ECE 330 Exam 1: Spring 2019 NAME Solution
90 minutes

Section (Check one) MWF 2pm__________ MWF 3pm__________

1. __________/25 2. __________/25

3. __________/25 4. __________/25 TOTAL _________/100

USEFUL INFORMATION

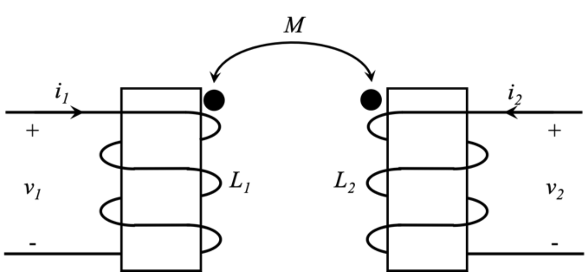
sin(x)=cos(x-90°)	$\bar{V} = \bar{Z}\bar{I}$	$\bar{S} = \bar{V}\bar{I}^* = P + jQ$	$\bar{S}_{3\varphi} = \sqrt{3}V_L I_L \angle\theta$
0<θ<180° (lag)	$I_L = \sqrt{3}I_\varphi$ (delta)	$\bar{Z}_Y = \bar{Z}_\Delta/3$	
-180°<θ<0 (lead)	$V_L = \sqrt{3}V_\varphi$ (wye)	$\mu_0 = 4\pi \times 10^{-7}$H/m	

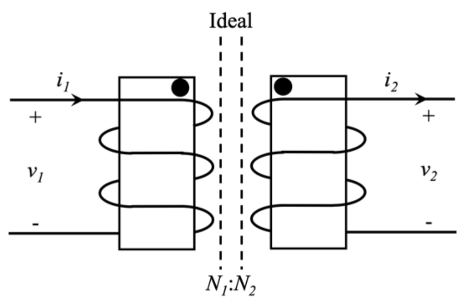
ABC phase sequence has A at 0, B at -120°, and C at +120°

$\int \underline{H} \cdot \underline{dl} = \int \underline{J_f} \cdot \hat{n} dA$ $\int \underline{E} \cdot \underline{dl} = -\frac{d}{dt}\left(\int \underline{B} \cdot \hat{n} dA\right)$ $\mathcal{R} = \frac{l}{\mu A}$ $Ni = \mathcal{R}\varphi$

$\varphi = BA$ $\lambda = N\varphi = Li$ (if linear) $v = \frac{d\lambda}{dt}$ $k = \frac{M}{\sqrt{L_1 L_2}}$

1hp=746 W



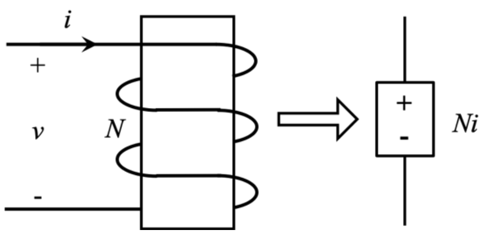


$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$

$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$

$a = \frac{N_1}{N_2}$ $N_1 i_1 = N_2 i_2$

$\frac{V_1}{V_2} = \frac{N_1}{N_2}$



Problem 1 (25 Points)

3 single-phase loads are connected in parallel to a generator that supplies 240 V. The loads are given as:

1) A purely resistive load that consumes 100 kW
2) A load that consumes 300 kVA at a power factor of 0.8 lagging
3) A load with impedance Z=30+j40 Ω

a) What is the total power supplied by the source? (12 points)

$\bar{S}_1 = 100\angle 0^\circ$ kVA

$\theta_2 = \cos^{-1}(0.8) \Rightarrow \theta_2 = 36.87^\circ$

$\bar{S}_2 = 300\angle 36.87^\circ$ kVA

$\bar{Z}_3 = 30 + j40\,\Omega$

$= 50\angle 53.13^\circ\,\Omega$

$\bar{V} = \bar{Z}_3 \bar{I}_3 \Rightarrow \bar{I}_3 = \frac{\bar{V}}{\bar{Z}_3}$

$\bar{S}_3 = \bar{V}\bar{I}^* = \frac{V^2}{\bar{Z}^*} \Rightarrow \bar{S}_3 = \frac{240^2}{50\angle -53.13^\circ} \Rightarrow \bar{S}_3 = 1.152\angle 53.13^\circ$ kVA

