ECE 330 Exam 1: Fall 2019 NAME Solution

90 minutes J. Schuh and R. Zhang

Section (Check one) MWF 10am_________ MWF 2pm__________

1. __________/25 2. __________/25

3. __________/25 4. __________/25 TOTAL _________/100

USEFUL INFORMATION

sin(x)=cos(x-90°) $\bar{V} = \bar{Z}\bar{I}$ $\bar{S} = \bar{V}\bar{I}^* = P + jQ$ $\bar{S}_{3\varphi} = \sqrt{3}V_L I_L \angle\theta$

0<θ<180° (lag) $I_L = \sqrt{3}I_\varphi$ (delta) $\bar{Z}_Y = \bar{Z}_\Delta/3$

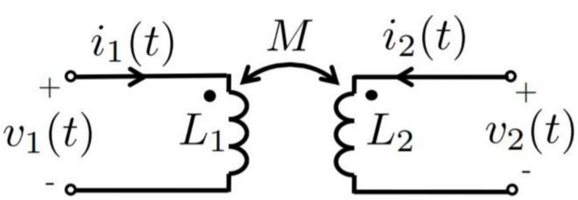
-180°<θ<0 (lead) $V_L = \sqrt{3}V_\varphi$ (wye) $\mu_0 = 4\pi \times 10^{-7}$H/m

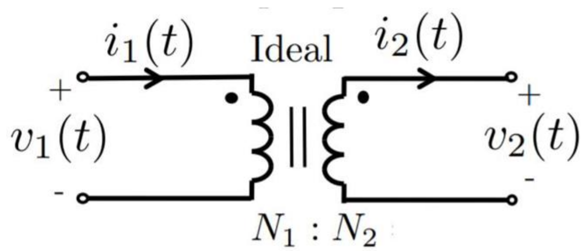
ABC phase sequence has A at 0, B at -120°, and C at +120°

$\int \underline{H} \cdot \underline{dl} = \int \underline{J_f} \cdot \hat{n} dA$ $\int \underline{E} \cdot \underline{dl} = -\frac{d}{dt}\left(\int \underline{B} \cdot \hat{n} dA\right)$ $\mathcal{R} = \frac{l}{\mu A}$ $Ni = \mathcal{R}\varphi$

$\varphi = BA$ $\lambda = N\varphi = Li$ (if linear) $v = \frac{d\lambda}{dt}$ $k = \frac{M}{\sqrt{L_1 L_2}}$

1hp=746 W



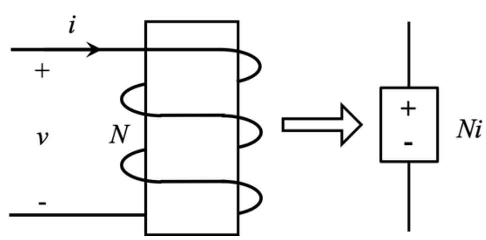


$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$

$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$

$a = \frac{N_1}{N_2}$ $N_1 i_1 = N_2 i_2$

$\frac{V_1}{V_2} = \frac{N_1}{N_2}$



Problem 1 (25 Points)

A single phase source is serving three loads connected in parallel, through a feeder with impedance $\bar{Z}_{\text{feeder}} = 1 + j2\ \Omega$.

- Load 1 is an impedance of $30 + j40\ \Omega$.
- Load 2 draws 200 W at a lagging PF of 0.8.
- Load 3 draws 1 A at a leading PF of 0.6.

The voltage across the loads is $\bar{V}_{\text{load}} = 100\angle 0°$ V.

a) Compute the current phasors $\bar{I}_1$, $\bar{I}_2$, and $\bar{I}_3$ for each of the three loads (20 points total).

$\bar{V}_L = 100\angle 0° \text{ V}$

$\bar{I}_1 = \dfrac{\bar{V}_L}{\bar{Z}_1}$

$\bar{Z}_1 = 30 + j40\,\Omega = 50\angle 53.13°\,\Omega$

$\bar{I}_1 = \dfrac{100\angle 0°}{50\angle 53.13°} \Rightarrow \boxed{\bar{I}_1 = 2\angle -53.13° \text{ A}}$

