ECE 330 Exam 1: Fall 2021 NAME______________________________
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Section (Check one) MWF 10am_________ MWF 12pm________ MWF 2pm__________

1. __________/25 2. __________/25

3. __________/25 4. __________/25 TOTAL _________/100

USEFUL INFORMATION

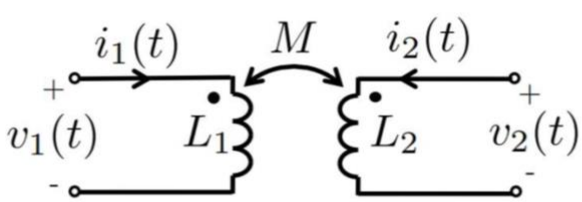
sin(x)=cos(x-90°)	$\bar{V} = \bar{Z}\bar{I}\bar{S} = \bar{V}\bar{I}^{*} = P + jQ$	$\bar{S}_{3\varphi} = \sqrt{3}V_L I_L \angle\theta$
$0<\theta<180°$ (lag)	$I_L = \sqrt{3}I_\varphi$ (delta)	$\bar{Z}_Y = \bar{Z}_\Delta/3$
$-180°<\theta<0$ (lead)	$V_L = \sqrt{3}V_\varphi$ (wye)	$\mu_0 = 4\pi \times 10^{-7}$H/m

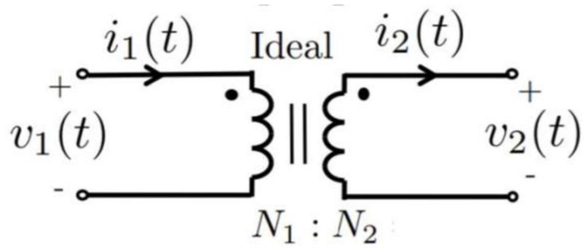
ABC phase sequence has A at 0, B at -120°, and C at +120°

$\int \underline{H} \cdot \underline{dl} = \int \underline{J_f} \cdot \hat{n} dA$ $\int \underline{E} \cdot \underline{dl} = -\frac{d}{dt}\left(\int \underline{B} \cdot \hat{n} dA\right)$ $\mathcal{R} = \frac{l}{\mu A}$ $Ni = \mathcal{R}\varphi$

$\varphi = BA$ $\lambda = N\varphi = Li$ (if linear) $v = \frac{d\lambda}{dt}$ $k = \frac{M}{\sqrt{L_1 L_2}}$

1hp=746 W



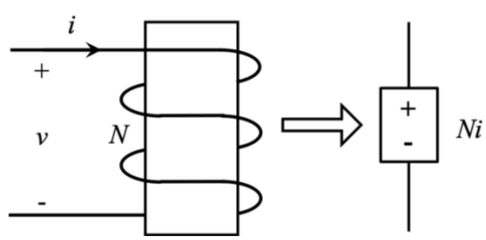


$v_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$

$v_2 = M\frac{di_1}{dt} + L_2\frac{di_2}{dt}$

$a = \frac{N_1}{N_2}$ $N_1 i_1 = N_2 i_2$

$\frac{V_1}{V_2} = \frac{N_1}{N_2}$



<u>Problem 1 (25 points)</u>

$\bar{Z}_f = 0.22\angle 63.4^\circ\ \Omega$

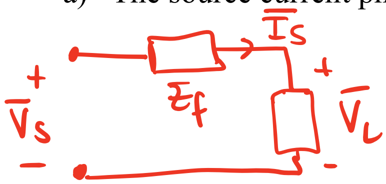
$S = \frac{P}{PF} = \frac{10k}{0.8} = 12.5\,kVA$

$\theta = \cos^{-1}(0.8) = 36.9^\circ$

A feeder with an impedance of 0.1+j0.2 Ω supplies a single-phase 10 kW 0.8 lagging power factor load. The voltage across the load is $v_L(t) = \sqrt{2}\,\sin(377t + 40^\circ)$ V. Calculate:

a) The source current phasor $\overline{I}_s$

$= \sqrt{2}\cos(377t \underbrace{-90^\circ + 40^\circ}_{-50^\circ}) \Rightarrow \bar{V}_L = \frac{\sqrt{2}}{\sqrt{2}}\angle -50^\circ = 1\angle -50^\circ\ V$