$\bar{S}_{TOT} = \bar{S}_1 + \bar{S}_2 + \bar{S}_3$

$= (100 + j0) + (240 + j180)$

$+ (0.691 + j0.922)$

$\bar{S}_{TOT} = 340.691 + j180.922$ kVA

$= 385.75\angle 27.97^\circ$ kVA

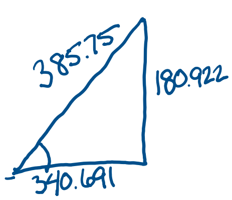
b) What is the source current? (5 points)

$\bar{S}_{TOT} = \bar{V}\bar{I}_s^*$

$\bar{I}_s^* = \frac{\bar{S}_{TOT}}{\bar{V}} \Rightarrow \bar{I}_s^* = 1607.3\angle 27.97^\circ$ A

$\bar{I}_s = 1607.3\angle -27.97^\circ$ A

c) A capacitor is connected in parallel with the loads. How much power should it supply to achieve an overall power factor of 0.975 lagging? (8 points)



$\theta_n = \cos^{-1}(0.975)$

$\theta_n = 12.84^\circ$

$Q_n = P\tan(\theta_n)$

$Q_n = 77.65$ kVAR

$Q_n = Q_{old} + Q_c$

$Q_c = Q_n - Q_{old} \Rightarrow Q_c = -103.272$ kVAR

103.272 kVAR of capacitance

Problem 2 (25 points)

Three 3-phase loads are connected in parallel to a wye-connected source with 480 V line to line. The loads are given as:

1) A wye connected load consuming 250 kVA at a 0.707 lagging power factor
2) A wye connected load consuming 75 kW at a 0.9 lagging power factor
3) A delta connected load with impedance Z=100+j100 Ω

a) What is the total line current supplied by the source? (13 points)

$\bar{S}_1 = 250\angle 45^\circ$ kVA

$P_2 = S_2 (PF_2) \Rightarrow S_2 = \frac{P_2}{PF_2} \Rightarrow S_2 = 83.33$ kVA $\quad \theta_2 = \cos^{-1}(0.9) \Rightarrow \theta_2 = 25.84$

$\bar{S}_2 = 83.33\angle 25.84^\circ$ kVA

$\bar{I}_{\phi 3} = \frac{\bar{V}_L}{\bar{Z}_3} \Rightarrow \bar{I}_{\phi 3} = 3.39\angle -45^\circ$ A

$\bar{S}_3 = 3(480)(3.39)\angle 45^\circ \Rightarrow \bar{S}_3 = 4.882\angle 45^\circ$ kVA

$\bar{S}_{TOT} = \bar{S}_1 + \bar{S}_2 + \bar{S}_3$

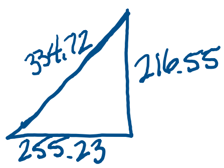
$= (176.78 + j176.78) + (75 + j36.32) + (3.45 + j3.45)$ kVA

$= 255.23 + j216.55$ kVA

$\bar{S}_{TOT} = 334.72\angle 40.31^\circ$ kVA $= \sqrt{3} V_L I_L \angle\theta$

$\boxed{\bar{I}_L = 402.61\angle -40.31^\circ}$

b) A delta connected capacitor bank is added in parallel to the three loads to make the overall power factor 0.99 lagging. What is the new line current? (12 points)