$I_3 = 1 \text{ A}$

$\theta_{i3} = +\cos^{-1}(0.6)$

$\theta_{i3} = 53.13°$

$\boxed{\bar{I}_3 = 1\angle 53.13° \text{ A}}$

$P_2 = V I_2 (PF_2)$

$I_2 = \dfrac{P_2}{V(PF_2)} \Rightarrow I_2 = \dfrac{200}{100(0.8)} \Rightarrow I_2 = 2.5 \text{ A}$

$\theta_{i2} = -\cos^{-1}(0.8) = -36.87°$

$\boxed{I_2 = 2.5\angle -36.87° \text{ A}}$

c) What is the source voltage $\bar{V}_{source}$? (3 points)

$\bar{I}_{tot} = \bar{I}_1 + \bar{I}_2 + \bar{I}_3$

$= (1.2 - j1.6) + (2 - j1.5) + (0.6 + j0.8)$

$= 3.8 - j2.3 A$

$= 4.44 \angle -31.18° A$

$\bar{V}_s = \bar{V}_{load} + \bar{Z}_{line} \bar{I}_{tot}$

$= 100\angle 0° + (2.24\angle 63.43°)(4.44\angle -31.18°)$

$= 100\angle 0° + 9.9456\angle 32.25°$

$\bar{V}_s = 108.41 + j5.307 V$

$= 108.54\angle 2.80° V$

d) What is the power factor <u>at the source</u>? State whether it is leading or lagging. (1 point)

$\theta_s = (2.80 - [-31.18]) = 33.98$

PF = 0.829 lagging

e) What is the power factor <u>at the load</u>? State whether it is leading or lagging. (1 point)

$\theta = 31.18°$

PF = 0.856 lagging

Problem 2 (25 Points)

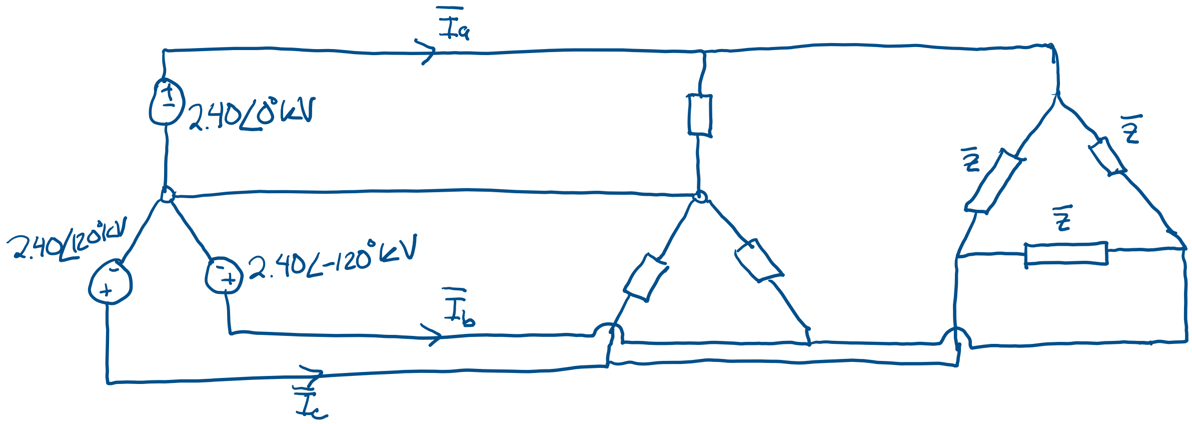
A Wye-connected three-phase generator delivers 1200 kVA at 0.6 PF lagging and 4160 volt (line-to-line) to the following loads in parallel.

- Load 1 is connected in Wye, and draws a total of 300 kW at unity power factor.
- Load 2 is an impedance load connected in Delta, with unknown per-phase impedance $\bar{Z}$.

a) Draw the three-phase circuit diagram. Label <u>all relevant instances</u> of the following:

- Phase-to-neutral voltage phasor (e.g. $10\angle 50°$ V).
- Per-phase impedance for load 2 as symbolic expressions (e.g. $\bar{Z}/\sqrt{5}\ \Omega$).

