{R_g \cdot 2R_x}{R_g + 2R_x}$$

$$R_x = \frac{x}{\mu_0 b D} \qquad R_g = \frac{g}{\mu_0 b D} \Rightarrow R_{total} = \frac{\frac{2gx}{(\mu_0 b D)^2}}{\frac{g+2x}{\mu_0 b D}} = \frac{2gx}{\mu_0 b D (g+2x)}$$

$\mathscr{R}_{total} = \dfrac{2gx}{\mu_0 b D (g+2x)}$

b) Find an expression for the flux linkage, λ in terms of the given variables.

$$\lambda = N\phi$$

$$\phi = \frac{Ni}{\mathcal{R}_{total}} = \frac{N\mu_0 bD(g+2x)}{2gx} i$$

$$\lambda = \underbrace{\frac{N^2\mu_0 bD(g+2x)}{2gx}}_{L(x)} i$$

λ= $\frac{N^2\mu_0 bD(g+2x)}{2gx} i$

—

c) Find an expression for the voltage, v(t) in terms of the given variables and t.

$$v(t) = i \cdot R + \frac{d\lambda}{dt}$$

$$\frac{d\lambda}{dt} = \frac{\partial\lambda}{\partial i}\frac{di}{dt} + \frac{\partial\lambda}{\partial x}\frac{dx}{dt} = L(x)\frac{di}{dt} + i\frac{\partial L}{\partial x}\frac{dx}{dt} =$$

$$= \frac{N^2\mu_0 bD(g+2x)}{2gx}\frac{di}{dt} - \frac{N^2\mu_0 bD i}{2x^2}\frac{dx}{dt}$$

$v(t)$= $i \cdot R + \frac{N^2\mu_0 bD(g+2x)}{2gx}\frac{di}{dt} - \frac{N^2\mu_0 bD i}{2x^2}\frac{dx}{dt}$

Problem 2 (25 Points)

The co-energy of a device is given by

$$W_m{}'(i,x) = \frac{i^5}{24x} + \frac{i^3}{6x} + \frac{i}{x}$$

Find:

a) The flux linkage λ as a function of i and x

$$\lambda = \frac{\partial W_m{}'}{\partial i} = \frac{5i^4}{24x} + \frac{3i^2}{6x} + \frac{1}{x}$$

λ=____

b) The force of electric origin $f^e(i,x)$. State the unit for force.

$$f^e = \left.\frac{\partial W_m{}'}{\partial x}\right|_{i-\text{const}} = -\frac{i^5}{24x^2} - \frac{i^3}{6x^2} - \frac{i}{x^2} \quad [N]$$

$f^e(i,x) =$____

c) The energy stored in a coupling field W_m as a function of i and x. State the unit for energy.

