ECE 330 Exam #1, Spring 2015 Name: Solutions
90 Minutes

Section (Check One) MWF 10am _______ TTh 9:30am _______

1. ________ / 25 2. ________ / 25

3. ________ / 25 4. ________ / 25 Total ________ / 100

Useful information

$\sin(x) = \cos(x - 90^\circ)$ $\quad \bar{V} = \bar{Z}\bar{I}$ $\quad \bar{S} = \bar{V}\bar{I}^*$ $\quad \bar{S}_{3\phi} = \sqrt{3} V_L I_L \angle\theta$

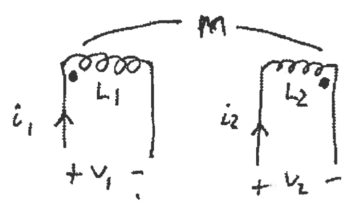
$0 < \theta < 180^\circ$ (lag)
$-180^\circ < \theta < 0$ (lead)

$I_L = \sqrt{3} I_\phi$ (delta)
$V_L = \sqrt{3} V_\phi$ (wye)

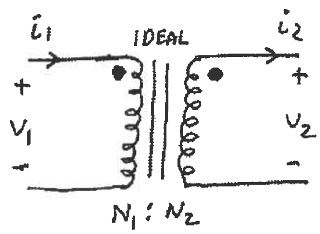
$\bar{Z}_Y = \bar{Z}_\Delta / 3$ $\quad \mu_0 = 4\pi \cdot 10^{-7}$ H/m

$\int_C \mathbf{H} \cdot \mathbf{dl} = \int_S \mathbf{J} \cdot \mathbf{n} da$ $\quad \int_C \mathbf{E} \cdot \mathbf{dl} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot \mathbf{n} da$ $\quad \Re = \frac{l}{\mu A}$ $\quad MMF = Ni = \phi\Re$

$\phi = BA$ $\quad \lambda = N\phi$ $\quad v = d\lambda/dt$ $\quad k = \frac{M}{\sqrt{L_1 L_2}}$ $\quad$ 1 hp = 746 Watts

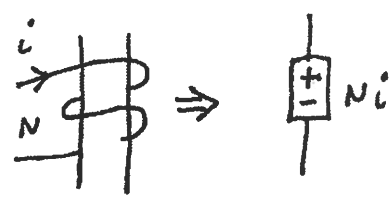


$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$



$a = \frac{N_1}{N_2}$ $\quad N_1 i_1 = N_2 i_2$

$\frac{V_1}{V_2} = \frac{N_1}{N_2}$



Problem 1. (25 points)

Three single-phase loads are connected in parallel across a 60Hz source of 240 Volts (RMS).

Load #1: 2.4 kW at 0.8 power factor lagging
Load #2: 20 Amps (RMS) with 2 kVAR reactive power
Load #3: Two resistors connected in series, each being 10 Ω

a) Find the complex power consumed by each load.
b) Find the total complex power consumed and the total current RMS magnitude.
c) Find the value of capacitive VARs that should be added in parallel to these three loads to make the overall power factor to be unity.

a) Load 1

$$\bar{S}_1 = \frac{2.4k}{0.8} \angle \cos^{-1}(0.8) = 3000 \angle 36.9^\circ \text{ VA}$$

Load 2

$$|\bar{S}_2| = 240 \times 20 = 4800 \text{ VA}$$

$$\bar{S}_2 = 4800 \angle \sin^{-1}\left(\frac{2000}{4800}\right) = 4800 \angle 24.6^\circ \text{ VA}$$

Load 3

$$\bar{S}_3 = \bar{V}\bar{I}^* = \frac{V^2}{\bar{Z}^*} = \frac{240^2}{10 \times 2} = 2880 \angle 0^\circ \text{ VA}$$

b) $\bar{S}_1 + \bar{S}_2 + \bar{S}_3 = 10365 \angle 21.5^\circ$ VA

$$|\bar{I}| = \frac{|\bar{S}|}{|\bar{V}|} = \frac{10365}{240} = 43.2A$$

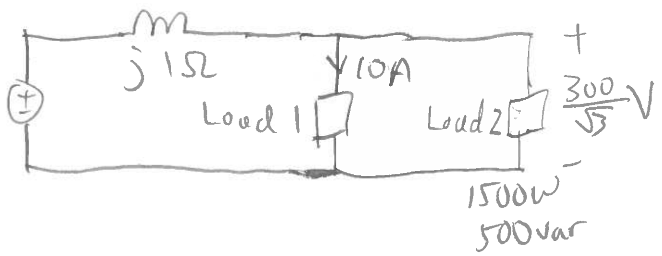
c) $\bar{Q} = 10365 \times \sin 21.5^\circ = 3799$ var

Problem 2. (25 points)

A balanced, 60 Hz, 3 phase source supplies power to two loads in parallel through a feeder with impedance j1 Ω. The first load is 4-wire, Wye connected and draws 10 Amps line current with a power factor of 0.8 lagging. The second load is 3-wire Delta connected and consumes 1500 W and 500 var. The voltage at the load is 300V.

