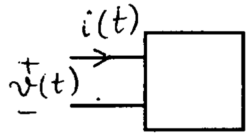
Problem 1 (50 pts.)

a) If v(t) = 200cos(377t-10°) and I(t) = -10sin(377t-130°), find the complex power P+jQ, PF (specify lead or lag)

P+jQ = 866 + j500, PF = 0.866, Lead or Lag (Circle one)



$$\bar{V} = \frac{200}{\sqrt{2}} \angle -10^\circ$$

$$i(t) = -10\sin(377t - 130^\circ) = 10\cos(377t - 40^\circ)$$

$$\bar{I} = \frac{10}{\sqrt{2}} \angle -40^\circ$$

$$\bar{S} = \bar{V}\bar{I}^* = \left(\frac{200}{\sqrt{2}} \angle -10^\circ\right)\left(\frac{10}{\sqrt{2}} \angle 40^\circ\right) = 1000\angle 30^\circ = 866 + j500$$

$$PF = \cos 30^\circ = 0.866 \text{ lag}$$

b) Two loads in parallel, 30 kVA at 0.8 PF lag and 16 kW at 0.8 PF lead, are supplied by a source $\bar{V} = 100\angle 0^\circ$ V. Find the total current (magnitude) supplied by the source and the combined PF (specify lead or lag)

I = 404.5 A, PF = 0.9889, Lead or Lag (Circle one)

$$\bar{S}_T = \bar{S}_1 + \bar{S}_2 = 30(0.8 + j0.6) + \frac{16}{0.8}(0.8 - j0.6)$$

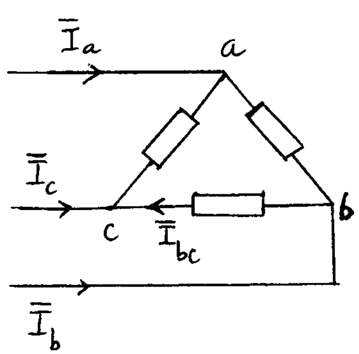
$$= 24 + j18 + 16 - j12 = 40 + j6 \text{ kVA}$$

$$= 40.45 \angle 8.53^\circ \text{ kVA}$$

$$I = \frac{S_T}{V} = \frac{40450}{100} = 404.5 \text{ A}$$

$$PF = \cos 8.53^\circ = 0.9889 \quad \text{lag}$$

c) For the sequence a-b-c if $\bar{I}_a = 30\angle -10^\circ$, the phasor $\bar{I}_{bc}$ is $17.3\angle -100^\circ$



$$\bar{I}_{ab} = \bar{I}_a \times \frac{1}{\sqrt{3}} \angle 30^\circ = \frac{30}{\sqrt{3}} \angle 20^\circ$$

$$\bar{I}_{bc} = \frac{30}{\sqrt{3}} \angle -100^\circ = 17.3 \angle -100^\circ$$

d) Two loads of 100+j100 kVA and 60-j20 kVA are connected in parallel. What is the kVAr of capacitor required to improve the PF to 0.9 lagging

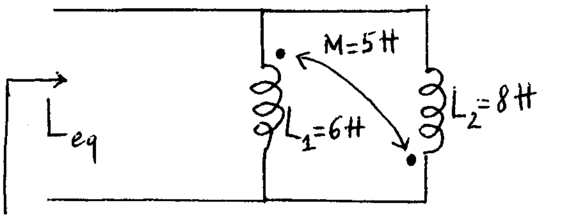
$$\bar{S}_T = 100+j100 + 60-j20 = 160+j80 \text{ kVA}$$

$$P = 160 \text{ kW}, \quad Q_{old} = 80 \text{ kVAr}$$

$$Q_{new} = P\tan(\cos^{-1}0.9) = 77.5 \text{ kVAr}$$

$$Q_{cap} = Q_{new} - Q_{old} = 77.5 - 80 = \boxed{-2.5 \text{ kVAr}}$$

e) Find the equivalent inductance of the circuit below, L_{eq} = 23/24 = 0.9583 H



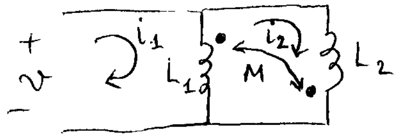
Left loop: $v = L_1\frac{d}{dt}(i_1-i_2) - M\frac{di_2}{dt}$