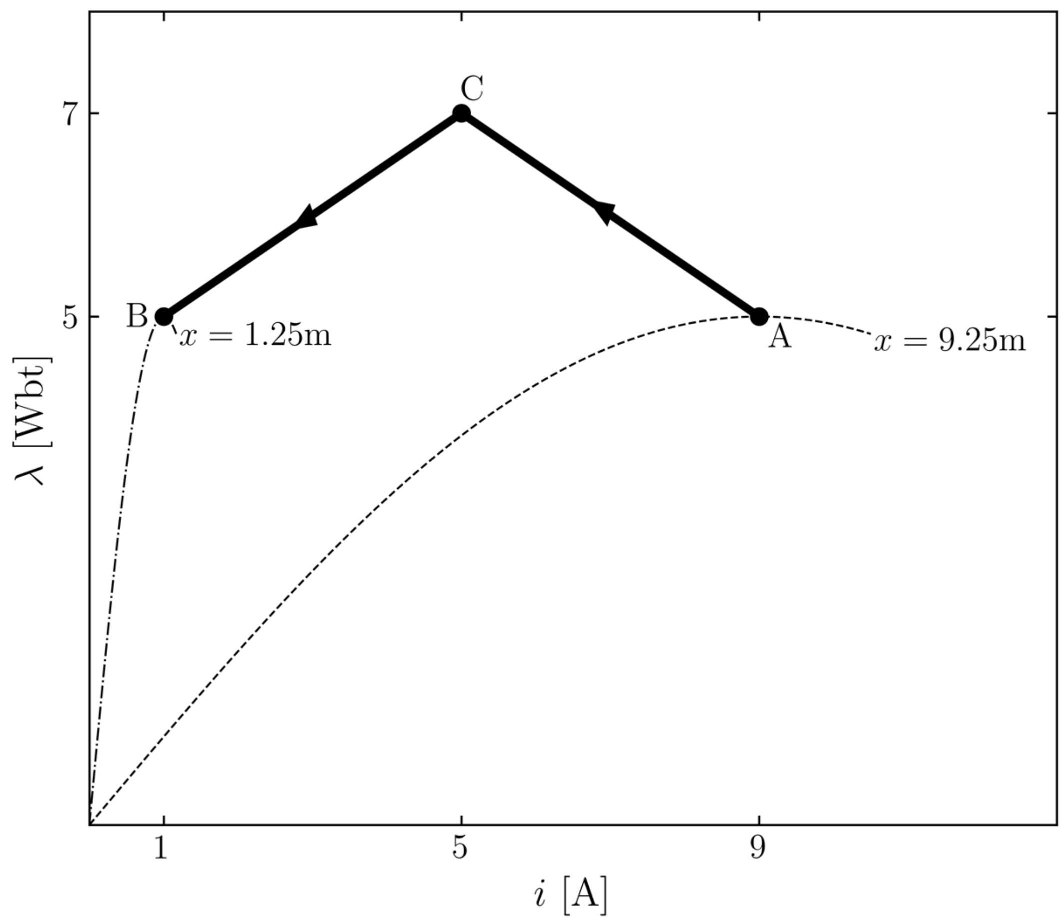
$$W_m + W_m' = i\lambda$$

$$W_m = i\lambda - W_m' = i\left(\frac{5i^4}{24x} + \frac{3i^2}{6x} + \frac{1}{x}\right) - \frac{i^5}{24x} - \frac{i^3}{6x} - \frac{i}{x} =$$

$$= \frac{4i^5}{24x} + \frac{2i^3}{6x} = \frac{i^5}{6x} + \frac{i^3}{3x} \; [J]$$

$Wm=$ $\frac{i^5}{6x} + \frac{i^3}{3x} \; [J]$

Problem 3 (25 Points)



