ECE 330 Exam 1: Fall 2019 NAME Solution

90 minutes J. Schuh and R. Zhang

Section (Check one) MWF 10am________ MWF 2pm________

1. ________/25 2. ________/25

3. ________/25 4. ________/25 TOTAL ________/100

USEFUL INFORMATION

sin(x)=cos(x-90°) $\bar{V} = \bar{Z}\bar{I}$ $\bar{S} = \bar{V}\bar{I}^* = P + jQ$ $\bar{S}_{3\varphi} = \sqrt{3}V_L I_L \angle\theta$

$0<\theta<180°$ (lag) $I_L = \sqrt{3}I_\varphi$ (delta) $\bar{Z}_Y = \bar{Z}_\Delta/3$

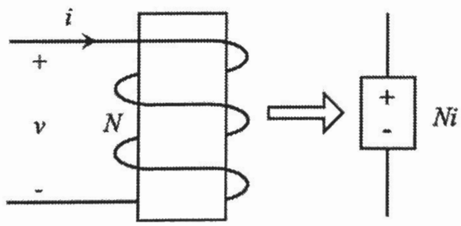
$-180°<\theta<0$ (lead) $V_L = \sqrt{3}V_\varphi$ (wye) $\mu_0 = 4\pi \times 10^{-7}$H/m

ABC phase sequence has A at 0, B at -120°, and C at +120°

$\int \underline{H} \cdot \underline{dl} = \int \underline{J_f} \cdot \hat{n} dA$ $\int \underline{E} \cdot \underline{dl} = -\frac{d}{dt}\left(\int \underline{B} \cdot \hat{n} dA\right)$ $\mathcal{R} = \frac{l}{\mu A}$ $Ni = \mathcal{R}\varphi$

$\varphi = BA$ $\lambda = N\varphi = Li$ (if linear) $v = \frac{d\lambda}{dt}$ $k = \frac{M}{\sqrt{L_1 L_2}}$

1hp=746 W



$W_m = \int_0^\lambda i d\hat{\lambda}$ $W_m' = \int_0^i \lambda d\hat{\imath}$ $W_m + W_m' = i\lambda$ $f^e = -\frac{\partial W_m}{\partial x} = \frac{\partial W_m'}{\partial x}$

$x \to \theta, f^e \to T^e$

$EFE_{a\to b} = \int_a^b i d\lambda$ $EFM_{a\to b} = \int_a^b -f^e dx$ $EFE_{a\to b} + EFM_{a\to b} = W_{mb} - W_{ma}$

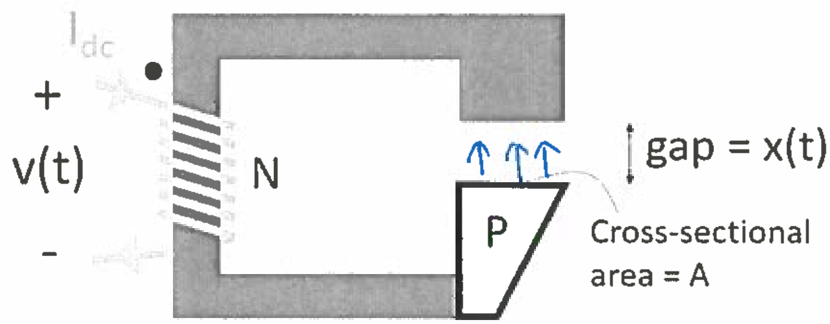
$i = \frac{\partial W_m}{\partial \lambda}$ $\lambda = \frac{\partial W_m'}{\partial i}$

$\dot{x}_1 = f_1(x_1, x_2)$ $\dot{x}_2 = f_2(x_1, x_2)$ $x(t_0 + \Delta t) \approx x(t_0) + \Delta t \left.\frac{dx}{dt}\right|_{t=t_0}$

Problem 1 (25 Points)

A magnetic structure is shown below, with a movable member P. The area of the air gap is A, and the size of the gap is x(t). The magnetic structure is excited by a coil of N turns carrying a positive, constant current i(t) = Idc > 0, in the polarity shown.