$\bar{S}_L = \bar{V}_L \cdot \bar{I}_s^* \Rightarrow \bar{I}_s = \left(\frac{\bar{S}_L}{\bar{V}_L}\right)^* = \left(\frac{12.5k\angle 36.9^\circ}{1\angle -50^\circ}\right)^* = 12.5k\angle -86.9^\circ\ A$

$\overline{I}_s$= $12.5\angle -86.9^\circ$ kA

b) The source (sending end) voltage phasor $\overline{V}_s$

$\bar{V}_s = \bar{V}_f + \bar{V}_L = \underbrace{\bar{I}_s \cdot \bar{Z}_f}_{} + \underbrace{\bar{V}_L}_{0.64 - j0.76\ V} = 2521.64 - j1096.76 = 2750\angle -23.26^\circ\ V$

$12.5k\angle -86.9^\circ \cdot 0.22\angle 63.4^\circ =$

$= 2750\angle -23.5^\circ\ V =$

$= 2521 - j1096\ V$

$\overline{V}_s$= $2750\angle -23.26^\circ$ V

c) The total complex power supplied by the source, $\overline{S_t}$

$$\overline{S}_t = \overline{V}_s \cdot \overline{I}_s^* = 2750 \angle -23.26^\circ \cdot 12.5k \angle 86.9^\circ = 34 \angle 63.64^\circ \text{ MVA}$$

$\overline{S_t}$= $34 \angle 63.64^\circ$ MVA

d) The power factor at the source (sending end). Specify whether it is lagging or leading.

$$PF = \cos(63.4^\circ) = 0.44$$

PF= 0.44

Circle one: Lagging / Leading

Problem 2 (25 points)

Two 3-phase loads are connected in parallel to a wye-connected source of 208V line to line. The loads are given as:

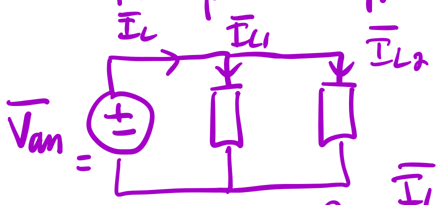
$\nearrow I_L$

Load 1: wye-connected, 10 A (line current), 3200 W (3-phase), lagging power factor
Load 2: delta-connected, 8 A (phase current), 0.9 power factor lagging.

$\searrow I_\phi$

a) What is the total line current $\overline{I_L}$ being supplied by the source?

Per-phase eq. circuit:



$= \frac{208}{\sqrt{3}}\angle 0° \, V \quad P = \frac{3200}{3} = 1066 W$

$\nearrow 12.5 - j6 A$

$I_{L2} = \sqrt{3}\, I_\phi = 8\sqrt{3} A \; ; \; \overline{I_{L2}} = 8\sqrt{3} \angle -25.8° A$

$PF = 0.9 \text{ lag} \Rightarrow \theta = \cos^{-1}(0.9) = 25.8° \Rightarrow \theta_i = -25.8°$

$\overline{I_{L1}} : \; I_{L1} = 10 A \; ; \; P = V \cdot I \cdot \underbrace{\cos\theta}_{PF} \Rightarrow PF = \frac{P}{V \cdot I} = \frac{1066}{120 \cdot 10} = 0.88$

$\theta = \cos^{-1}(0.88) = 28.4°$

$\overline{I_{L1}} = 10 \angle -28.4° A =$

$= 8.8 - j4.8 A$

$\overline{I_L}$= $\overline{I_{L1}} + \overline{I_{L2}} = 21.3 - j10.8 = 23.8 \angle -26.5° A$

b) How many capacitive VARs PER-PHASE needed to bring overall power factor to unity? Justify your answer. Answers without justification will receive no credit.