A certain electro-mechanical system whose flux linkage is given as

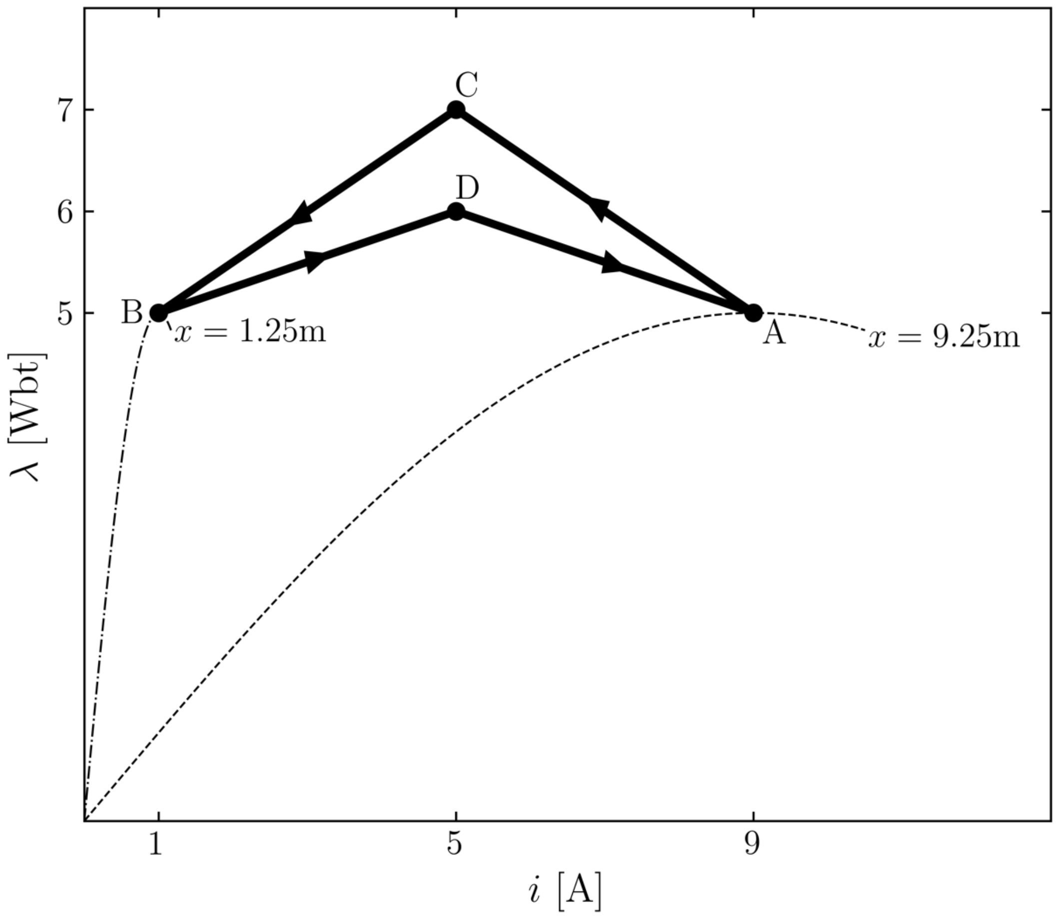
$$\lambda = 5 \sin\left(\frac{\pi i}{2(x - 0.25)}\right)$$

moves from point A (i=9 A, λ=5 Wbt) to B (i=1 A, λ=5 Wbt) through point C (i=5 A, λ=7 Wbt) along the lines shown. The values for x at points A and B are given in the figure.

a) What is the energy at points A and B?

W_{ma} =____

W_{mb} =____



c) If the system moves from point B back to point A through point D (i=9 A, λ=6 Wbt) along the lines shown, what is the EFE over this cycle? Does the system behave like a motor or a generator over this cycle?

$EFE|_{cycle}$ =____

Circle One: Motor / Generator

3. a) $\lambda = 5\sin\left(\frac{\pi i}{2(x-0.25)}\right)$

$$W_m' = \int_0^i \lambda \, di = \int_0^i 5\sin\left(\frac{\pi \hat{i}}{2(x-0.25)}\right) d\hat{i}$$

$$= 5\left(-\cos\left(\frac{\pi \hat{i}}{2(x-0.25)}\right)\right)\left(\frac{2(x-0.25)}{\pi}\right)\Bigg|_0^i$$

$$= -\frac{10(x-0.25)}{\pi}\cos\left(\frac{\pi \hat{i}}{2(x-0.25)}\right)\Bigg|_0^i$$

$$= -\frac{10(x-0.25)}{\pi}\left(\cos\left(\frac{\pi i}{2(x-0.25)}\right) - 1\right)$$

$$W_m' = \frac{10(x-0.25)}{\pi}\left[1 - \cos\left(\frac{\pi i}{2(x-0.25)}\right)\right]$$

$$W_m + W_m' = i\lambda$$

$$W_m = 5i\sin\left(\frac{\pi i}{2(x-0.25)}\right) - \frac{10(x-0.25)}{\pi}\left[1 - \cos\left(\frac{\pi i}{2(x-0.25)}\right)\right]$$

$$W_{mA} = 5(9)\sin\left(\frac{\pi(9)}{2(9.25-0.25)}\right) - \frac{10(9.25-0.25)}{\pi}\left[1 - \cos\left(\frac{\pi(9)}{2(9.25-0.25)}\right)\right]$$