Neglect fringing and assume that the permeability of the structure to be infinite. Recall that the permeability of free-space is $\mu_0 = 4\pi \times 10^{-7}$ [H/m] by definition.



Up not because
* Flux direction / current direction.

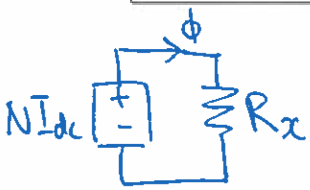
This is an electro-magnet!! Reverse the current / flux and member "p" still moves upwards.

a) What are the magnitude and direction of the force of electric origin acting on the member P? State your answer as a function of μ_0, A, x(t), N, and Idc. State the unit for force. (9 points for magnitude, 0.5 point for correct direction, 2.5 points for justification of correct direction.)

means absolute value

Magnitude	$= + \frac{1}{2} \frac{\mu_0 A N^2 I_{dc}^2}{x^2}$ [Newtons]
Direction	= Upwards / Downwards

Up because:
* wants to close airgap
* wants to minimize reluctance
* x points down, $|f^e|$ is down, $-|f^e|$ is up.



$R_x = \frac{x}{\mu_0 A}$ (by def)

$N I_{dc} = R_x \cdot \phi$ ("Ohm's" law)

$\Rightarrow \phi = \frac{N I_{dc}}{R_x} = \frac{\mu_0 A N I_{dc}}{x}$

$\lambda = N\phi = \frac{\mu_0 A N^2 I_{dc}}{x}$ (by def)

(call this λ_{dc})

Co-Energy

fixed

$W_m' = \int_0^{I_{dc}} \lambda(\hat{i}, x)\, d\hat{i} = \int_0^{I_{dc}} \left(\frac{\mu_0 A N^2}{x}\right) \hat{i}\, d\hat{i}$

$= \frac{1}{2}\left(\frac{\mu_0 A N^2}{x}\right) I_{dc}^2$

$f^e = + \frac{\delta W_m'}{\delta x} = \frac{-1}{2}\left(\frac{\mu_0 A N^2}{x^2}\right) I_{dc}^2$

(by def) $\quad \left(\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}\right)$

Energy (not recommended) $\quad W_m = \int_0^{\lambda_{dc}} i(\hat{\lambda}, x)\, d\hat{\lambda} = \int_0^{\lambda_{dc}} \left(\frac{x}{\mu_0 A N^2}\right) \hat{\lambda}\, d\hat{\lambda}$

$= \frac{1}{2}\left(\frac{x}{\mu_0 A N^2}\right) \lambda_{dc}^2$

$f^e = \frac{-\delta W_m}{\delta x} = \frac{-1}{2} \frac{\lambda_{dc}^2}{\mu_0 A N^2} = \frac{-1}{2} \frac{\mu_0 A N^2}{x^2} I_{dc}^2$

b) Let Idc = 10 A, N = 100 turns, and A = 10 cm^2. Suppose that the member P is moving downwards at a rate of 2 m/s. In other words, x(t) has a derivative of dx/dt = +2 [m/s].

What is the voltage v(t) seen at the terminals of the coil when x(t) = 2 mm? State your answer as a numerical value to three significant figures. State the unit for voltage. (10 points)

v(t) =	−62.8 V

$$v = \frac{d\lambda}{dt} = \frac{d}{dt}\left[\frac{\mu_0 A N^2 I_{dc}}{x}\right]$$

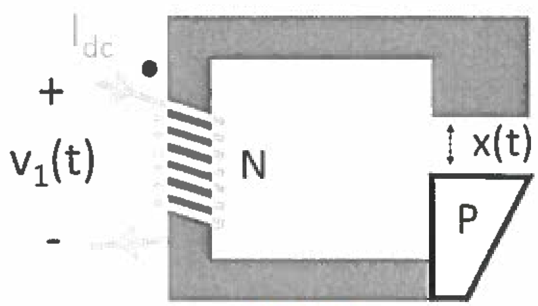
$$= \underbrace{\frac{\mu_0 A N^2}{x}\cancel{\frac{d I_{dc}}{dt}}^{\,0}}_{=0} - \frac{\mu_0 A N^2 I_{dc}}{x^2}\frac{dx}{dt}$$