Right loop: $0 = L_1\frac{d}{dt}(i_2-i_1) + L_2\frac{di_2}{dt} + M\frac{di_2}{dt} + M\frac{d}{dt}(i_2-i_1)$

$$\Rightarrow 0 = (L_1+L_2+2M)\frac{di_2}{dt} - (L_1+M)\frac{di_1}{dt}$$

$$\Rightarrow \frac{di_2}{dt} = \frac{L_1+M}{L_1+L_2+2M}\frac{di_1}{dt}$$

Apply a test voltage at terminals



Substitute into the first equation:

$$v = L_1\frac{di_1}{dt} - (L_1+M)\frac{di_2}{dt} = \left[L_1 - \frac{(L_1+M)^2}{L_1+L_2+2M}\right]\frac{di_1}{dt}$$

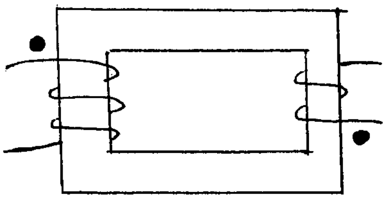
$$v = \frac{L_1L_2-M^2}{L_1+L_2+2M}\frac{di_1}{dt} \Rightarrow L_{eq} = \frac{L_1L_2-M^2}{L_1+L_2+2M} = \frac{23}{24}$$

f) Two coils which are coupled have L1 = 81 mH, L2 = 36 mH, and the coupling coefficient k = 0.9. Find the mutual inductance.

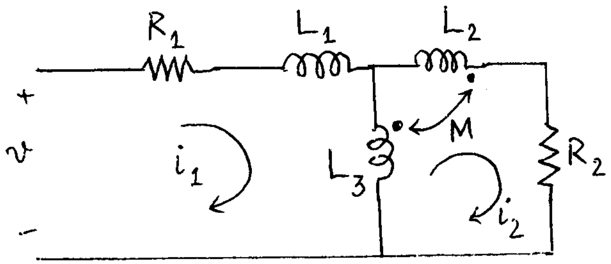
M = 48.6 mH

$$M = k\sqrt{L_1L_2} = 0.9\sqrt{81\times 36} = 48.6$$

g) Put dot markings on the coils



h) Write the loop equations. Note that L1 is not coupled to other coils

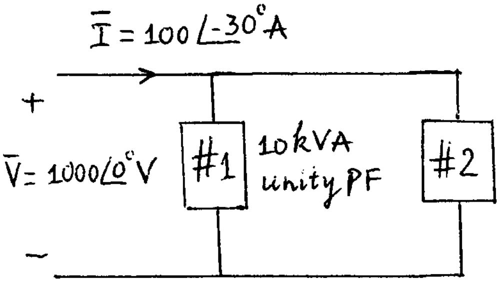


$$v = R_1 i_1 + L_1 \frac{di_1}{dt} + L_3 \frac{d}{dt}(i_1 - i_2) - M \frac{di_2}{dt}$$

$$0 = L_2 \frac{di_2}{dt} + R_2 i_2 + L_3 \frac{d}{dt}(i_2 - i_1) + M \frac{d}{dt}(i_2 - i_1) + M \frac{di_2}{dt}$$

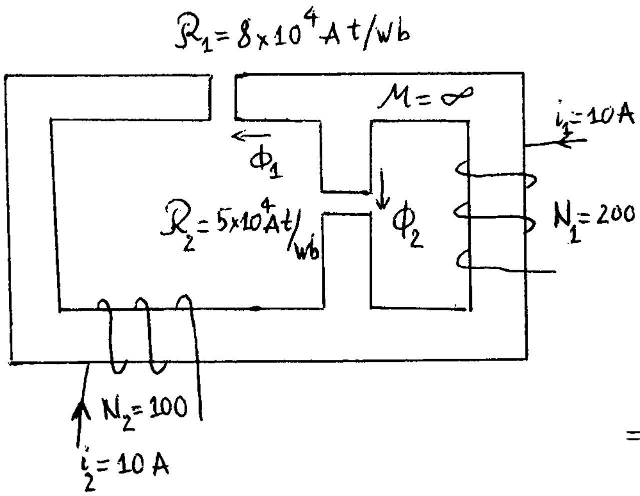
i) Specify the load #2 in term of real power, PF (lead or lag)