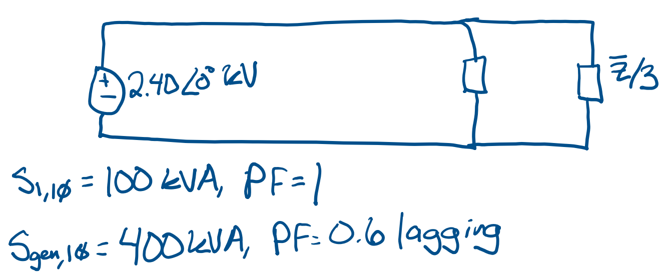
Include polarity markers (+/-) in all of your voltage labels. (11 points)



b) Draw the single-phase equivalent circuit for phase a. Label <u>one instance</u> of the following:

- Voltage phasor (e.g. $10\angle 50°$ V).
- Power for the generator in terms of kVA and PF (e.g. 10 kVA at 0.1 PF leading).
- Power for load 1 in terms of kVA and PF (e.g. 10 kVA at 0.1 PF leading).
- Impedance for load 2 as a symbolic expression (e.g. $\bar{Z}/\sqrt{5}\ \Omega$).

All quantities should be stated in per-phase values. (6 points)



d) How much three-phase kVARs do we need to bring the combined power factor to unity? How would you connect the capacitors (in Wye or Delta) and why? (3 points)

$\bar{S}_{3\phi} = 1200\angle 53.13^\circ$ kVA

$= 720 + j960$ kVA

Add 960 kVAR of capacitance

Delta: lower capacitance needed to get given power; lower phase current

Wye: lower phase voltage

Either: complex power equation is the same for Wye and Delta

e) Compute the numerical value for the unknown impedance $\bar{Z}$. (5 points)

$\bar{S}_2 = \bar{S}_{tot} - \bar{S}_1$

$= 420 + j960$ kVA

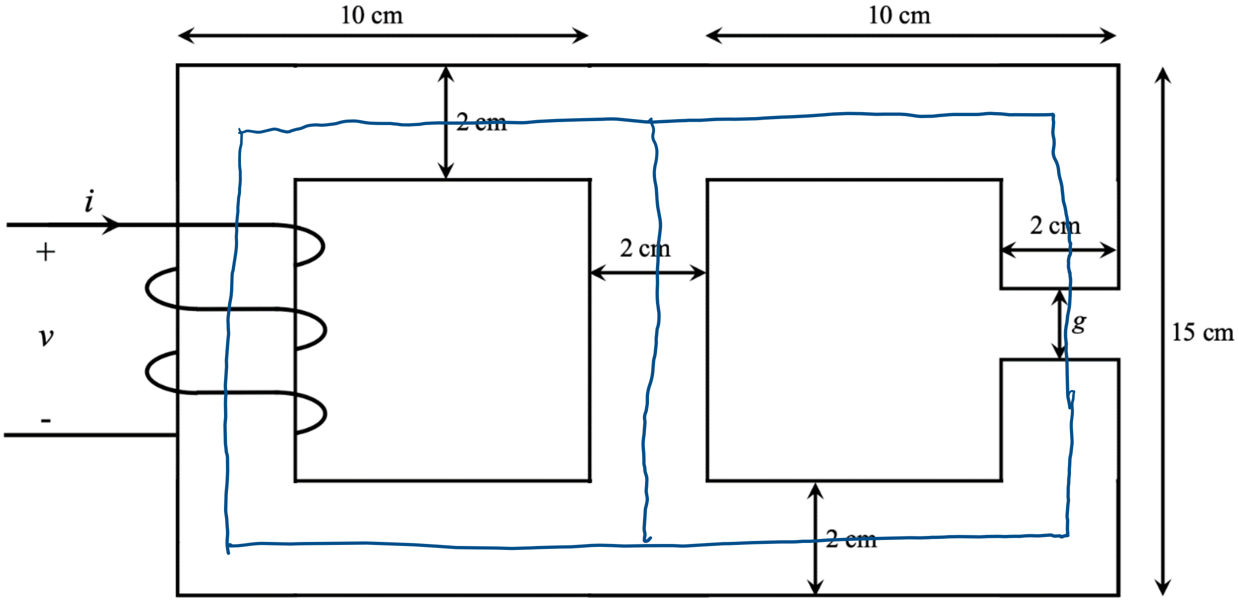
$= 1047.85\angle 66.37^\circ$

$\bar{S}_2 = 3\bar{V}_L \bar{I}_\phi^* \Rightarrow \bar{S}_2 = \frac{3V_L^2}{\bar{Z}^*} \Rightarrow \bar{Z} = \left(\frac{3V_L^2}{\bar{S}_2}\right)^*$

