ECE430
Spring 2006
Exam 2
April 12, 2006

Name: KEY

Section (C for Kimball MWF, F for Tate TR)_____________

1:	____________
2:	____________
3:	____________
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Total:	________

Equations:

$$\bar{S}_{1\phi} = \bar{V}\bar{I}^* = \frac{|\bar{V}|^2}{\bar{Z}^*} = |\bar{I}|^2\,\bar{Z}$$

$$\bar{S}_{3\phi} = 3\bar{V}_\phi \bar{I}_\phi^* = \sqrt{3}V_L I_L \measuredangle\theta$$

$$P_{3\phi} = \sqrt{3}V_L I_L \cos\theta$$

$$Q_{3\phi} = \sqrt{3}V_L I_L \sin\theta$$

$$pf = \cos\left(\measuredangle\bar{V} - \measuredangle\bar{I}\right)$$

$$\theta > 0 \rightarrow lagging,\ \theta < 0 \rightarrow leading$$

$$P^2 + Q^2 = S^2$$

$$X_c = -\frac{1}{\omega C}$$

$$X_L = \omega L$$

$$wye, abc\ sequence: \bar{V}_L = \bar{V}_\phi\left(\sqrt{3}\measuredangle 30°\right), \bar{I}_\phi = \bar{I}_L$$

$$delta, abc\ sequence: \bar{V}_\phi = \bar{V}_L, \bar{I}_L = \bar{I}_\phi\left(\sqrt{3}\measuredangle -30°\right)$$

$$\bar{Z}_\Delta = 3\bar{Z}_Y$$

$$\bar{Z}_1 \,||\, \bar{Z}_2 = \left(\bar{Z}_1^{-1} + \bar{Z}_2^{-1}\right)^{-1}$$

$$\Re = \frac{l}{\mu A}$$

$$\mu_0 = 4\pi \times 10^{-7}\ H/m$$

$$\lambda = N\Phi = Li$$

$$L = N^2\wp = \frac{N^2}{\Re}$$

$$mmf\ (source) = Ni$$

$$mmf\ (drop) = \Phi\Re$$

$$\sum mmf = 0\ around\ loop$$

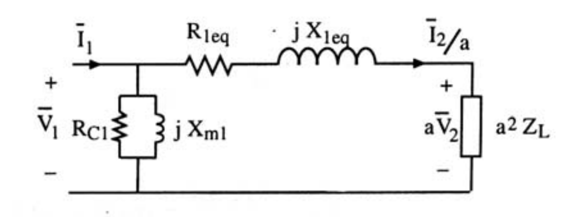
$$\oint \vec{H}\bullet d\vec{l} = \int \vec{J}\bullet\hat{n}\,da$$

$$\oint \vec{E}\bullet d\vec{l} = -\frac{\partial}{\partial t}\int \vec{B}\bullet\hat{n}\,da$$

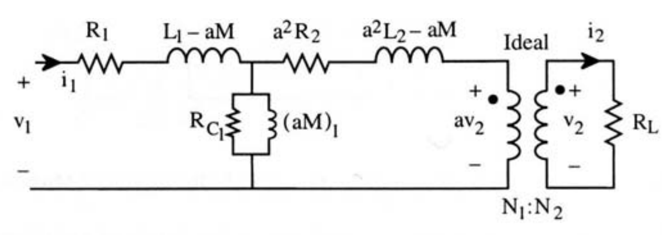
$$\oint \vec{B}\bullet\hat{n}\,da = 0$$

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

$$v = \frac{d\lambda}{dt}$$



Transformer Approximate Equivalent Circuit



Transformer Equivalent Circuit

$$W_m = \int_0^{\lambda} i \, d\hat{\lambda}$$

$$W_m{}' = \int_0^{i} \lambda \, d\hat{i}$$

$$T^e = \frac{\partial W_m{}'}{\partial \theta} = -\frac{\partial W_m}{\partial \theta}$$

