Zero-state impulse response of T.L.

(solve for v+ and v- when source voltage is a delta function)

$$V^+(t) = \sum_n \tau_S (\Gamma_L \Gamma_S)^n \delta(t - n\frac{2l}{v}) \equiv h^+(t)$$

$$V^-(t) = \sum_n \tau_S \Gamma_L (\Gamma_L \Gamma_S)^n \delta(t - (n+1)\frac{2l}{v}) \equiv h^-(t)$$

convolve impulse response function above with arbitrary source voltage f(t) to yield **general zero-state solution** and add appropriate delays to find voltage at arbitrary z:

$$V(z,t) = \sum_n \tau_S (\Gamma_L \Gamma_S)^n [f(t - n\frac{2l}{v} - \frac{z}{v}) + \cdots$$

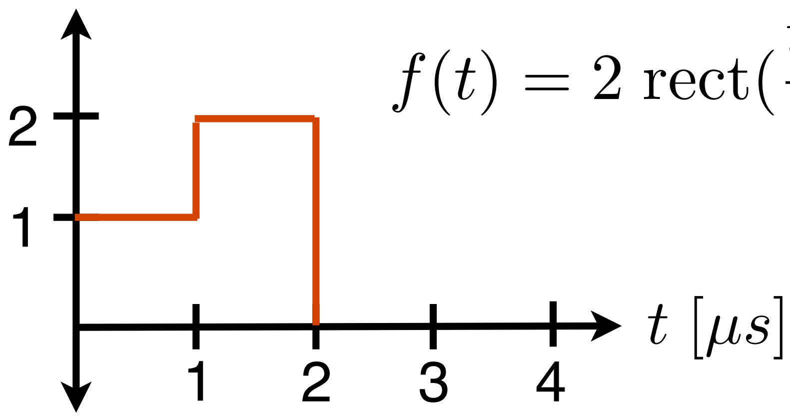
$$\cdots \Gamma_L f(t - (n+1)\frac{2l}{v} + \frac{z}{v})]$$

EXAMPLE 1: Consider a T.L. characterized by:

> length $l = 2400 \text{ m}$
> tx phase speed $v_p = c = 3 \times 10^8 \text{ m/s}$
> one-way travel time $T = l/v_p = 8\ \mu\text{s}$

> characteristic impedance $Z_0 = 50\ \Omega$
> load (resistive) $R_L = 100\ \Omega$
> internal (Thevenin) resistance $R_s = 0\ \Omega$

> source (input) voltage:



$$f(t) = 2\ \text{rect}\left(\frac{t - 3/2}{1\ \mu\text{s}}\right) + \text{rect}\left(\frac{t - 1/2}{1\ \mu\text{s}}\right)\ \text{V}$$

EXAMPLE 1:

injection and reflection:

$$\tau_s = \frac{50}{0+50} = 1$$

....so, v+ is not scaled initially relative to f(t)