$\bar{I}_\phi = \frac{V_L}{\bar{Z}}$

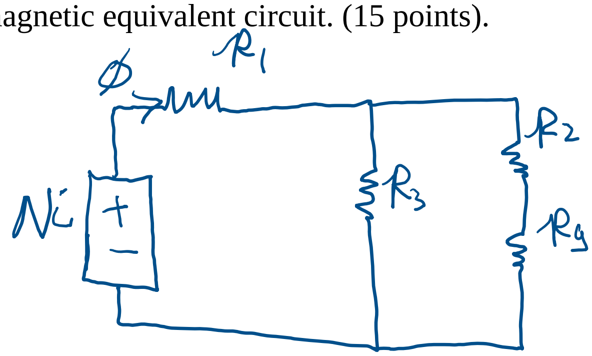
$\bar{Z} = 49.54\angle 66.37^\circ\ \Omega$

Problem 3 (25 Points)



A coil is wrapped 300 times around an iron core with the given dimensions and a depth of 2 cm into the page. The iron has a permeability $\mu=2000\mu_0$ and contains an air gap of g=2 mm. Fringing effects can be neglected.

a) Draw the magnetic path through the iron core on the figure above and draw the corresponding magnetic equivalent circuit. (15 points).



$$R_1 = \frac{33\text{cm}}{2000\mu_0(4\text{cm}^2)} \Rightarrow R_1 = 3.283\times10^5 \text{ At/Wb}$$

$$R_2 = \frac{33\text{cm} - 2\text{mm}}{2000\mu_0(4\text{cm}^2)} \Rightarrow R_2 = 3.263\times10^5 \text{ At/Wb}$$

$$R_3 = \frac{13\text{cm}}{2000\mu_0(4\text{cm}^2)} \Rightarrow R_3 = 1.293\times10^5 \text{ At/Wb}$$

$$R_g = \frac{2\text{mm}}{\mu_0(4\text{cm}^2)} \Rightarrow R_g = 3.979\times10^6 \text{ At/Wb}$$

b) What is the total flux ϕ through the circuit? (7 points)

$Ni = \phi R_{tot}$

$R_{tot} = \left(\frac{1}{R_3} + \frac{1}{R_2 + R_g}\right)^{-1} + R_1$

$= 4.538 \times 10^5$ At/Wb

$300i = 4.538 \times 10^5 \phi$

$\phi = 6.610 \times 10^{-4} i$

c) What is the inductance of the coil? (3 points)