$$f^e = \frac{\partial W_m{}'}{\partial x} = -\frac{\partial W_m}{\partial x}$$

$$\underset{a \to b}{EFE} = \int_a^b i \, d\lambda$$

$$\underset{a \to b}{EFM} = -\int_a^b f^e dx$$

For $\dot{x}_1 = f_1(x_1, x_2)$ and $\dot{x}_2 = f_2(x_1, x_2)$,

$$\begin{bmatrix} \Delta\dot{x}_1 \\ \Delta\dot{x}_2 \end{bmatrix} \approx \begin{bmatrix} \left.\frac{\partial f_1}{\partial x_1}\right|_{x=x^e} & \left.\frac{\partial f_1}{\partial x_2}\right|_{x=x^e} \\ \left.\frac{\partial f_2}{\partial x_1}\right|_{x=x^e} & \left.\frac{\partial f_2}{\partial x_2}\right|_{x=x^e} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix}$$

$$x(t_0 + \Delta t) \approx x(t_0) + \Delta t \cdot \left.\frac{dx}{dt}\right|_{t=t_0}$$

For $\underline{\dot{x}} = \underline{A}\underline{x}$, the eigenvalues λ of the system are given by $\left|\lambda \underline{I} - \underline{A}\right| = 0$

Problem 1 (25 points)

In class, when discussing rotational systems, we started with an inductance that was a square wave, then approximated it with a sinusoid. A better approximation is to include harmonics. For example, consider a system with the following λ-i characteristic:

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_r \end{bmatrix} = \begin{bmatrix} L_s & 0 & M\left(\cos\theta - 0.1\cos(3\theta)\right) \\ 0 & L_s & M\left(\sin\theta + 0.1\sin(3\theta)\right) \\ M\left(\cos\theta - 0.1\cos(3\theta)\right) & M\left(\sin\theta + 0.1\sin(3\theta)\right) & L_r \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_r \end{bmatrix}$$

a) Find the co-energy W_m' (10 points)

b) Find the torque of electric origin T^e (10 points)

c) Suppose $i_a = i_b = i_r = 1$ A, $L_s = L_r = 1$ H, $M = 0.9$ H, and $\theta = 60°$. What is the difference in torque between considering only the fundamental (θ variation) and including harmonics (3θ variation)? (5 points)

1. a) $W_m' = \int \lambda \, di = \frac{1}{2} L_s i_a^2 + \frac{1}{2} L_s i_b^2 + \frac{1}{2} L_r i_r^2$

$$+ M(\cos\theta - 0.1\cos 3\theta)\, i_a i_r$$

$$+ M(\sin\theta + 0.1\sin 3\theta)\, i_b i_r$$

b) $T_e = \frac{\partial W_m'}{\partial \theta} = (-M\sin\theta + 0.3M\sin 3\theta)\, i_a i_r$

$$+ (M\cos\theta + 0.3M\cos 3\theta)\, i_b i_r$$

c) $T^e(\theta \text{ only}) - T^e(\theta, 3\theta) \Rightarrow (0.3M\sin 3\theta)\, i_a i_r + (0.3M\cos 3\theta)\, i_b i_r$

$$\Delta T^e = 0.3\, \overset{M}{(0.9)}\, \overset{i_r}{(1)} \left(\overset{i_a}{(1)} \sin 180^\circ + \overset{i_b}{(1)} \cos 180^\circ \right) = -0.27$$

Problem 2

A system is described with the following state space equations:

$$\dot{x}_1 = -2x_1 + 4x_2$$
$$\dot{x}_2 = -2x_1 - 8x_2$$

a) Is this system stable? (10 points)

b) What steady-state values will $x_1(t)$ and $x_2(t)$ reach as t goes to infinity? (5 points)

c) Use Forward Euler to calculate $x_1(t)$ and $x_2(t)$ at t = 0.2 seconds using a step size of 0.1 seconds. Use the following initial conditions: $x_1(0) = 4$, $x_2(0) = 1$. (10 points)

a) $A = \begin{bmatrix} -2 & 4 \\ -2 & -8 \end{bmatrix}$ $\quad \lambda I - A = \begin{bmatrix} \lambda+2 & -4 \\ 2 & \lambda+8 \end{bmatrix}$ $\quad |\lambda I - A| = \lambda^2 + 10\lambda + 16 + 8$