$\theta_n = \cos^{-1}(0.99)$

$\theta_n = 8.11^\circ$

$Q_n = 255.23 \tan(8.11^\circ)$

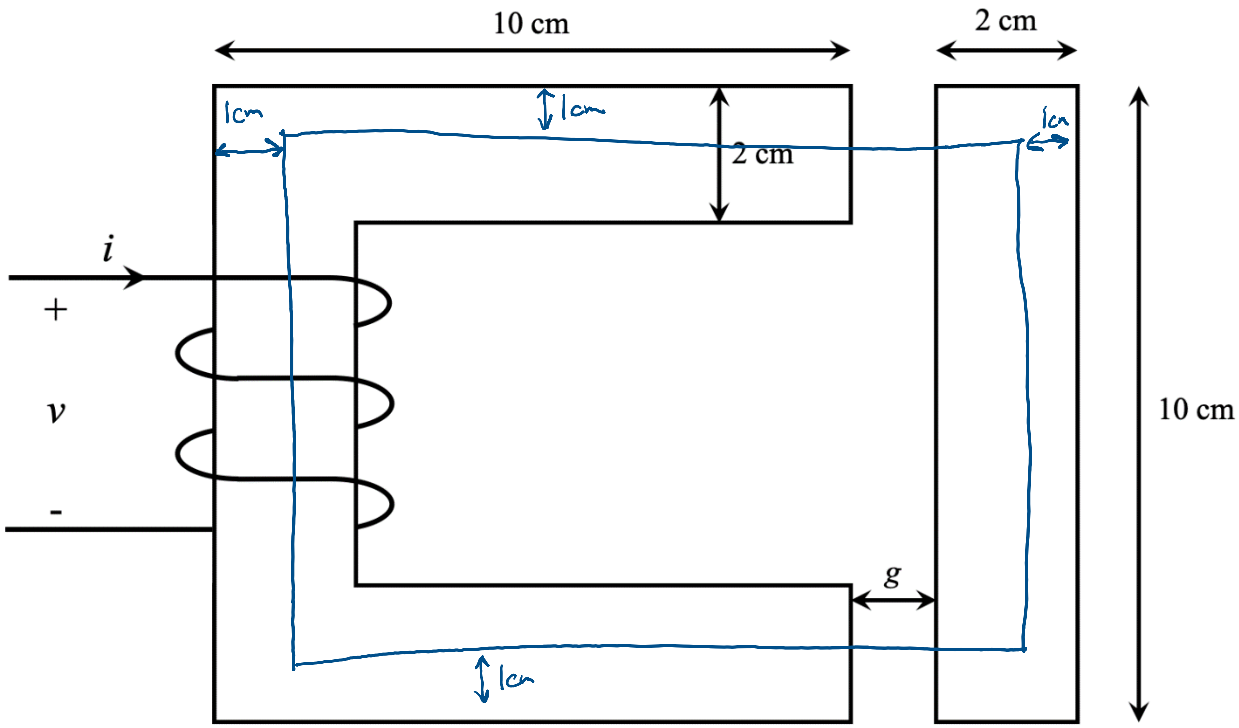
$= 36.37$ kVAR

$\bar{S}_n = 255.23 + j36.37$ kVA

$= 257.81\angle 8.11^\circ = \sqrt{3} V_L I_L \angle\theta$

$\boxed{\bar{I}_L = 310.1\angle -8.11^\circ}$

Problem 3 (25 Points)



A coil is wound 150 times around an iron core (dimensions given above) with relative permeability 2500 that has an air gap *g* of 20 mm and depth into the page of 3 cm. With the voltage polarity and current direction as defined above:

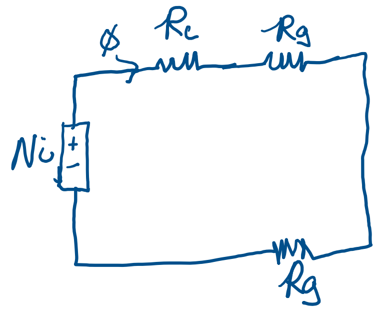
a) What is the reluctance path length inside the iron core? (7 points)

$9+9+8=26$ cm

$1+1+8=10$ cm

$l_c = 36$ cm

b) Draw the magnetic equivalent circuit and determine the reluctance values for the iron and air gap (neglect fringing). (8 points)



$$R_c = \frac{l_c}{\mu_0 \mu_r A} \Rightarrow \boxed{R_c = 1.910 \times 10^5 \text{ At/Wb}}$$

$A = 6 cm^2$

$$R_g = \frac{g}{\mu_0 A} \Rightarrow \boxed{R_g = 2.65 \times 10^7 \text{ At/Wb}}$$

c) What is the inductance of the coil? (10 points)