$\overline{S_{t,1\phi}} = \overline{V_{an}} \cdot \overline{I_L}^{*} = \frac{208}{\sqrt{3}} \angle 0° \cdot 23.8 \angle 26.5° = 2856 \angle 26.5° =$

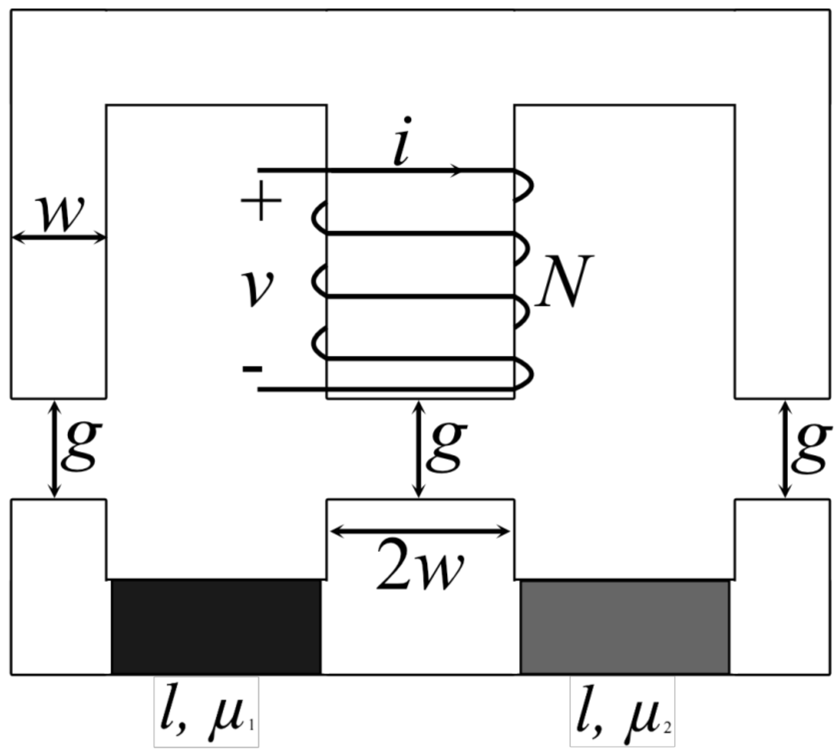
$= \underbrace{2556}_{P} + j\underbrace{1274}_{Q} \; VA$

$PF = 1 \rightarrow \theta_v = \theta_i \rightarrow Q = 0 \rightarrow$ add 1274 VARs of capacitance per-phase

Q_c= 1274 VARs of capacitance per-phase

Reason:

Problem 3 (25 points)



An iron core with infinite permeability, width *w*=2 cm, air gap length *g*=3 mm, and depth into the page *d*=3 cm has a coil wrapped around it as given above with *N*=250. The iron core also has two inserts in it with length *l*=10 cm and permeabilities $\mu_1 = 2500\mu_0$ and $\mu_2 = 1250\mu_0$. Fringing effects can be neglected.

a) Draw the magnetic equivalent circuit for this system. (8 points)

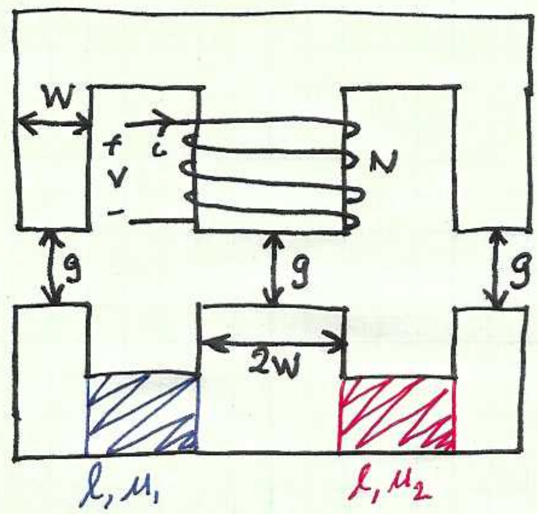
b) What is the flux through the coil? (12 points)

ϕ=____

c) What is the voltage drop across the coil in terms of the unknown current i? (5 points)

v=____

3)



$d = 3$ cm

$W = 2$ cm

$l = 10$ cm

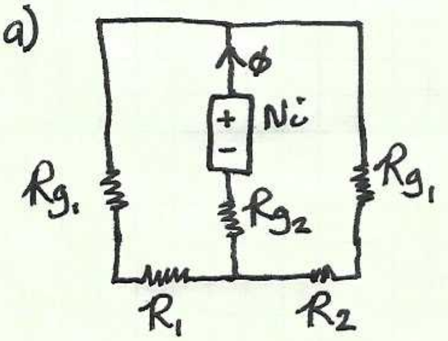
$g = 3$ mm

$\mu_1 = 2500\mu_0$

$\mu_2 = 1250\mu_0$

$N = 250$

a)



