ECE 330 Exam #2, Spring 2019 Name: Solution
90 Minutes

Section (Check One) MWF 2pm _______ MWF 3pm _______

1. ________ / 25 2. ________ / 25

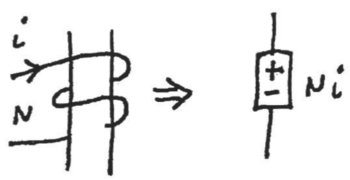
3. ________ / 25 4. ________ / 25 Total ________ / 100

Useful information

$\sin(x) = \cos(x - 90^\circ)$ $\bar{V} = \bar{Z}\bar{I}$ $\bar{S} = \bar{V}\bar{I}^*$ $\mu_0 = 4\pi \cdot 10^{-7}$ H/m

$\int_C \mathbf{H} \cdot \mathbf{dl} = \int_S \mathbf{J} \cdot \mathbf{n} da$ $\int_C \mathbf{E} \cdot \mathbf{dl} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot \mathbf{n} da$ $MMF = Ni = \phi \Re$

$\Re = \frac{l}{\mu A}$ $B = \mu H$ $\phi = BA$ $\lambda = N\phi$ $\lambda = Li$ (if linear)



$W_m = \int_0^\lambda i \, d\hat{\lambda}$ $W_m' = \int_0^i \lambda \, d\hat{i}$ $W_m + W_m' = \lambda i$ $f^e = \frac{\partial W_m'}{\partial x} = -\frac{\partial W_m}{\partial x}$ $x \to \theta$

$f^e \to T^e$

$\underset{a \to b}{EFE} = \int_a^b i \, d\lambda$ $\underset{a \to b}{EFM} = -\int_a^b f^e dx$ $\underset{a \to b}{EFE} + \underset{a \to b}{EFM} = W_{mb} - W_{ma}$ $\lambda = \frac{\partial W_m'}{\partial i}$ $i = \frac{\partial W_m}{\partial \lambda}$

$$\dot{x}_1 = f_1(x_1, x_2) \text{ and } \dot{x}_2 = f_2(x_1, x_2)$$

$$x(t_0 + \Delta t) \approx x(t_0) + \Delta t \cdot \left. \frac{dx}{dt} \right|_{t = t_0}$$

Problem 1 (25 points)

A single-phase generator consists of a coil on the stator and a coil on the rotor with a mutual inductance variation as 0.1cos(θ) (Henries) where θ is the angle from the stator field axis to the rotor field axis. The rotor is being driven at a constant speed of 377 radians per second and the rotor coil has a constant dc current i_r = 5 A. The self inductances of the stator and rotor are both constants and you may assume a linear magnetic core.

a) Compute the open-circuit stator voltage (i_s = 0) as a function of time (recall that dθ/dt=ω and assume some angle θ= θo at time zero)

$$\lambda_s = L_s i_s + 0.1\cos(\theta) i_r$$
$$\lambda_r = 0.1\cos(\theta) i_s + L_r i_r$$
$$\theta = 377t + \theta_0$$

$$V_s = \frac{d\lambda_s}{dt}$$

$$V_s = \cancel{L_s \frac{di_s}{dt}}^{\to 0} + \cancel{0.1\cos(\theta)\frac{di_r}{dt}}^{\to 0} - 0.1\sin(\theta)\frac{d\theta}{dt} i_r$$

$$V_s = -0.1(377)(5)\sin(377t+\theta_0) \Rightarrow \boxed{V_s = -188.5\sin(377t+\theta_0)\ V}$$

$$V_s = -188.5\sin(377t+\theta_0)$$
$$V_s = 188.5\sin(-[377t+\theta_0])$$
$$V_s = 188.5\cos\left(-[377t+\theta_0] - \frac{\pi}{2}\right)$$
$$\boxed{V_s = 188.5\cos\left(377t + \left[\theta_0 + \frac{\pi}{2}\right]\right) V}$$

b) What is the torque of electrical origin when i_s = 10 Amps and θ = 45°?

$$T^e = \frac{\partial W_m'}{\partial \theta}$$

$$W_m' = \int_0^{i_s} \lambda_s(\hat{i}_s, i_r = 0, \theta)\, d\hat{i}_s + \int_0^{i_r} \lambda_r(i_s, \hat{i}_r, \theta)\, d\hat{i}_r$$

$$= \int_0^{i_s} L_s \hat{i}_s\, d\hat{i}_s + \int_0^{i_r} (0.1\cos(\theta) i_s + L_r \hat{i}_r)\, d\hat{i}_r$$

$$= \frac{1}{2} L_s i_s^2 + 0.1\cos(\theta) i_s i_r + \frac{1}{2} L_r i_r^2$$

$$T^e = \frac{\partial W_m'}{\partial \theta} \Rightarrow T^e = -0.1\sin(\theta) i_s i_r$$

$$T^e = -0.1(10)(5)\sin(45^\circ) \Rightarrow \boxed{T^e = -3.536\,Nm}$$

(There is an extra page at the end if you need it)