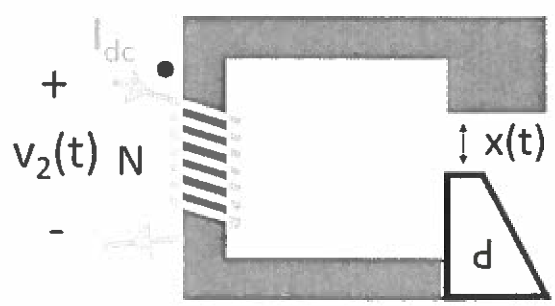
$$= -\frac{(4\pi\times10^{-7})(10\,cm^2)(100)^2\,10}{(2mm)^2}\cdot 2 \Rightarrow V = -20\pi$$

don't drop this

c) Suppose that the member P is moving upwards at a rate of 1 m/s in both of the following diagrams. That is, x(t) has a derivative of dx/dt = −1 [m/s] in both of these diagrams.

Which structure should see a higher terminal voltage? (Note that +1 is higher than −4.) Circle the label of your choice and indicate your reasoning below. (0.5 point for correct choice, 2.5 points for correct justification)





Circle one: Fig. p Fig. d

We have $v = -\underbrace{\frac{\mu_0 N^2 I_{dc}}{x^2}}_{\text{positive}} \cdot A \cdot \underbrace{\frac{dx}{dt}}_{\text{negative}}$

So both v_1 and v_2 are positive.

Moreover, $A_p > A_d$. Hence,

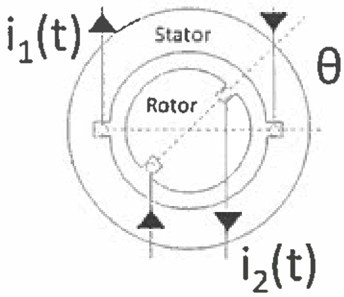
$v_1 > v_2 > 0$.

Note (Fringing)

fringing increases area, but cannot compete with Fig. P

Problem 2 (25 Points)

The following machine has the following linear flux linkage vs current characteristic (1 = stator, 2 = rotor):



$$\lambda_1 = +3i_1 \qquad -2i_2\sin\theta \qquad [\text{Wb-t}]$$

$$\lambda_2 = -2i_1\sin\theta \qquad +4i_2 \qquad [\text{Wb-t}]$$

a) Find the co-energy associated with the coupling field, as a function of θ, i1, i2. State the unit for co-energy. (12 points)

Co-energy $= \frac{3}{2}i_1^2 + 2i_2^2 - 2i_1 i_2 \sin\theta$ [Joules]

$$W_m' = \int_0^{i_1} \lambda_1(\underbrace{\hat{i}_1}_{\text{variable}}, \underbrace{i_2=0}_{\text{fixed at zero}}, \theta)\, d\hat{i}_1 + \int_0^{i_2} \lambda_2(\underbrace{i_1}_{\text{fixed at max}}, \underbrace{\hat{i}_2}_{\text{variable}}, \theta)\, d\hat{i}_2$$

"charge first coil" + "charge second coil w/ first coil charged"

$$= \int_0^{i_1}\left[+3\hat{i}_1 - 2(0)\sin\theta\right]d\hat{i}_1 + \int_0^{i_2}\left[-2i_1\sin\theta + 4\hat{i}_2\right]d\hat{i}_2$$

$$= \frac{3}{2}i_1^2 - 0 + -2i_1i_2\sin\theta + \frac{4i_2^2}{2}$$

Noting linearity we have short cuts

$$\left(= \frac{1}{2}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}^T \underbrace{\begin{bmatrix} +3 & -2\sin\theta \\ -2\sin\theta & +4 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}}_{\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}} \right)$$ optional, but sanity check if in doubt.

$$\left(= \frac{1}{2}i_1\lambda_1 + \frac{1}{2}i_2\lambda_2 \right)$$ optional, but another sanity check if in doubt.

b) Suppose that i1 = 1 A, i2 = 2 A, and θ = 90 degrees. Recall that sin(+90°) = 1, sin(-90°) = -1 and cos(+90°) = cos(-90°) = 0.