$$\Gamma_L = \frac{100-50}{100+50} = \frac{1}{3}$$

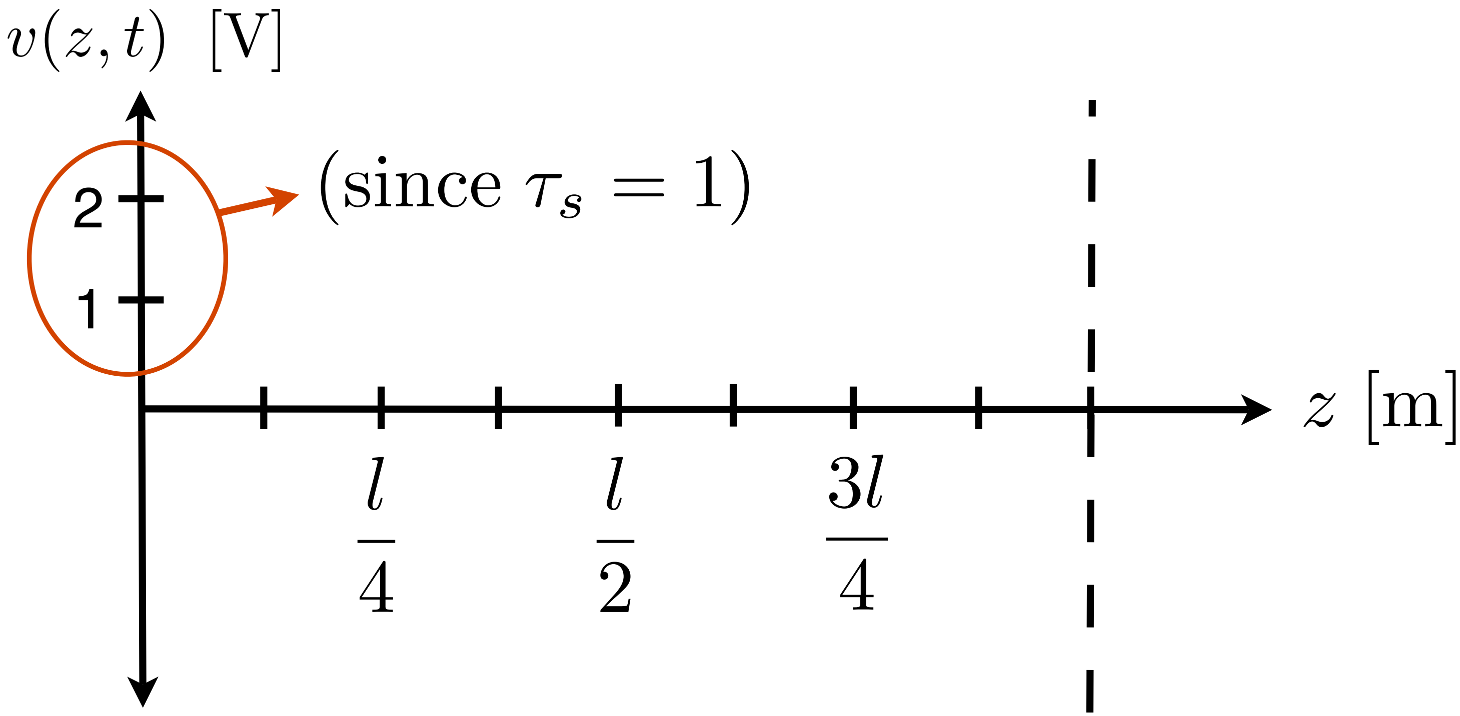
reflection at the load end scales v- by a factor of 1/3 relative to v+

$$\Gamma_S = \frac{0-50}{0+50} = -1$$

reflection at the source end flips the sign of v+ relative to v-.