Problem 2 (25 Points)

The flux linkages for a 2-coil system are given as

$$\lambda_1 = \left(\frac{0.0085}{1+\frac{x}{l}}\right) i_1 - \left(\frac{0.0021}{1+\frac{x}{l}}\right) i_2$$

$$\lambda_2 = -\left(\frac{0.0021}{1+\frac{x}{l}}\right) i_1 + \left(\frac{0.0021}{1+\frac{x}{l}}\right) i_2$$

For this system, find:

a) The co-energy. (9 points)
b) The force of electric origin. (9 points)
c) What is the force of electric origin when i_1=1 A, i_2=0.5 A, x=1 cm, and l=20 cm? (7 points)

a) $W_m' = \int_0^{i_1} \lambda_1(\hat{i}_1, i_2=0, x)\, d\hat{i}_1 + \int_0^{i_2} \lambda_2(i_1, \hat{i}_2, x)\, d\hat{i}_2$

$= \int_0^{i_1} \left(\frac{0.0085}{1+\frac{x}{l}}\right)\hat{i}_1\, d\hat{i}_1 + \int_0^{i_2}\left(-\left[\frac{0.0021}{1+\frac{x}{l}}\right] i_1 + \left[\frac{0.0021}{1+\frac{x}{l}}\right]\hat{i}_2\right) d\hat{i}_2$

$= \frac{1}{2}\left(\frac{0.0085}{1+\frac{x}{l}}\right) i_1^2 + \frac{1}{2}\left(\frac{0.0021}{1+\frac{x}{l}}\right) i_2^2 - \left(\frac{0.0021}{1+\frac{x}{l}}\right) i_1 i_2$

$$\boxed{W_m' = \left(\frac{0.00425}{1+\frac{x}{l}}\right) i_1^2 + \left(\frac{0.00105}{1+\frac{x}{l}}\right) i_2^2 - \left(\frac{0.0021}{1+\frac{x}{l}}\right) i_1 i_2}$$

b) $f^e = \frac{\partial W_m'}{\partial x} \Rightarrow \boxed{f^e = -\frac{1}{l}\left(\frac{0.00425}{[1+\frac{x}{l}]^2}\right) i_1^2 - \frac{1}{l}\left(\frac{0.00105}{[1+\frac{x}{l}]^2}\right) i_2^2 + \frac{1}{l}\left(\frac{0.0021}{[1+\frac{x}{l}]^2}\right) i_1 i_2}$

c) $f^e = \frac{-\frac{1}{l}}{[1+\frac{x}{l}]^2}\left(0.00425 i_1^2 + 0.00105 i_2^2 - 0.0021 i_1 i_2\right)$

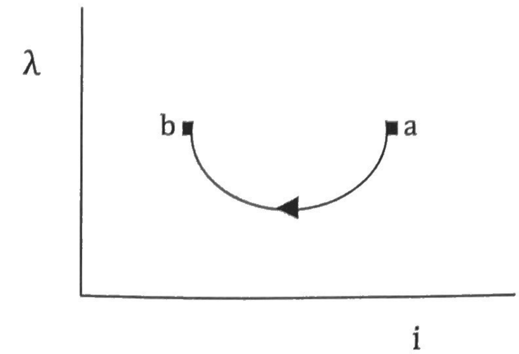
$f^e = \frac{-1}{0.2(1+\frac{0.01}{0.2})^2}\left(0.00425(1)^2 + 0.00105(0.5)^2 - 0.0021(1)(0.5)\right)$

$$\boxed{f^e = -0.0157\text{ N}}$$

(There is an extra page at the end if you need it)

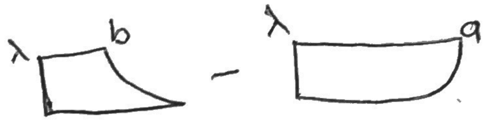
Problem 3 (25 pts.)

An electromechanical system with $\lambda = L(x)i$ is operated through the transition from a to b as shown below - note that the flux linkage at a is equal to the flux linkage at b:



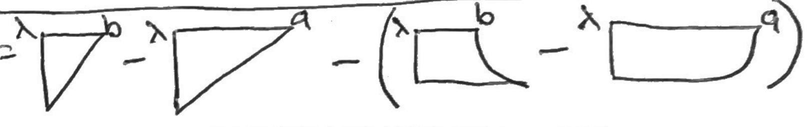
a) Find the energy transferred from the electrical system into the coupling field as the system moves from a to b as shown. (give a graphical answer)
b) Find the energy transferred from the mechanical system into the coupling field as the system moves from a to b as shown (give a graphical answer)
c) If the device moves back from b to a along a constant flux linkage path, find the energy transferred from the electrical system into the coupling field for this motion and the energy transferred from the mechanical system into the coupling field during this motion.
d) If you consider the combined motion of a) to c) above to be one cycle, is this a motor or a generator? Explain why.