$= \lambda^2 + 10\lambda + 24$

$= (\lambda+6)(\lambda+4)$

$\lambda = -4, -6$; $Re\{\lambda\} < 0$ for all λ, so stable.

b) $\begin{bmatrix} -2 & 4 \\ -2 & -8 \end{bmatrix}\begin{bmatrix} x_1^e \\ x_2^e \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} x_1^e \\ x_2^e \end{bmatrix} = \begin{bmatrix} -8/24 & -4/24 \\ 2/24 & -\frac{2}{24} \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow \boxed{x_1^e = x_2^e = 0}$

c) $\begin{bmatrix} x_1(0.1) \\ x_2(0.1) \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix} + (0.1)\begin{bmatrix} -2\cdot 4 + 4\cdot 1 \\ -2\cdot 4 - 8\cdot 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix} + 0.1\begin{bmatrix} -4 \\ -16 \end{bmatrix} = \begin{bmatrix} 3.6 \\ -0.6 \end{bmatrix}$

$\begin{bmatrix} x_1(0.2) \\ x_2(0.2) \end{bmatrix} = \begin{bmatrix} 3.6 \\ -0.6 \end{bmatrix} + (0.1)\begin{bmatrix} -2\cdot 3.6 + 4\cdot(-0.6) \\ -2\cdot 3.6 - 8\cdot(-0.6) \end{bmatrix} = \begin{bmatrix} 3.6 \\ -0.6 \end{bmatrix} + 0.1\begin{bmatrix} -9.6 \\ -2.4 \end{bmatrix}$

$\begin{bmatrix} x_1(0.2) \\ x_2(0.2) \end{bmatrix} = \begin{bmatrix} 2.64 \\ -0.84 \end{bmatrix}$

Problem 3

Consider the system:

$$\dot{x}_1 = -x_2^2 + 1 - x_1$$
$$\dot{x}_2 = Kx_1$$

where K is a real number.

a) Compute all equilibria (5 points)
b) Write the differential equations of the linearized system at each equilibrium (10 points)
c) For each equilibrium, determine the range of values of K such that the system is stable (10 points)

a) $0 = -x_2^{e\,2} + 1 - x_1^e \qquad x_1^e = 0 \qquad x_2^e = \pm 1$

$0 = K \cdot x_1^e$

b)
$$\begin{bmatrix} \Delta\dot{x}_1 \\ \Delta\dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & -2x_2^e \\ K & 0 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix}$$

For $(0,1)$: $\begin{bmatrix} -1 & -2 \\ K & 0 \end{bmatrix}$ $\qquad (0,-1)$: $\begin{bmatrix} -1 & 2 \\ K & 0 \end{bmatrix}$

c) For $(0,1)$:

$|\lambda I - A| = (\lambda+1)(\lambda) + 2K = 0$

$\lambda^2 + \lambda + 2K = 0$

$\lambda = \dfrac{-1 \pm \sqrt{1-8K}}{2}$

Need $Re\{\lambda\} < 0$:

$Re\{-1 + \sqrt{1-8K}\} < 0$

$Re\{\sqrt{1-8K}\} < 1$

$K > 0$

For $(0,-1)$:

$|\lambda I - A| = (\lambda+1)(\lambda) - 2K = 0$

$\lambda = \dfrac{-1 \pm \sqrt{1+8K}}{2}$

Need $Re\{\lambda\} < 0$:

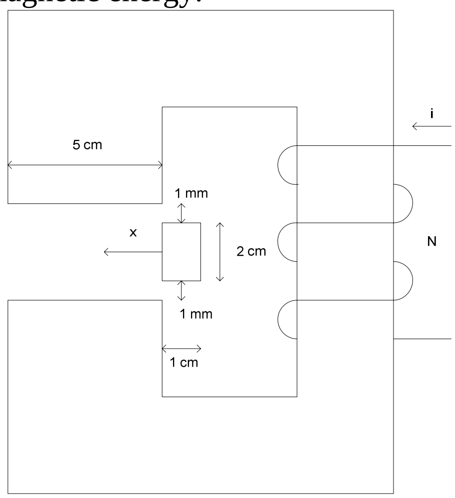
$Re\{-1 + \sqrt{1+8K}\} < 0$

$Re\{\sqrt{1+8K}\} < 1$

$K < 0$

Problem 4 (25 points)

Consider the system shown below. Something similar was built in 1996 by Matt Greuel and Dan Logue as a senior design project. The idea is to use the moving member as a projectile, accelerated by magnetic energy.



The flux linkage is found to be:

$$\lambda(i,x)=\begin{cases}\mu_0\left(\dfrac{2x+0.01}{44\times10^{-6}}\right)i & x\in[0,0.01\,m]\\ \mu_0\left(\dfrac{0.03}{44\times10^{-6}}\right)i & x\in[0.01\,m,0.05\,m]\end{cases}$$

a) Find co-energy W_m' for the two intervals in x given (8 points)

b) Find force of electric origin f^e for the two intervals in x given (7 points)

c) Find EFE and EFM as the system proceeds through the following sequence (10 points):

   i. From x = i = 0 to x = 0, i = 10 A

   ii. From x = 0 to x = 0.01 m with constant current (i = 10 A)

   iii. From i = 10 to i = 0 with constant position (x = 0.01 m)

4. a) $x \in [0, 0.01]$: $w_m' = \mu_0 \left(\frac{2x+0.01}{2 \cdot 44 \times 10^{-6}} \right) i^2 = (0.02856x + 142.8e{-6})\, i^2$

$x \in [0.01, 0.05]$: $w_m' = \mu_0 \left(\frac{0.03}{2 \cdot 44 \times 10^{-6}} \right) i^2 = 428.4e{-6}\ i^2$

b) $x \in [0, 0.01]$: $f^e = 0.02856\ i^2$

$x \in [0.01, 0.05]$: $f^e = 0$

c) i) $EFE = \int_0^{\lambda_f} \left. \frac{44e{-6}}{\mu_0 (2x+0.01)} \right|_{x=0} \lambda d\lambda = \frac{1}{2} \frac{44e{-6}}{0.01 \mu_0} \lambda_f^2$

$\lambda_f = \mu_0 \cdot \frac{2(0)+0.01}{44e{-6}} \cdot 10 = 2.856e{-3}$

$EFE = 14.28\ mJ$

$\Delta x = 0 \Rightarrow EFM = 0$

ii) $EFM = -\int f^e dx = -\int_0^{0.01} 0.02856\, i^2 \Big|_{i=10} dx = -(0.02856)(10^2)(0.01)$

$EFM = -28.56\ mJ$

$EFE = \int i d\lambda = \int 10\, d\lambda = 10 \lambda \Big|_{\lambda_0}^{\lambda_f}$

this λ_0 = above $\lambda_f = 2.856e{-3}$

this $\lambda_f = \mu_0 \left(\frac{0.03}{44e{-6}} \right)(10) = 8.568e{-3}$

$EFE = 57.12\ mJ$

iii) $EFE = \int i d\lambda = \int_{\lambda_0}^{0} \frac{44e{-6}}{0.03 \mu_0} \lambda d\lambda = \frac{44e{-6}}{2 \times 0.03 \mu_0} \lambda^2 \Big|_{\lambda_0}^{0}$

this λ_0 = above $\lambda_f = 8.568e{-3}$

$EFE = -42.84\ mJ$

$EFM = 0 \qquad (\Delta x = 0)$