a) Draw the per-phase equivalent circuit with all given voltages, currents, powers, and impedances labeled.
b) What is the voltage at the source?
c) What is the complex power delivered by the source?
d) What is the per-phase impedance of the Delta connected load?

a)



b) $\bar{I}_1 = 10 \angle -\cos^{-1} 0.8 = 10 \angle -36.9° \text{ A}$

$\bar{S}_2 = 3 \bar{V} \bar{I}_2^*$

$\bar{I}_2 = \left(\frac{\bar{S}_2}{3\bar{V}}\right)^* = \left(\frac{1500 + j500}{3 \times \frac{300}{\sqrt{3}}}\right)^* = 3.04 \angle -18.4° \text{ A}$

$\bar{I}_{source} = \bar{I}_1 + \bar{I}_2 = 12.9 \angle -32.6° \text{ A}$

$\bar{V}_{source} = \frac{300}{\sqrt{3}} + 12.9 \angle -32.6 \times j1 = 180.5 \angle 3.45° \text{ V}$

c) $\bar{S}_{source} = 3 \bar{V} \bar{I}^* = 3 \times 180.5 \angle 3.45° \times 12.9 \angle +32.6°$

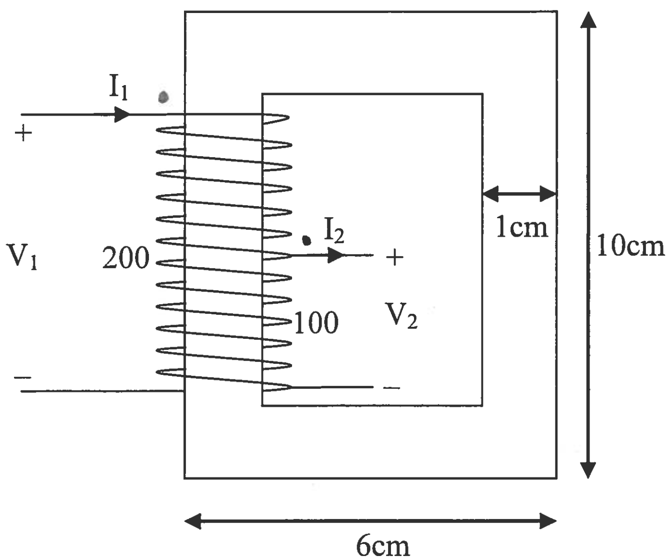
$= 6985 \angle 36.05° \text{ VA}$

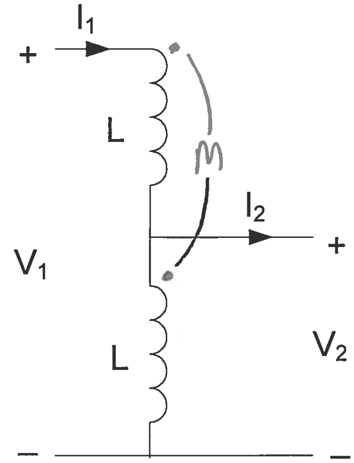
d) $\bar{Z}_{2,Y} = \frac{\bar{V}_2}{\bar{I}_2} = \frac{\frac{300}{\sqrt{3}}}{3.04 \angle -18.4°} = 56.98 \angle 18.4° \ \Omega$

$\bar{Z}_{2,\Delta} = 3 \bar{Z}_{2,Y} = 171 \angle 18.4° \ \Omega$

Problem 3. (25 points)

An autotransformer is a transformer with only one winding. Portions of the same winding act as both the primary and secondary sides of the transformer, as shown in the figure below. The primary side has 200 turns, and the secondary terminals are created by a tap at 100 turns. The electric circuit for an autotransformer is also shown. The inductances of both halves of the coil are identical since they both contain 100 turns. The coefficient of coupling k is 0.95, μ_r=1000 for the core, and the depth of the core is 1cm.





a) Place dots on the windings for both diagrams. Label the mutual inductance M in the circuit diagram.
b) Calculate the inductance L.
c) Find the mutual inductance M.
d) Write the KVL loop equations for the electric circuit in time domain. Do not substitute for L and M.
e) Write the KVL loop equations for the electric circuit in phasor domain, assuming we are using this transformer on a 60Hz system. Do not substitute for L and M.

b) $\lambda = N\phi = Li$

$$L = \frac{N\phi}{i} = \frac{N \times \frac{Ni}{\mathcal{R}}}{i} = \frac{N^2}{\mathcal{R}} = \frac{100^2}{2.23E6} = 4.48\,mH$$

$$\mathcal{R} = \frac{l}{\mu A} = \frac{(0.09+0.05)\times 2}{1000\mu_0 \times 0.01^2} = 2.23E6\ \frac{At}{wb}$$

Blank page provided next for work

Blank page for work

c) $k = \frac{M}{\sqrt{L \times L}} = \frac{M}{L}$

$M = 0.95 \times 4.48 = 4.26\ \text{mH}$

d) $v_1 = L\frac{di_1}{dt} + M\frac{d(i_1 - i_2)}{dt} + L\frac{d(i_1 - i_2)}{dt} + M\frac{di_1}{dt}$