What are the magnitude and direction of the torque of electric origin acting on the rotor? State the unit for torque. (7 points for correct magnitude, 0.5 point for correct direction, 2.5 points for justification of correct direction)

Magnitude	=	0 Nm
Direction	=	Clockwise / Anticlockwise

any choice, not well defined.

$$\tau^e = +\frac{\delta W_m'}{\delta \theta} = -2 i_1 i_2 \cos\theta$$

but $\theta = 90^\circ$ and $\cos\theta = 0$.

If $\tau^e > 0$ due to error, then anticlockwise
< 0 due to error, then clockwise.

Reason: * min gap, * min reluctance, * θ is anticlockwise, and τ^e defined to increase θ.

c) Suppose that θ = 0 degrees. Recall that sin(0°) = 0 and cos(0°) = 1. Find the energy associated with the coupling field, as a function of λ1, λ2. State the unit for energy. (3 points)

Energy	=	$\frac{\lambda_1^2}{6} + \frac{\lambda_2^2}{8}$ Joules.

from part a

Integration (easiest)

$\lambda_1 = 3 i_1 - 2\sin\theta\, i_2$

$\lambda_2 = -2\sin\theta\, i_1 + 4 i_2$

but $\theta = 0$ and $\sin\theta = 0$,

$\lambda_1 = 3 i_1$, $\lambda_2 = 4 i_2$,

$$W_m = \int_0^{\lambda_1} \left(\frac{\hat\lambda_1}{3}\right) d\hat\lambda_1 + \int_0^{\lambda_2}\left(\frac{\hat\lambda_2}{4}\right) d\hat\lambda_2$$

$$= \frac{\lambda_1^2}{6} + \frac{\lambda_2^2}{8}$$

From co-energy

$$W_m = i_1\lambda_1 + i_2\lambda_2 - \left(\frac{3 i_1^2}{2} \underbrace{- 2 i_1 i_2 \sin\theta}_{=0} + 2 i_2^2\right)$$

$$= \frac{3 i_1^2}{2} + 2 i_2^2 \; (= W_m')$$

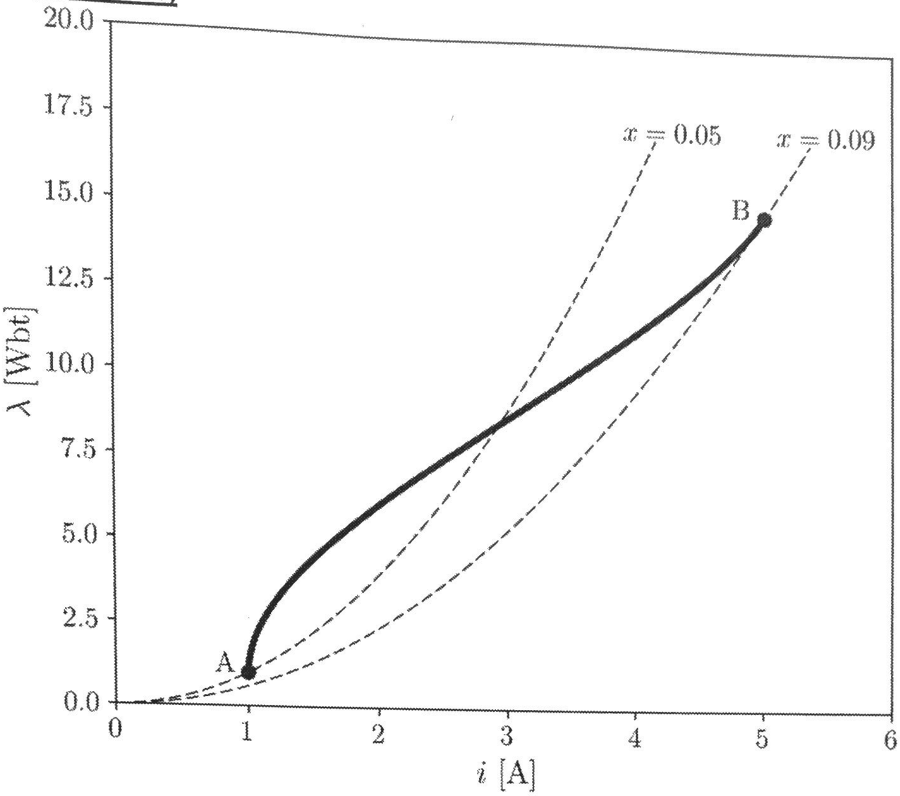
same as co-energy b/c linearity.