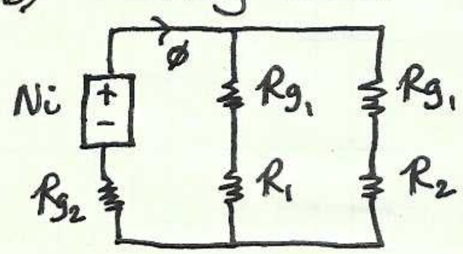
$$R_{g_1} = \frac{3\text{mm}}{\mu_0(6\text{cm}^2)} = 3.979\times10^6 \text{ At/Wb}$$

$$R_{g_2} = \frac{3\text{mm}}{\mu_0(12\text{cm}^2)} = 1.989\times10^6 \text{ At/Wb}$$

$$R_1 = \frac{10\text{cm}}{2500\mu_0(6\text{cm}^2)} = 5.305\times10^4 \text{ At/Wb}$$

$$R_2 = \frac{10\text{cm}}{1250\mu_0(6\text{cm}^2)} = 1.061\times10^5 \text{ At/Wb}$$

b) Redrawing circuit



$$R_{eq} = R_{g_2} + \left(\frac{1}{R_{g_1}+R_1} + \frac{1}{R_{g_1}+R_2}\right)^{-1}$$

$$R_{eq} = 4.019\times10^6 \text{ At/Wb}$$

$$\phi = \frac{Ni}{Rq} \Rightarrow \boxed{\phi = 6.221\times10^{-5}\, i \text{ Wb}}$$

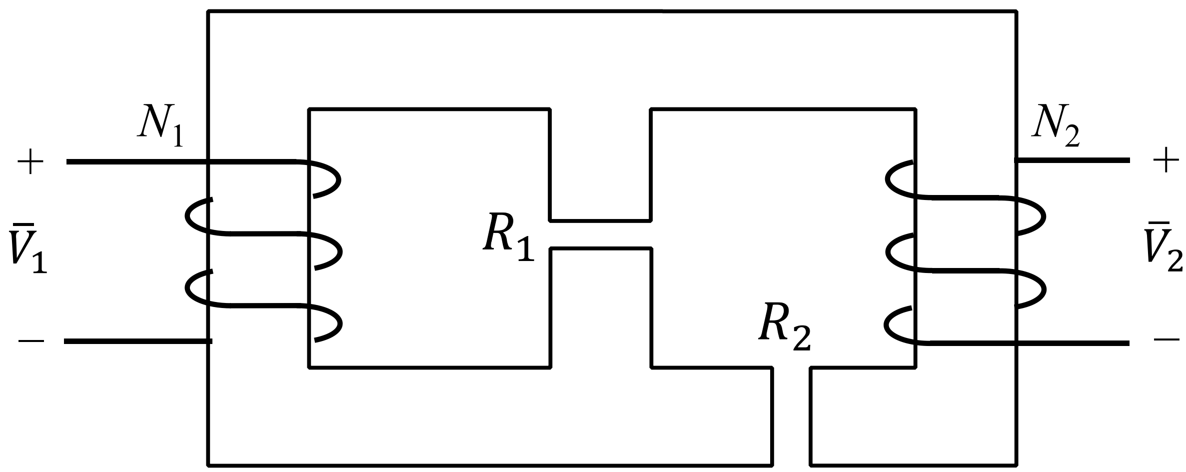
c) $V = \frac{d\lambda}{dt}$

$$\boxed{V = 0.0156\frac{di}{dt} \text{ V}}$$

$\lambda = N\phi \Rightarrow \lambda = 0.0156i$

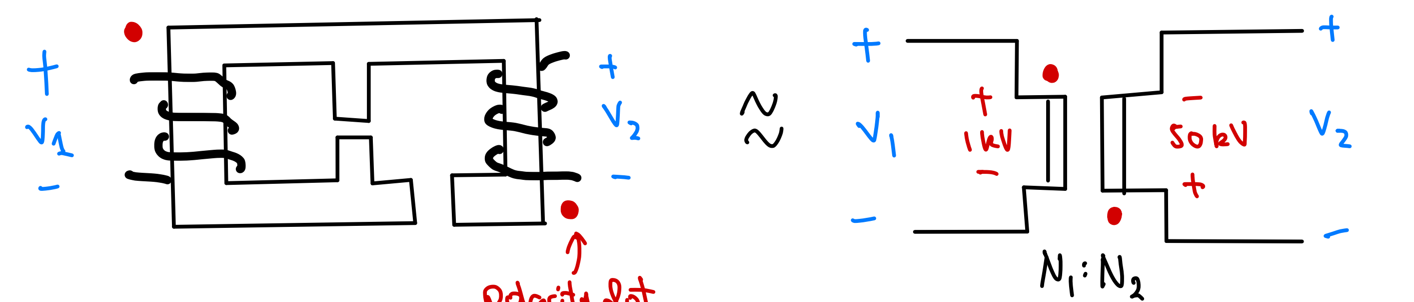
Problem 4 (25 Points)