$v_1 = (2L + 2M)\frac{di_1}{dt} - (L + M)\frac{di_2}{dt}$

$v_2 = L\frac{d(i_1 - i_2)}{dt} + M\frac{di_1}{dt}$

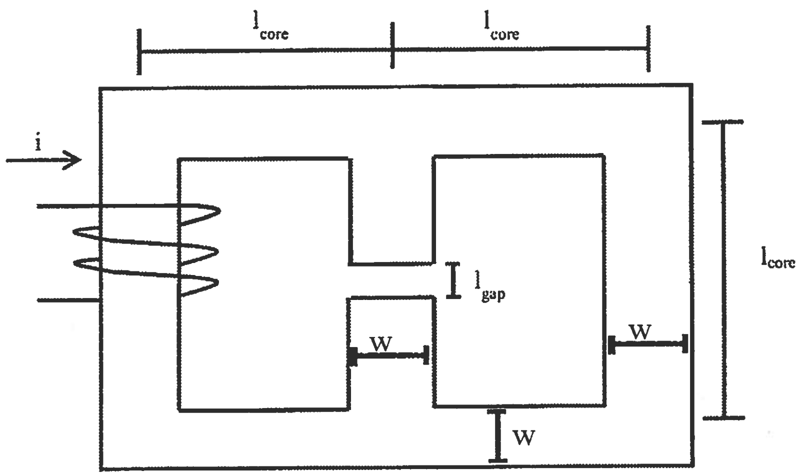
$v_2 = (L + M)\frac{di_1}{dt} - L\frac{di_2}{dt}$

e) $\bar{V}_1 = 2j\omega(L+M)\bar{I}_1 - j\omega(L+M)\bar{I}_2$

$\bar{V}_2 = j\omega(L+M)\bar{I}_1 - j\omega L\bar{I}_2$

Problem 4. (25 points)

a) Consider the inductor below. The dimensions are: l_{core}=10cm, l_{gap}=1mm, w=1cm. The current i=10A. The depth of the core is 1cm, $\mu_{core}=1000\mu_0$, and the coil has 100 turns. Solve for the flux ϕ in the center leg of the core. Do not neglect fringing.

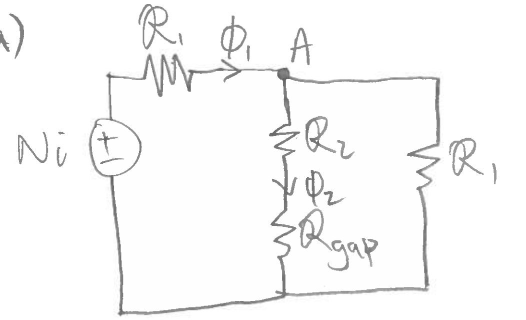


b) An ideal transformer has a winding ratio of 2000:200. The primary (high voltage) side is connected to 120V. The secondary side serves a load with impedance 1+j0.5 Ω.

i. Find the current on the primary winding.

ii. What does the load impedance look like on the primary side?

a)



$Ni = 100 \times 10 = 1000$ At

$R_1 = \frac{l}{\mu A} = \frac{0.3}{1000\mu_0 \times 0.01^2} = 2.39\text{E}6 \frac{At}{Wb}$

$R_2 = \frac{0.1 - 0.001}{1000\mu_0 \times 0.01^2} = 7.88\text{E}5 \frac{At}{Wb}$

$R_{gap} = \frac{0.001}{\mu_0 \times (0.01 + 0.001)^2} = 6.58\text{E}6 \frac{At}{Wb}$

$R_{eq} = R_1 + R_1 // (R_2 + R_{gap})$

$= 4.19\text{E}6 \frac{At}{Wb}$

$\Phi_1 = \frac{1000}{R_{eq}} = 2.39\text{E-}4$ Wb

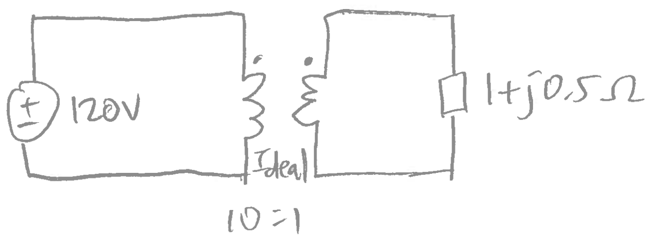
Blank page provided next for work

Blank page for work

$mmf_A = 1000 - \mathcal{R}_1 \Phi_1 = 428.8 \text{ At}$

$$\Phi_2 = \frac{mmf_A}{\mathcal{R}_2 + \mathcal{R}_{gap}} = \frac{428.8}{7.88E5 + 6.58E6} = 5.82E{-5} \text{ Wb}$$

b) i)



$\bar{V}_2 = 12V$

$$\bar{I}_2 = \frac{12}{1+j0.5} = 10.7\angle{-26.6°}$$

$I_1 = 1.07 \text{ A}$

ii) $\bar{Z}_1 = a^2 \bar{Z}_2 = 100 + j50 \ \Omega$