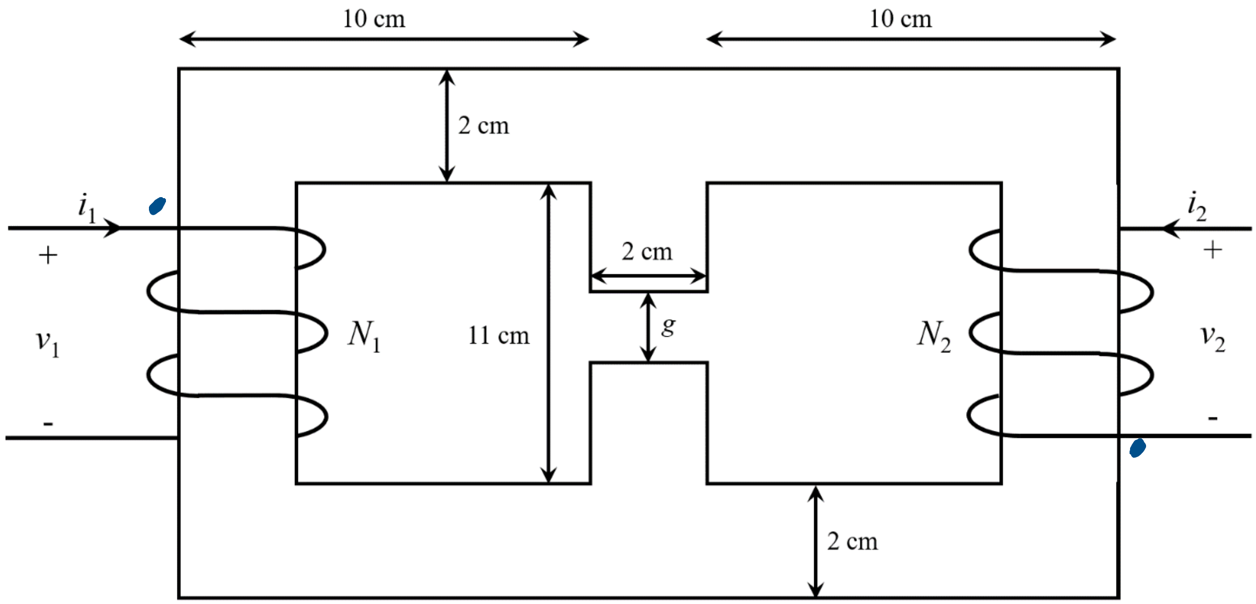
$\lambda = N\phi = Li$

$\lambda = 300(6.610 \times 10^{-4})i$

$\lambda = 0.198 i$

$L = 0.198$ H

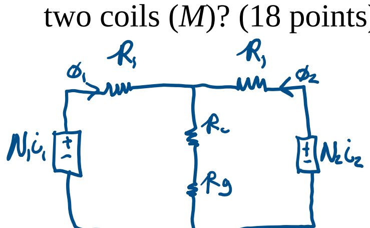
Problem 4 (25 Points)



Two coils are wrapped around an iron core with the given dimensions and depth of 3 cm into the page. Coil 1 contains 400 turns and Coil 2 contains 200 turns. The iron has a permeability μ=1500μ_0 and contains an air gap of g=4 mm. Fringing effects can be neglected.

a) Draw on the figure above where the dot marks should go for each coil. (2 points)

b) What is the self-inductance for each coil (L_1 and L_2) and the mutual inductance between the two coils (M)? (18 points)



$R_1 = \frac{33\text{cm}}{1500\mu_0(6\text{cm}^2)} \Rightarrow R_1 = 2.918\times10^5$ At/Wb

$R_c = \frac{13\text{cm} - 4\text{mm}}{1500\mu_0(6\text{cm}^2)} \Rightarrow R_c = 1.114\times10^5$ At/Wb

$R_g = \frac{4\text{mm}}{\mu_0(6\text{cm}^2)} = R_g = 5.305\times10^6$ At/Wb

$N_1 i_1 = R_1\phi_1 + (R_c+R_g)(\phi_1+\phi_2)$

$-\ N_2 i_2 = R_1\phi_2 + (R_c+R_g)(\phi_1+\phi_2)$

$N_1 i_1 - N_2 i_2 = R_1(\phi_1 - \phi_2)$

$\phi_1 = \frac{N_1 i_1 - N_2 i_2}{R_1} + \phi_2$

$\phi_1 = 7.034\times10^{-4} i_1 - 1.669\times10^{-4} i_2$

$\lambda_1 = N_1\phi_1$

$\lambda_1 = 0.2814 i_1 - 0.06676 i_2$

$L_1 = 0.2814$ H $L_2 = 0.01758$ H $M = 0.0667$ H

$N_2 i_2 = (R_1+R_c+R_g)\phi_2 + (R_c+R_g)\phi_1$

$N_2 i_2 = (R_1+R_c+R_g)\phi_2 + (R_c+R_g)\left[\frac{N_1 i_1 - N_2 i_2}{R_1} + \phi_2\right]$

$= (R_1+2R_c+2R_g)\phi_2 + \left(\frac{R_c+R_g}{R_1}\right)[N_1 i_1 - N_2 i_2]$

$-\left(\frac{R_c+R_g}{R_1}\right)N_1 i_1 + \left(1+\frac{R_c+R_g}{R_1}\right)N_2 i_2 = (R_1+2R_c+2R_g)\phi_2$

$\phi_2 = -6.674\times10^{-4} i_1 + 1.758\times10^{-4} i_2$

$\lambda_2 = N_2\phi_2$

$\lambda_2 = -0.06674 i_1 + 0.01758 i_2$

c) What is the open circuit voltage measured across Coil 2 if a current of $i_1 = \sqrt{2}(10)\cos(377t)$ is applied across Coil 1? Write your answer as a cosine function. (5 points)

$$V_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$V_2 = -M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

Open circuit: $i_2 = 0$

$$V_2 = -M \frac{di_1}{dt}$$

$$V_2 = -0.06676\left(-\sqrt{2}(10)(377)\sin(377t)\right)$$

$$V_2 = \sqrt{2}(251.69)\sin(377t)$$

$$\boxed{V_2 = \sqrt{2}(251.69)\cos(377t - 90°)}$$

(Blank page for extra work)