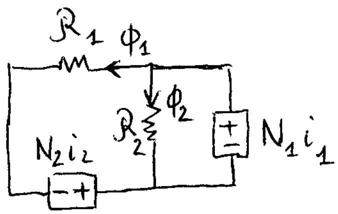
P = 76.6 kW, PF = 0.8374, Lead or Lag (Circle one)



$$\bar{S}_2 = \bar{V}\bar{I}^* - \bar{S}_1$$
$$= (1000\angle 0^\circ)(100\angle 30^\circ) - 10\times 10^3$$
$$= 76.6 + j50 \text{ kVA}$$
$$= 91.48\angle 33.13^\circ \text{ kVA}$$
$$PF = \cos 33.13^\circ = 0.8374 \text{ lag}$$

j) Find the fluxes flowed across the air gaps, Φ_1 = 0.0375 Wb, Φ_2 = 0.04 Wb





$$\Phi_2 = \frac{N_1 i_1}{\mathcal{R}_2} = \frac{200\times 10}{5\times 10^4} = 0.04 \text{ Wb}$$

$$N_1 i_1 - \mathcal{R}_1 \Phi_1 + N_2 i_2 = 0$$

$$\Rightarrow \Phi_1 = \frac{N_1 i_1 + N_2 i_2}{\mathcal{R}_1} = \frac{2000 + 1000}{8\times 10^4} = .0375 \text{ Wb}$$

Problem 2 (25 pts.)

The following balanced three phase loads are supplied from a wye-connected source whose line-line voltage is 2400 V. Load #1: 120 kVA at o.96 PF lag; Load #2: 180 kVA at 0.8 PF lag; and Load #3: 100 + j100 kVA.

(1) Find the total line current (magnitude) and PF of the total load
(2) What is the total capacitive kVAr needed to improve the PF to 0.9 PF lag
(3) What is the phase angle between $\bar{I}_{a}$ and $\bar{V}_{ab}$. Take $\bar{V}_{an}$ as reference.

(1) $\bar{S}_T = \bar{S}_1 + \bar{S}_2 + \bar{S}_3 = 120\angle\cos^{-1}0.96 + 180\angle\cos^{-1}0.8 + 100 + j100$

$= 359.2 + j241.6 \text{ kVA} = 432.89\angle 33.93^\circ \text{ kVA}$

$I_\ell = \dfrac{S_T}{\sqrt{3}\,V_{\ell,\ell}} = \dfrac{432890}{\sqrt{3}\;2400} = \boxed{104.14\text{ A}}$

$PF = \cos 33.93^\circ = \boxed{0.8297 \text{ lag}}$

(2) $P = 359.2 \text{ kW}$, $Q_{old} = 241.6 \text{ kVAr}$

$Q_{new} = P\tan(\cos^{-1}0.9) = 173.97 \text{ kVAr}$

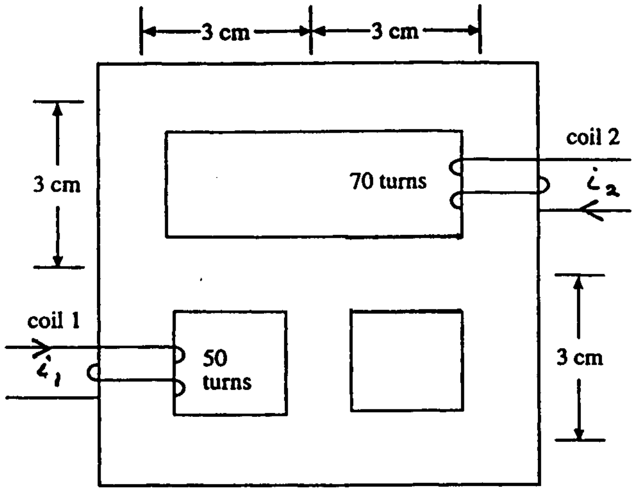
$Q_{cap} = Q_{new} - Q_{old} = \boxed{-67.63 \text{ kVAr}}$

(3) $\bar{V}_{an} = V_{an}\angle 0^\circ$, $\bar{V}_{ab} = V_{ab}\angle 30^\circ$