$$= 45\sin\left(\frac{\pi}{2}\right) - \frac{90}{\pi}\left[1 - \cos\left(\frac{\pi}{2}\right)\right]$$

$$= 45 - \frac{90}{\pi}$$

$$W_{mB} = 5(1)\sin\left(\frac{\pi(1)}{2(1.25-0.25)}\right) - \frac{10(1.25-0.25)}{\pi}\left[1 - \cos\left(\frac{\pi(1)}{2(1.25-0.25)}\right)\right]$$

$$= 5\sin\left(\frac{\pi}{2}\right) - \frac{10}{\pi}\left[1 - \cos\left(\frac{\pi}{2}\right)\right]$$

$$= 5 - \frac{10}{\pi}$$

$$\boxed{\begin{array}{l} W_{mA} = 16.352 \text{ J} \\ W_{mB} = 1.817 \text{ J} \end{array}}$$

b) $\underset{A\to B}{EFM} = W_{mB} - W_{mA} - \underset{A\to B}{EFE}$

$$\underset{A\to B}{EFE} = \underset{A\to C}{EFE} + \underset{C\to B}{EFE}$$

$$= \frac{7-5}{2}(9+5) + \left[-\left(\frac{7-5}{2}\right)(5+1)\right]$$

$$= \frac{2}{2}(14) - \frac{2}{2}(6)$$

$$= 14-6$$

$$= 8J$$

$$\underset{A\to B}{EFM} = 1.817 - 16.352 - 8$$

$$\boxed{\underset{A\to B}{EFM} = -22.535J}$$

c) $\underset{cycle}{EFE} = \underset{A\to B}{EFE} + \underset{B\to A}{EFE}$

$$= 8 + \frac{6-5}{2}(5+1) + \left[-\frac{(6-5)}{2}(5+9)\right]$$

$$= 8 + \frac{1}{2}(6) - \frac{1}{2}(14)$$

$$= 8+3-7$$

$$\boxed{\underset{cycle}{EFE} = 4J}$$

$$\boxed{\underset{cycle}{EFE} > 0 : \text{Motor}}$$

Problem 4 (25 Points)
You are designing a novel single-phase, doubly-fed machine as a part of an electric vehicle project for a prominent automotive manufacturer based in Michigan.

The machine was measured to have the following mutual inductance characteristic

$$\lambda_1 = 3i_1 + 2i_2 \sin\theta\,, \qquad \lambda_2 = 2i_1 \sin\theta + 4i_2,$$

where "1" denotes the stator of the machine, and "2" denotes the rotor.

The machine needs to pass a maximum-speed open-circuit test. For this test, the stator is held at open circuit, the rotor is set to carry a fixed DC current of $i_2(t) = 2$ amperes, and the machine is driven at its redline speed of 10,000 rpm (i.e. the angular frequency of the machine is $2\pi \times 10^4$ rad/s).

Please justify your answers below. Correct answers without justification will receive no credit.

a) What is the maximum open circuit voltage seen at the stator terminals?