$$\phi = \frac{Ni}{2R_g + R_c}$$

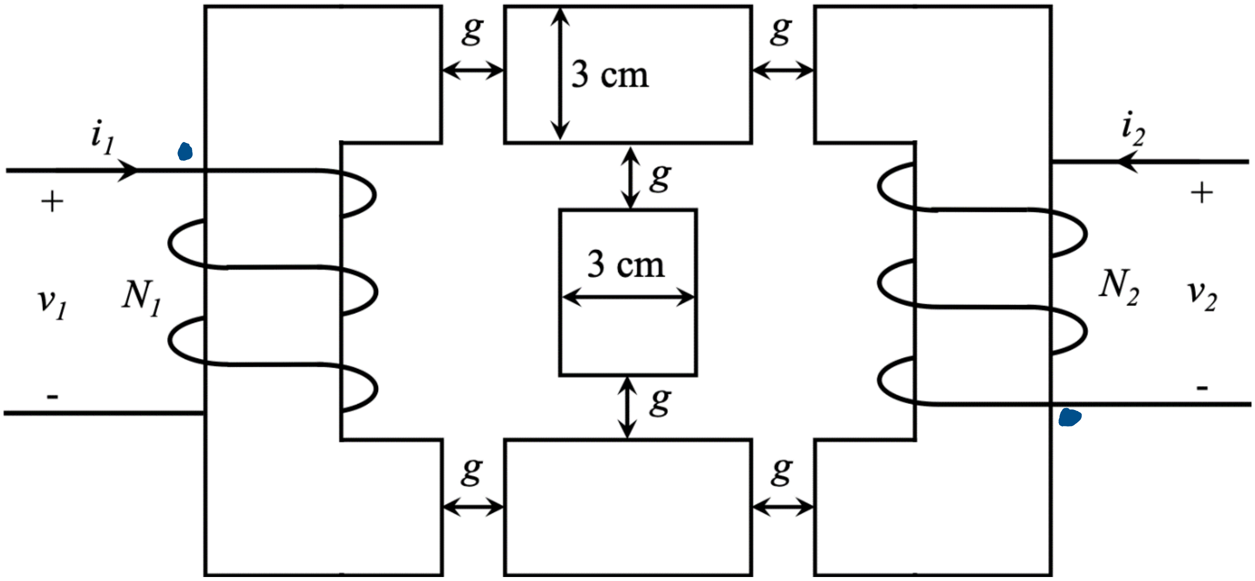
$$\phi = 2.82 \times 10^{-6} \, i$$

$$\lambda = N\phi$$

$$\lambda = 0.423 \times 10^{-3} \, i = Li$$

$$\boxed{L = 0.423 \text{ mH}}$$

Problem 4 (25 Points)

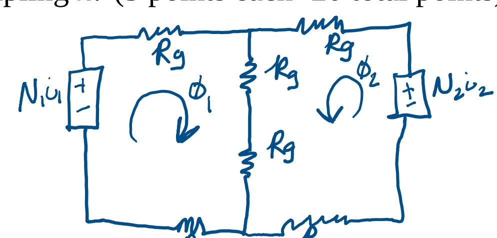


Two coils are wrapped around an iron core with infinite permeability as shown. Coil 1 has 200 turns while coil 2 has 100 turns. The air gap *g* is 10 mm and the depth into the page is 3 cm. Neglect fringing.

a) Put the dot markings on the two coils. (5 points)

b) What is the self-inductance of the two coils, the mutual inductance, and the coefficient of coupling *k*? (5 points each=20 total points)

$R_g = \frac{g}{\mu_0 A} = 8.842\times10^6$ At/Wb



$N_1 i_1 = R_g\phi_1 + 2R_g(\phi_1+\phi_2) + R_g\phi_1 \Rightarrow N_1 i_1 = 4R_g\phi_1 + 2R_g\phi_2$

$N_2 i_2 = R_g\phi_2 + R_g(\phi_1+\phi_2) + R_g\phi_2 \Rightarrow N_2 i_2 = 2R_g\phi_1 + 4R_g\phi_2$

$\phi_2 = \frac{N_2 i_2 - 2R_g\phi_1}{4R_g}$

$N_1 i_1 = 4R_g\phi_1 + \frac{1}{2}(N_2 i_2 - 2R_g\phi_1)$

$N_1 i_1 = 3R_g\phi_1 + \frac{1}{2}N_2 i_2$

$\frac{N_1 i_1 - \frac{1}{2}N_2 i_2}{3R_g} = \phi_1 \Rightarrow \phi_1 = 7.54\times10^{-6} i_1 - 1.88\times10^{-6} i_2$

$\phi_2 = \frac{N_2 i_2 - \frac{2}{3}(N_1 i_1 - \frac{1}{2}N_2 i_2)}{4R_g} \Rightarrow \phi_2 = \frac{-\frac{2}{3}N_1 i_1 + \frac{4}{3}N_2 i_2}{4R_g} \Rightarrow \phi_2 = -3.77\times10^{-6} i_1 + 3.77\times10^{-6} i_2$

$\lambda_1 = N_1\phi_1 = 1.508\times10^{-3} i_1 - 0.377\times10^{-3} i_2$

$\lambda_2 = N_2\phi_2 = -0.377\times10^{-3} i_1 + 0.377\times10^{-3} i_2$

$L_1 = 1.508$ mH

$L_2 = 0.377$ mH

$M = 0.377$ mH

$k = \frac{M}{\sqrt{L_1 L_2}} = 0.5$