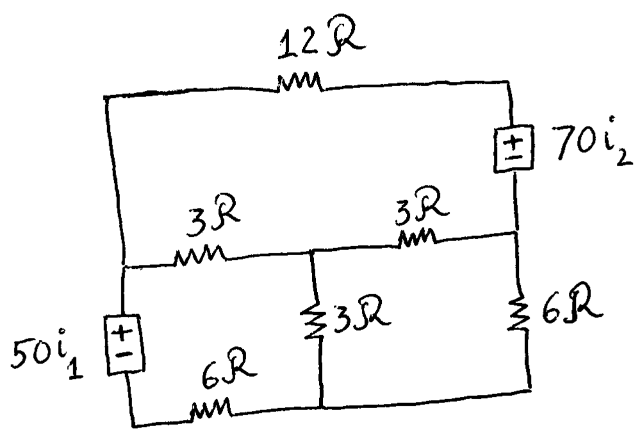
$\bar{I}_a = I_\ell\angle -33.93^\circ$

$\angle\bar{V}_{ab} - \angle\bar{I}_a = 30 + 33.93 = \boxed{63.93^\circ}$

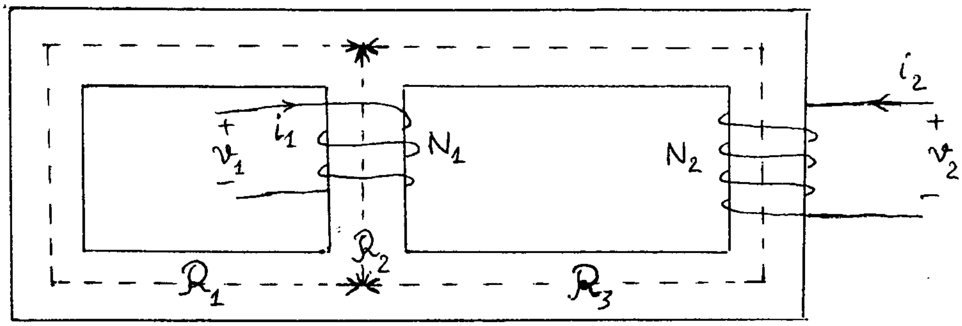
Problem 3 (25 pts.)



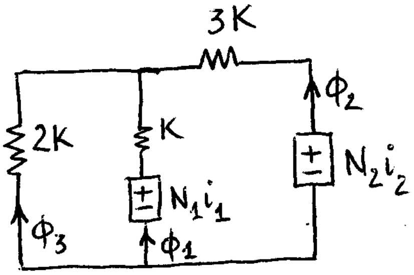
(1) Draw the equivalent circuit for the magnetic device shown above. The magnetic material of the core has reluctance $\Re$ per cm.



(2) Determine the self inductances L_1, L_2, and the mutual inductance M in terms of K, N_1, and N_2.



$\mathcal{R}_1 = 2K \quad \mathcal{R}_2 = K \quad \mathcal{R}_3 = 3K$



$$\phi_1 + \phi_2 + \phi_3 = 0$$

$$\Rightarrow \phi_3 = -\phi_1 - \phi_2$$

$$-N_1 i_1 + K\phi_1 - 2K\phi_3 = 0 \Rightarrow -N_1 i_1 + K\phi_1 - 2K(-\phi_1 - \phi_2) = 0$$

$$-N_1 i_1 + 3K\phi_1 + 2K\phi_2 = 0$$

$$\Rightarrow \phi_2 = \frac{1}{2K}\left(N_1 i_1 - 3K\phi_1\right)$$

$$-N_1 i_1 + K\phi_1 - 3K\phi_2 + N_2 i_2 = 0 \Rightarrow -N_1 i_1 + K\phi_1 - 3K\frac{1}{2K}(N_1 i_1 - 3K\phi_1) + N_2 i_2 = 0$$

$$-\frac{5}{2}N_1 i_1 + \frac{11K}{2}\phi_1 + N_2 i_2 = 0$$

$$\Rightarrow \phi_1 = \frac{5N_1}{11K} i_1 - \frac{2N_2}{11K} i_2$$

$$\lambda_1 = N_1 \phi_1 = \frac{5N_1^2}{11K} i_1 - \frac{2N_1 N_2}{11K} i_2 = L_1 i_1 + M i_2$$

$$\Rightarrow \boxed{L_1 = \frac{5N_1^2}{11K}, \quad M = -\frac{2N_1 N_2}{11K}}$$

$$\phi_2 = \frac{1}{2K}\left(N_1 i_1 - 3K\left(\frac{5N_1}{11K} i_1 - \frac{2N_2}{11K} i_2\right)\right), \quad \lambda_2 = N_2\phi_2 = -\frac{2N_1 N_2}{11K} i_1 + \frac{3N_2^2}{11K} i_2$$

$$= -\frac{2N_1}{11K} i_1 + \frac{3N_2}{11K} i_2 \qquad\qquad = M i_1 + L_2 i_2$$

$$\Rightarrow \boxed{L_2 = \frac{3N_2^2}{11K}}$$