a) $\underset{a\to b}{EFE} = \int_{\lambda_a}^{\lambda_b} i(\lambda, x)\,d\lambda =$



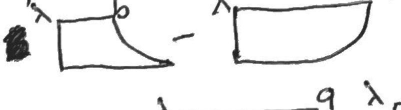
b) $\underset{a\to b}{\Delta W_m} = \underset{a\to b}{EFE} + \underset{a\to b}{EFM} \Rightarrow \underset{a\to b}{EFM} = \underset{a\to b}{\Delta W_m} - \underset{a\to b}{EFE}$

$\underset{a\to b}{EFM} =$

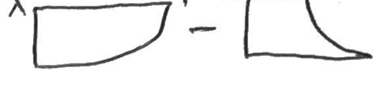


c) $\underset{cycle}{EFE} + \underset{cycle}{EFM} = 0$

$\underset{cycle}{EFE} =$



$\underset{cycle}{EFM} = -\underset{cycle}{EFE} =$



d) Generator. $\underset{cycle}{EFE} < 0$

$(\underset{cycle}{EFM} > 0)$

(There is an extra page at the end if you need it)

Problem 4 (25 points)

The equation of motion governing the angle of a pendulum connected to a torsional spring under gravity is given as

$$\frac{d^2\theta}{dt^2} = -\frac{\kappa}{ml^2}\theta + \frac{g}{l}\sin(\theta)$$

For this given system:

a) Write the equation of motion in state space form. (7 points)

b) Write the equation(s) you would have to solve (BUT DO NOT SOLVE) for determining the equilibrium positions for the pendulum. (7 points)

c) The function $\frac{\sin(\theta)}{\theta} \approx 1 - \frac{\theta^2}{6}$. Using this approximation, what are the equilibrium positions for the pendulum? (7 points)

d) Assuming that the pendulum starts at an initial position of $\theta = 0.01$ rad, $\dot{\theta} = 0$ rad/s, m=1 kg, l=0.1 m, $\kappa = 10$ Nm/rad, and g=9.8 m/s^2, find the state variables at t=0.002 s using a step size $dt = 0.001$ s. (4 points)

a) $\underline{x} = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}$

$x_1 = \theta$

$x_2 = \dot{\theta}$

$$\boxed{\begin{aligned} \frac{dx_1}{dt} &= x_2 \\ \frac{dx_2}{dt} &= \frac{-\kappa}{ml^2}x_1 + \frac{g}{l}\sin(x_1) \end{aligned}}$$

b) $\frac{d\underline{x}}{dt} = 0 \Rightarrow$ $x_2 = 0$, $\frac{-\kappa}{ml^2}x_1 + \frac{g}{l}\sin(x_1) = 0$ $\Rightarrow$ $\boxed{x_2 = 0, \quad \frac{\sin(x_1)}{x_1} = \frac{\kappa}{mgl}}$

c) $x_2 = 0$, $1 - \frac{x_1^2}{6} = \frac{\kappa}{mgl} \Rightarrow$ $x_2 = 0$, $\frac{x_1^2}{6} = 1 - \frac{\kappa}{mgl} \Rightarrow$ $x_2 = 0$, $x_1^2 = 6\left(1 - \frac{\kappa}{mgl}\right)$

$$\boxed{x_1 = \sqrt{6\left(1 - \frac{\kappa}{mgl}\right)}, \quad x_2 = 0} \qquad \boxed{x_1 = -\sqrt{6\left(1 - \frac{\kappa}{mgl}\right)}, \quad x_2 = 0}$$

(There is an extra page at the end if you need it)

d) $\frac{dx_1}{dt} = x_2$ $\qquad$ $\frac{dx_2}{dt} = -\frac{K}{ml^2} x_1 + \frac{g}{l}\sin(x_1)$

$\frac{x_1^n - x_1^{n-1}}{\Delta t} = x_2^{n-1}$ $\qquad$ $\frac{x_2^n - x_2^{n-1}}{\Delta t} = \frac{-K}{ml^2} x_1^{n-1} + \frac{g}{l}\sin(x_1^{n-1})$

$$\boxed{x_1^n = x_1^{n-1} + \Delta t\, x_2^{n-1}}$$

$$\boxed{x_2^n = x_2^{n-1} + \Delta t\left(-\frac{K}{ml^2} x_1^{n-1} + \frac{g}{l}\sin(x_1^{n-1})\right)}$$

$\frac{K}{ml^2} = \frac{10}{1(0.1)^2} = 1000$

$\frac{g}{l} = \frac{9.8}{0.1} = 98$

t	x_1	x_2
0	0.01	0
0.001	0.01	-0.00902
0.002	0.00999	-0.01804

(This is an extra page if you need it)