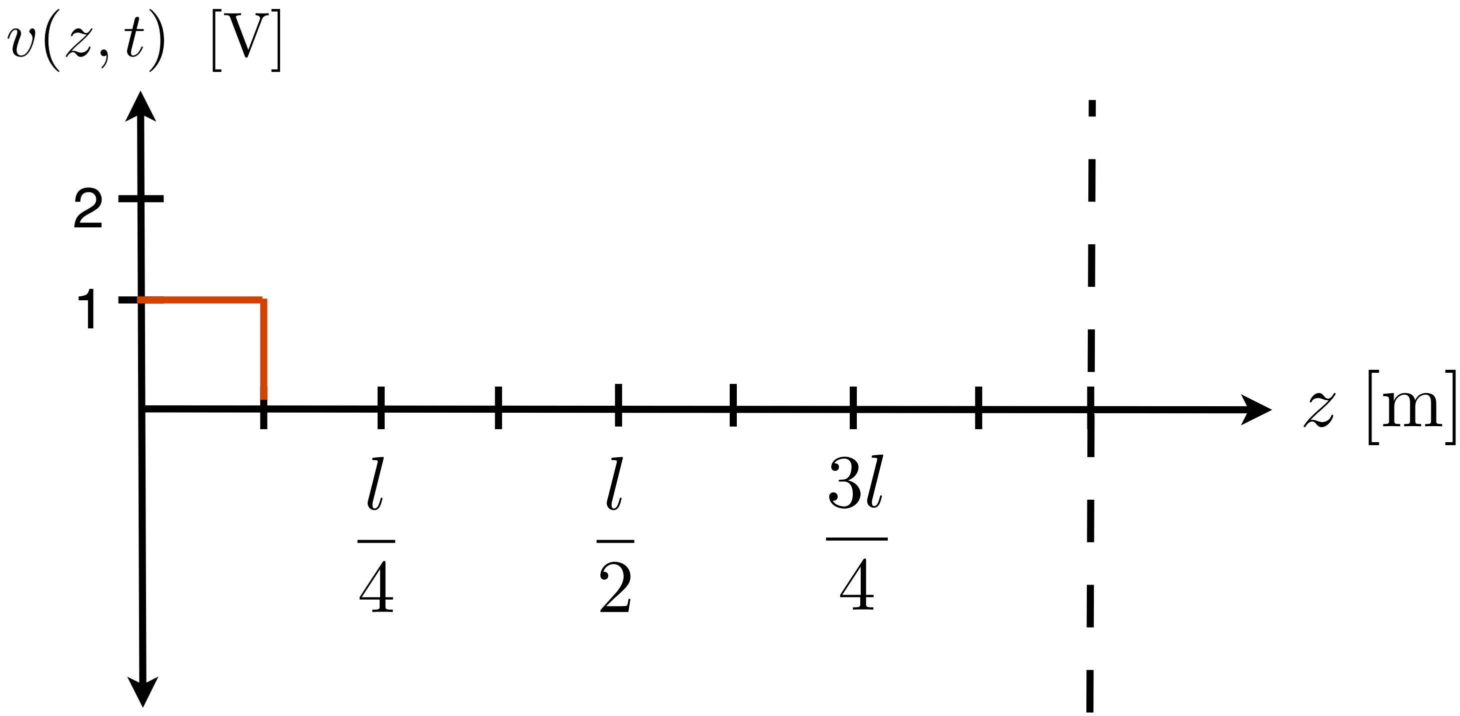
EXAMPLE 1:

$$t = 0 \times T = 0\ \mu\text{s}$$



$$n = 0$$

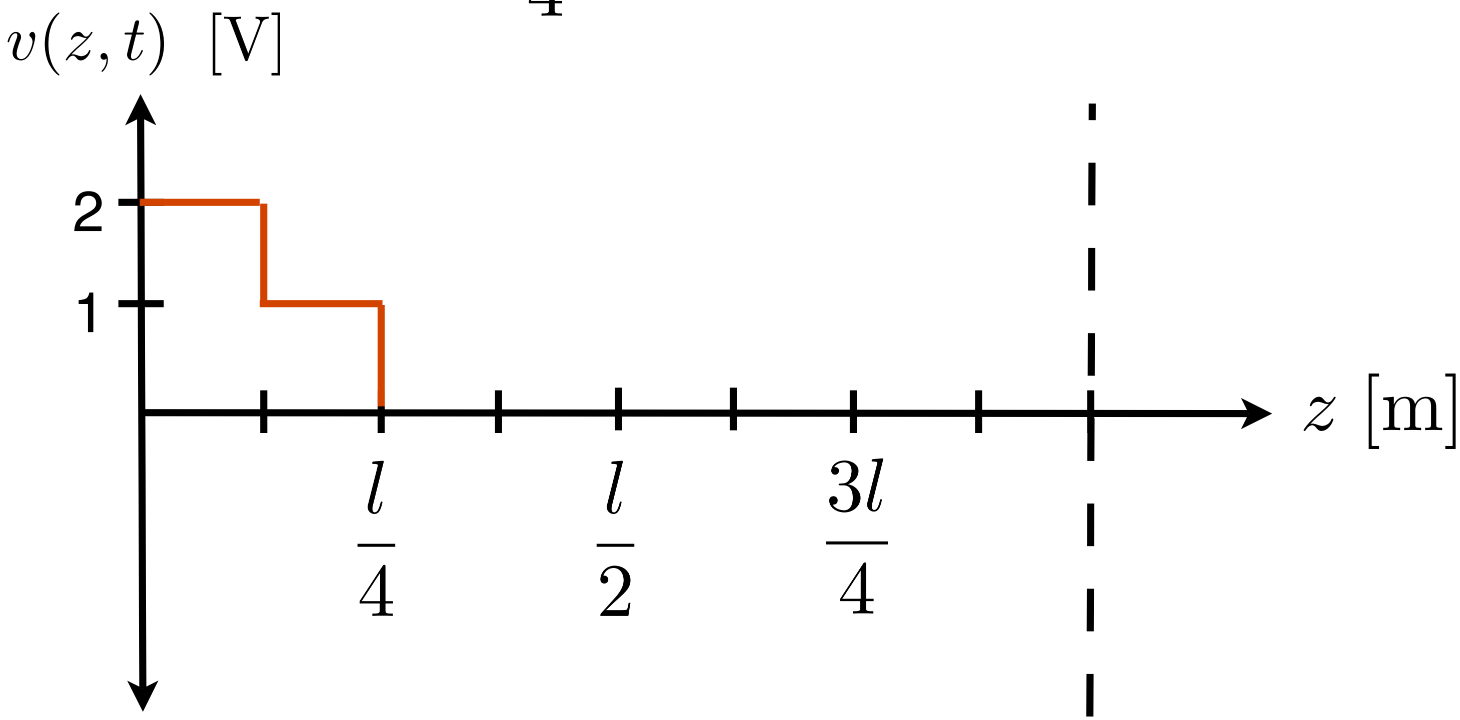
EXAMPLE 1:



$$n = 0$$

EXAMPLE 1:

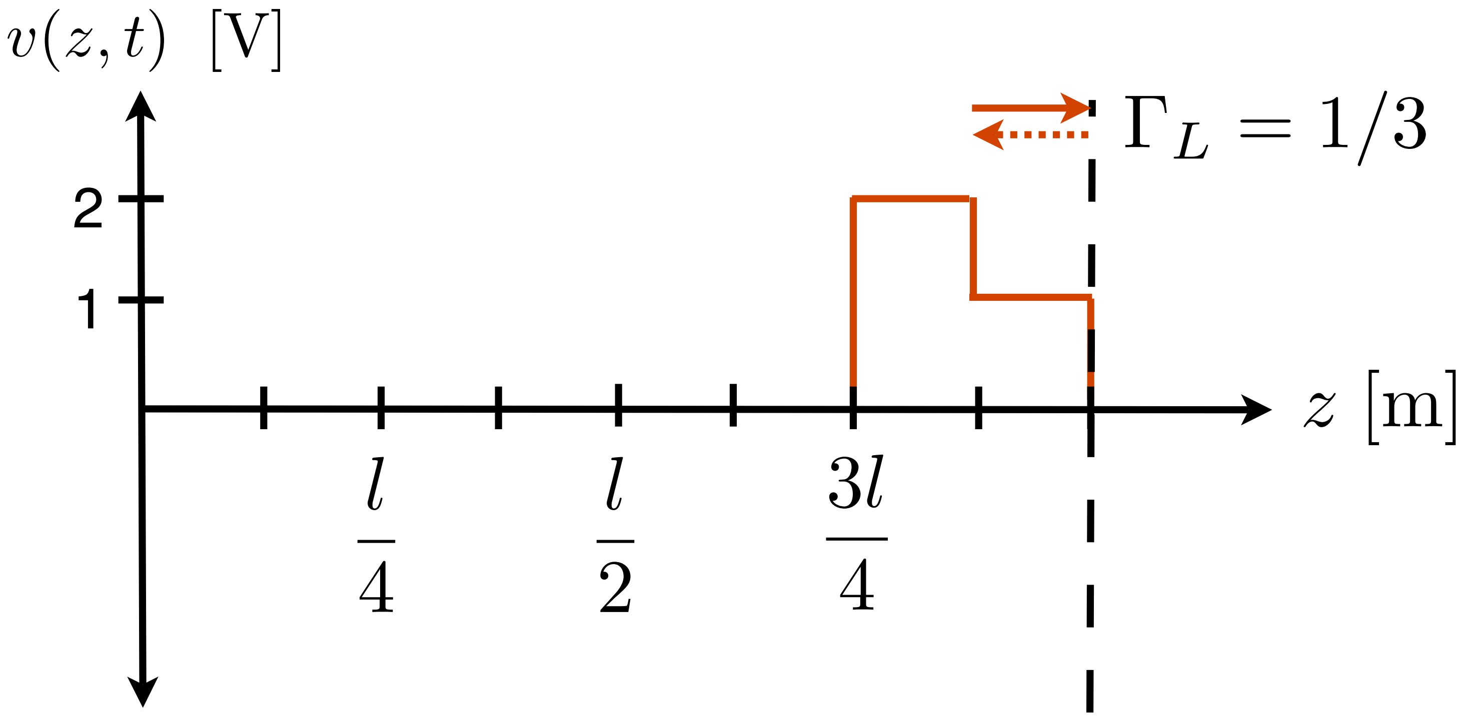
$$t = \frac{1}{4} \times T = 2\ \mu\text{s}$$