You are consulting for a wind farm company who is looking to install a high voltage transformer to minimize their transmission loss. A schematic of the transformer is shown below.



The first coil has $N_1 = 10$ turns while the second coil has $N_2 = 500$ turns. The transformer contains two air gaps, with reluctances $R_1 = 50$ At/Wb and $R_2 = 1000$ At/Wb as labeled. You may treat the permeability of the magnetic core as infinite.

a) Assuming for now that the transformer is ideal, what would the output voltage $\bar{V}_2$ be (at open circuit) with an input voltage $\bar{V}_1 = 1\angle 0°$ kV (rms) at 60 Hz? (Please specify both the phase and the magnitude of the output voltage $\bar{V}_2$, and briefly justify your answer.)



Polarity dot

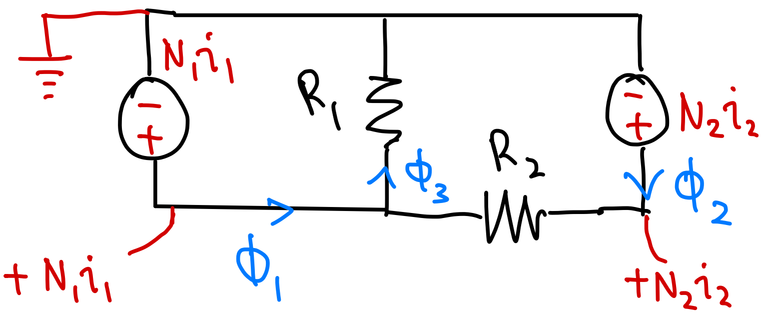
$\bar{V}_2 = -\frac{N_2}{N_1}\bar{V}_1 = -\frac{500}{10}\, 1\angle 0^\circ \text{ kV} = 50\angle 180^\circ \text{ kV} \quad (-50\angle 0^\circ \text{ kV})$

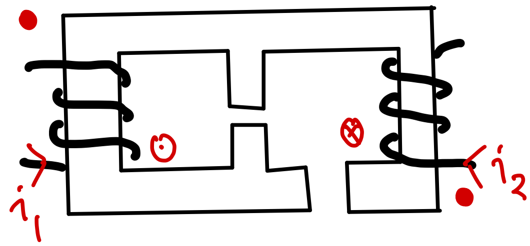
Reason: If ideal transformer, $V_1 : V_2$ determined purely by $N_1 : N_2$. But because way coils are wound, expect 180° phase shift.

Have to label current yourself.

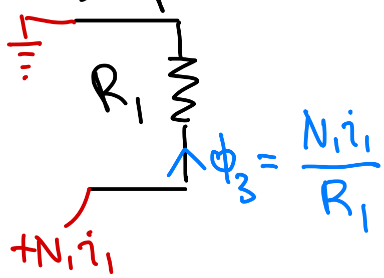
Doesn't matter, so pick easier choice.

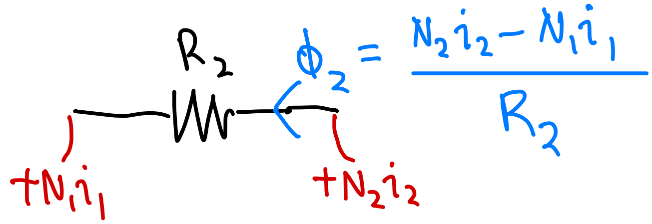
b) Compute the self inductances L1, L2, and the mutual inductance M of the two coils.