but $\lambda_1 = 3 i_1$, $\lambda_2 = 4 i_2$, so

$$W_m = \frac{3}{2}\left(\frac{\lambda_1}{3}\right)^2 + 2\left(\frac{\lambda_2}{4}\right)^2$$

$$= \frac{\lambda_1^2}{6} + \frac{\lambda_2^2}{8}$$

Problem 3 (25 Points)



A system with flux linkage given as $\lambda=\frac{0.06}{x+0.01}i^2$ moves from point A (i=1 A, λ=1 Wbt) to point B (i=5 A, λ=15 Wbt) as shown in the figure above along a path parameterized as

$$i=\frac{-1}{490}(\lambda-1)^3+\frac{24}{490}(\lambda-1)^2+1$$

Lines of constant x are also shown. For this system, determine:

a) The energy from the electrical (EFE) when going from point A to point B. Please write your final answer on the line provided. Hint: $\int (x-a)^n dx = \frac{1}{n+1}(x-a)^{n+1} + C$ (10 points).

$EFE=$ 39.2 J

$$EFE_{a\to b} = \int_{\lambda_a}^{\lambda_b} i\, d\lambda \Rightarrow$$

$$EFE_{a\to b} = \int_1^{15}\left(-\frac{1}{490}(\lambda-1)^3+\frac{24}{490}(\lambda-1)^2+1\right)d\lambda$$

$$= -\frac{1}{490}\left(\frac{1}{4}\right)(\lambda-1)^4+\frac{24}{490}\left(\frac{1}{3}\right)(\lambda-1)^3+\lambda\Big|_1^{15}$$

$$=\left(-\frac{1}{490}\left(\frac{1}{4}\right)(15-1)^4+\frac{24}{490}\left(\frac{1}{3}\right)(15-1)^3+15\right)$$

$$-(1)$$

$$= 40.2-1 \Rightarrow 39.2\text{ J}$$

b) The energy W_m at point A and B. Please write your final answers on the corresponding lines (10 points).

Energy at A= .6667 J

Energy at B= 50 J

$$W_m' = \int_0^i \lambda \, di \Rightarrow W_m' = \int_0^i \frac{0.06}{X+0.01} i^2 \, di \Rightarrow W_m' = \frac{0.02}{X+0.01} i^3$$

$$W_m + W_m' = i\lambda \Rightarrow W_m + \frac{0.02}{X+0.01} i^3 = \frac{0.06}{X+0.01} i^3$$

$$W_m = \frac{0.04}{X+0.01} i^3$$

$$W_{ma} = \frac{0.04}{0.05+0.01} (1)^3 = .6667 \text{ J}$$

$$W_{mb} = \frac{0.04}{0.09+0.01} (5)^3 = 50 \text{ J}$$

c) The energy from mechanical (*EFM*) when going from point A to point B. Please write your final answer on the line provided.
(5 points).