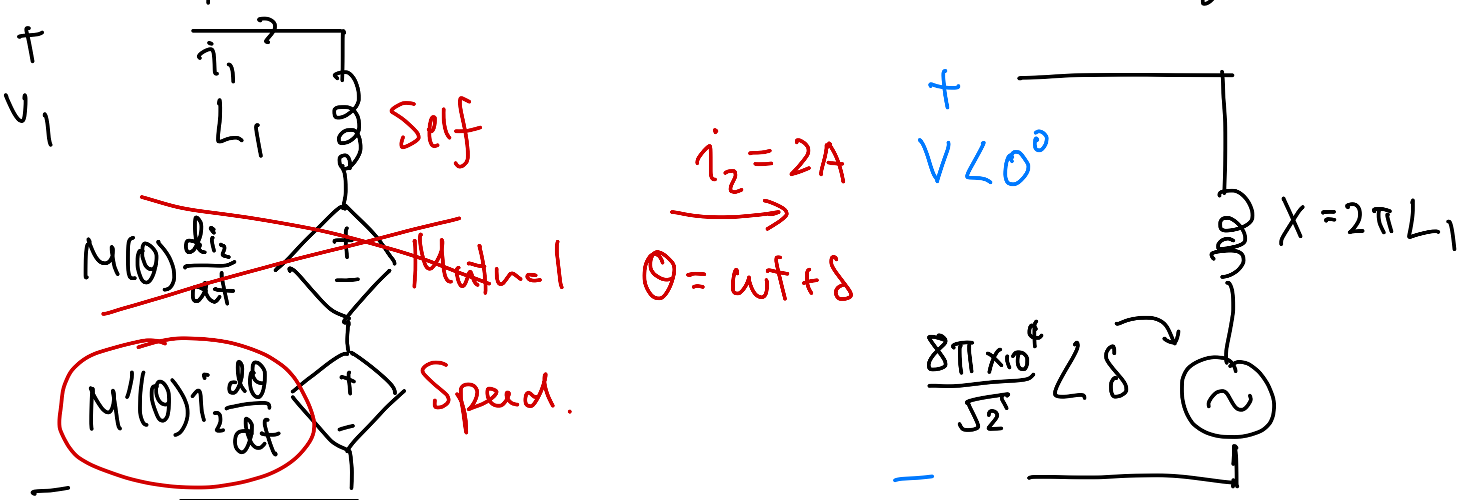
$$v_1 = \frac{d\lambda_1}{dt} = 3\frac{di_1}{dt} + 2\sin\theta\frac{di_2}{dt} + 2i_2\cos\theta\frac{d\theta}{dt}$$

Here: $i_1 = 0$ (open circuit)

$i_2 = 2$ A (provided)

$\omega = 2\pi \times 10^4$ (provided)

$v_1 = (2)(2)(2\pi\times10^4)\cos\theta$ at angle θ



$v_{oc} =$ $V_{1\,max} = 8\pi\times10^4$ V   maximized at $\theta = 0$ $(\cos\theta = 1)$

b) What is the maximum magnitude of the torque developed by the magnetic field of the machine?

Electrically linear: $L_1(\theta) = 3, \; M(\theta) = 2\sin\theta, \; L_2(\theta) = 4$

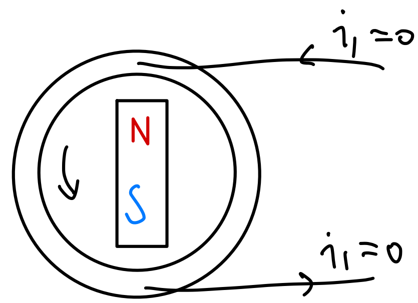
$$W_m'(i_1, i_2, \theta) = \frac{1}{2}\lambda_1(i_1, i_2, \theta)\, i_1 + \frac{1}{2}\lambda_2(i_1, i_2, \theta)\, i_2$$

$$= \frac{3}{2} i_1^2 + 2 i_2 i_1 \sin\theta + 2 i_2^2$$

$$\tau = + \frac{\delta W_m'}{\delta\theta} = 2 i_1 i_2 \cos\theta$$ but $i_1 = 0$ (open circuit)

$\tau = 0$ Nm $\rightarrow \tau_{max} = 0$ Nm.

Powerful magnet
no change in airgap.



$T^e_{\max} =$ 0 Nm

c) How much energy is stored in the magnetic field of the machine at $\theta = 0°$?

$W_m = W_m'$ (numerically)

$$W_m' = \cancel{\frac{3}{2} i_1^2} + \cancel{2 i_1 i_2 \sin\theta} + 2 i_2^2 = 2 i_2^2$$

($i_1 = 0$ open circuit)

$i_2 = 2$ A
(provided)

$$W_m(i_1 = 0, i_2 = 2A, \theta) = 8 J$$

$W_m =$ 8 J