Close-ups:





$$\phi_2 = \frac{N_2 i_2 - N_1 i_1}{R_2}, \Rightarrow \lambda_2 = \overbrace{\boxed{\frac{N_2^2}{R_2}}}^{L_2 \geq 0} i_2 - \overbrace{\boxed{\frac{N_1 N_2}{R_2}}}^{M \geq 0} i_1$$

$$\phi_1 + \phi_2 = \phi_3,$$

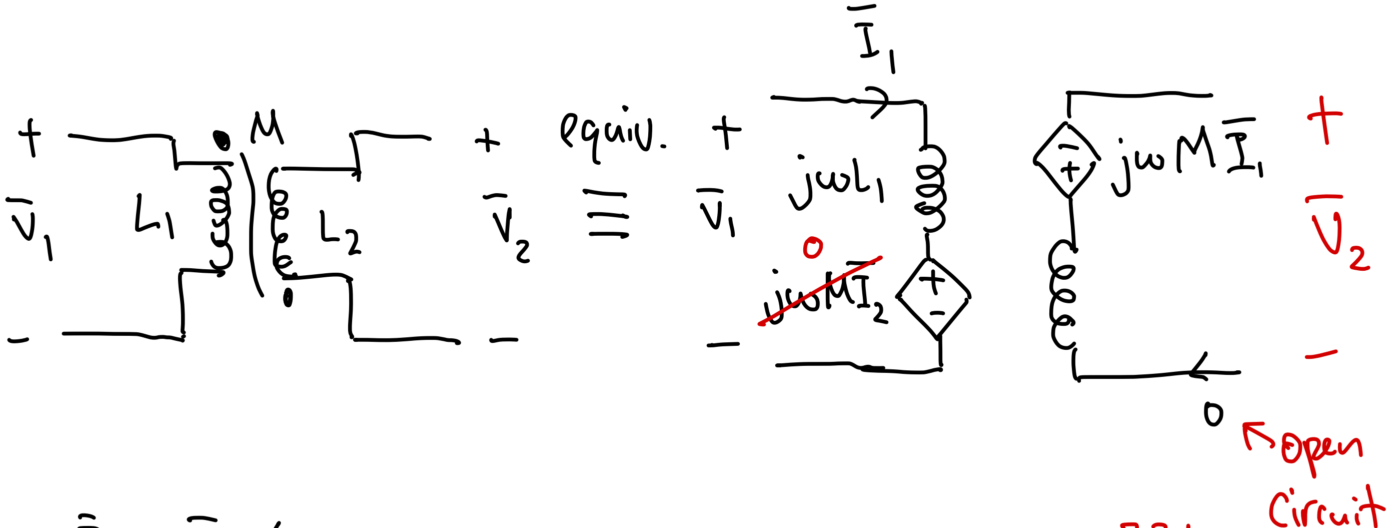
$$\phi_1 + \frac{N_2 i_2 - N_1 i_1}{R_2} = \frac{N_1 i_1}{R_1}, \Rightarrow \lambda_1 = \underbrace{\boxed{N_1^2\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}}_{L_1 \geq 0} i_1 - \boxed{\frac{N_1 N_2}{R_2}} i_2$$

L_1 = $N_1^2 (R_1^{-1} + R_2^{-1}) = 2.1$ H

L_2 = $N_2^2 / R_2 = 250$ H

M = $N_1 N_2 / R_2 = 5$ H

c) Use the self and mutual inductance values you have just computed above as a more realistic model of the transformer. What would the output voltage $\bar{V}_2$ be (at open circuit) with an input voltage $\bar{V}_1 = 1\angle 0°$ kV (rms) at 60 Hz? (Please specify both the phase and the magnitude of the output voltage $\bar{V}_2$, and briefly justify your answer.)



$$\bar{I}_1 = \bar{V}_1 / j\omega L_1$$

$$\bar{V}_2 = -j\omega M \bar{I}_1 = -\frac{\cancel{j\omega} M}{\cancel{j\omega} L_1}\bar{V}_1 = \overset{2.381}{\frac{5}{2.1}} \angle 180° \text{ kV}$$

$\bar{V}_2$= 2.381 $\angle 180°$ kV

Reason: $i_2 = 0$ because of open circuit, so $M\frac{di_2}{dt} = 0$, and i_1 is entirely determined by L_1. Once known, $V_2 = M\frac{di_1}{dt}$.

Finally, adjust for phasing: 180° due to polarity dots.