EFM= 10.1333 J

$$\underset{a\to b}{EFM} + \underset{a\to b}{EFE} = W_{mb} - W_{ma}$$

$$\underset{a\to b}{EFM} = W_{mb} - W_{ma} - \underset{a\to b}{EFE}$$

$$\underset{a\to b}{EFM} = 50 - 0.66667 - 39.2$$

$$\underset{a\to b}{EFM} = 10.1333 \text{ J}$$

Problem 4 (25 Points)

The equation of motion for a mass attached to a finitely extensible spring with maximum length l_e is given below. An external force is applied to the mass and air resistance is also included.

$$\ddot{x} = \frac{-\omega_n^2 x}{1-\left(\frac{x}{l_e}\right)^2} + \frac{F_e}{m} - 2\zeta\omega_n^2\dot{x}$$

a) Write the equation of motion in state space form. (4 points)

State Space Equations:

$x_1 = x$ $\qquad$ $\frac{dx_1}{dt} = x_2$

$x_2 = \dot{x}$ $\qquad$ $\frac{dx_2}{dt} = \frac{-\omega_n^2 x_1}{1-\left(\frac{x_1}{l_e}\right)^2} + \frac{F_e}{m} - 2\zeta\omega_n x_2$

b) Assuming that the applied force is constant in time, what are the equilibrium points for this system? Hint: quadratic formula $ax^2 + bx + c = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ (11 points)

Equilibrium Points:

$$\left(\frac{l_e^2}{2\left(\frac{F_e}{m}\right)}\left[-\omega_n^2 + \sqrt{\omega_n^4 + 4\left(\frac{F_e}{m l_e}\right)^2}\right], 0\right)$$

$$\left(\frac{l_e^2}{2\left(\frac{F_e}{m}\right)}\left[-\omega_n^2 - \sqrt{\omega_n^4 + 4\left(\frac{F_e}{m l_e}\right)^2}\right], 0\right)$$

$\frac{dx_1}{dt} = 0 \Rightarrow x_2 = 0$

$\frac{dx_2}{dt} = 0 \Rightarrow \frac{-\omega_n x_1}{1-\left(\frac{x_1}{l_e}\right)^2} + \frac{F_e}{m} = 0$

$$-\omega_n x_1 + \frac{F_e}{m}\left(1-\left(\frac{x_1}{l_e}\right)^2\right) = 0$$

$$-\omega_n x_1 + \frac{F_e}{m} - \frac{F_e}{m}\frac{x_1^2}{l_e^2} = 0$$

$$0 = \frac{F_e}{m}\frac{x_1^2}{l_e^2} + \omega_n x_1 - \frac{F_e}{m}$$

$$x_1 = \frac{-\omega_n^2 \pm \sqrt{\omega_n^4 - 4\left(\frac{F_e}{m l_e^2}\right)\left(-\frac{F_e}{m}\right)}}{2\left(\frac{F_e}{m l_e^2}\right)}$$

$$x_1 = \frac{-\omega_n^2 \pm \sqrt{\omega_n^4 + 4\left(\frac{F_e}{m l_e}\right)^2}}{2\left(\frac{F_e}{m l_e^2}\right)}$$

$$x = \frac{l_e^2}{2\left(\frac{F_e}{m}\right)}\left[-\omega_n^2 \pm \sqrt{\omega_n^4 + 4\left(\frac{F_e}{m l_e}\right)^2}\right]$$

c) Let $\omega_n^2 = 1\frac{1}{s^2}, \zeta = 0.5, \frac{F_e}{m} = 1\cos(t)\ m/s^2, l_e = 1\ m$. Using a time step of 0.02 s, determine the position and velocity of the mass at time t=0.06 s using the initial conditions $x_0 = 0, \dot{x}_0 = 1$. Keep 6 decimal points. (10 points)

t [s]	$x(t)$	$\dot{x}(t)$
0	0	1
0.02	0.02	1
0.04	0.04	0.999596
0.06	0.059992	0.998787

$$\frac{dx_1}{dt} = x_2$$

$$\frac{dx_2}{dt} = \frac{-x_1}{1-x_1^2} + \cos(t) - x_2$$

$$x_1^n = x_1^{n-1} + \Delta t\left(x_2^{n-1}\right)$$

$$x_2^n = x_2^{n-1} + \Delta t\left(\frac{-x_1^{n-1}}{1-\left(x_1^{n-1}\right)^2} + \cos\left(t^{n-1}\right) - x_2^{n-1}\right)$$

Blank Page for Extra Work