$$n = 0$$

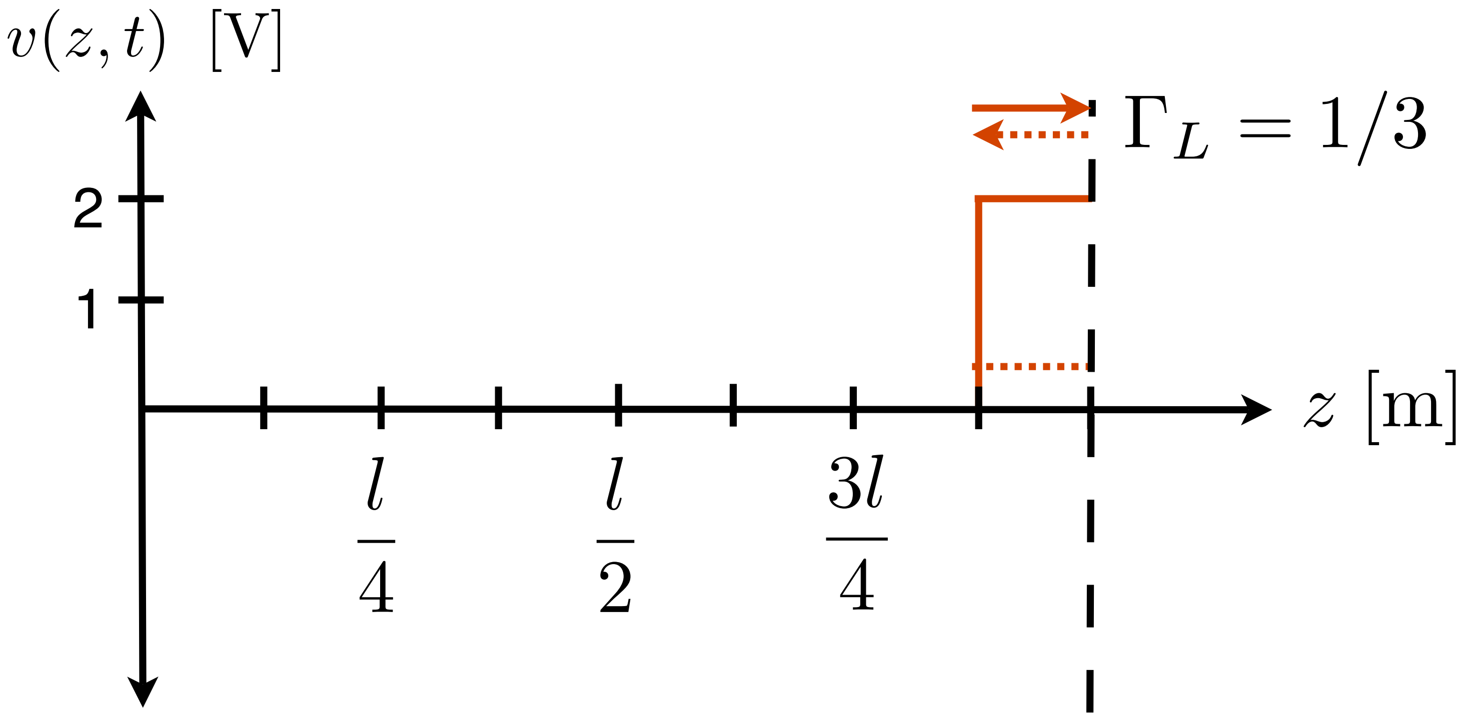
EXAMPLE 1:

$$t = T = 8\ \mu s$$



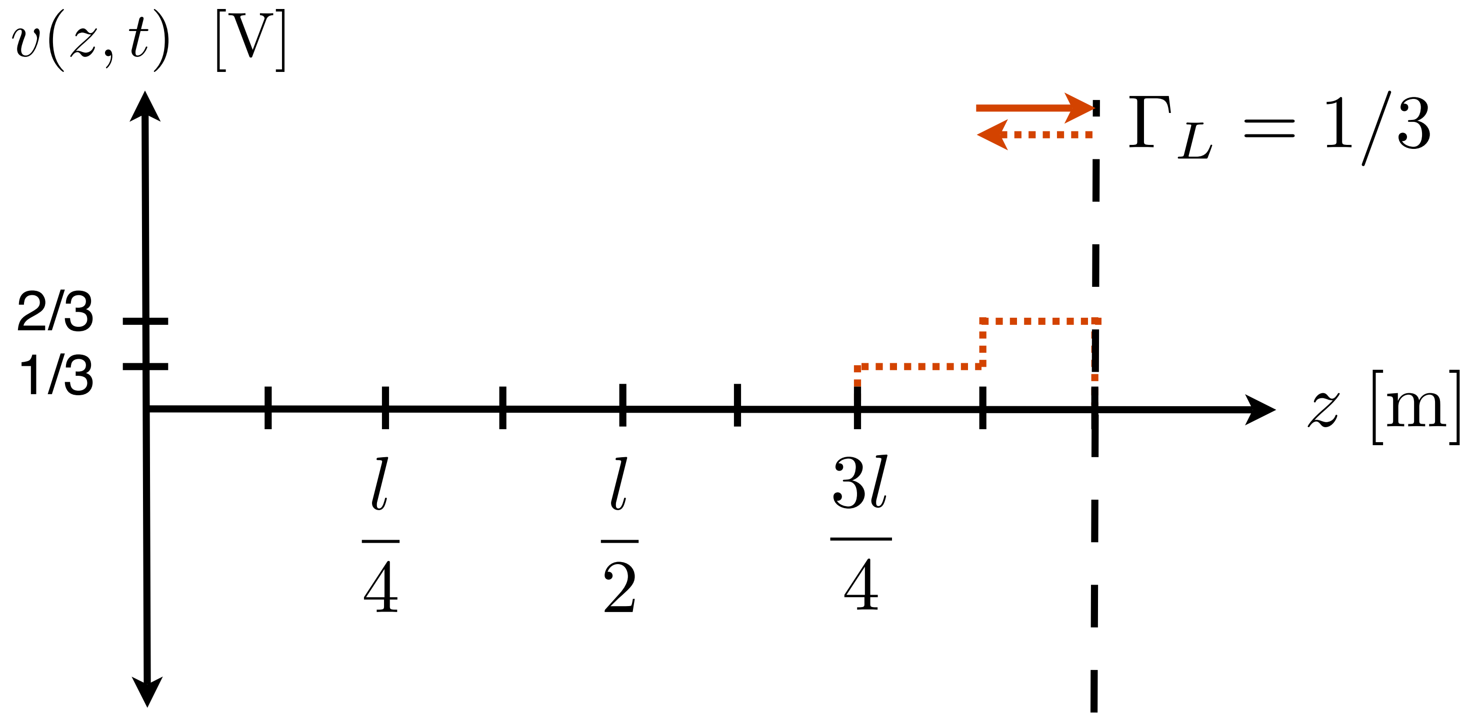
$$n = 0$$

EXAMPLE 1:



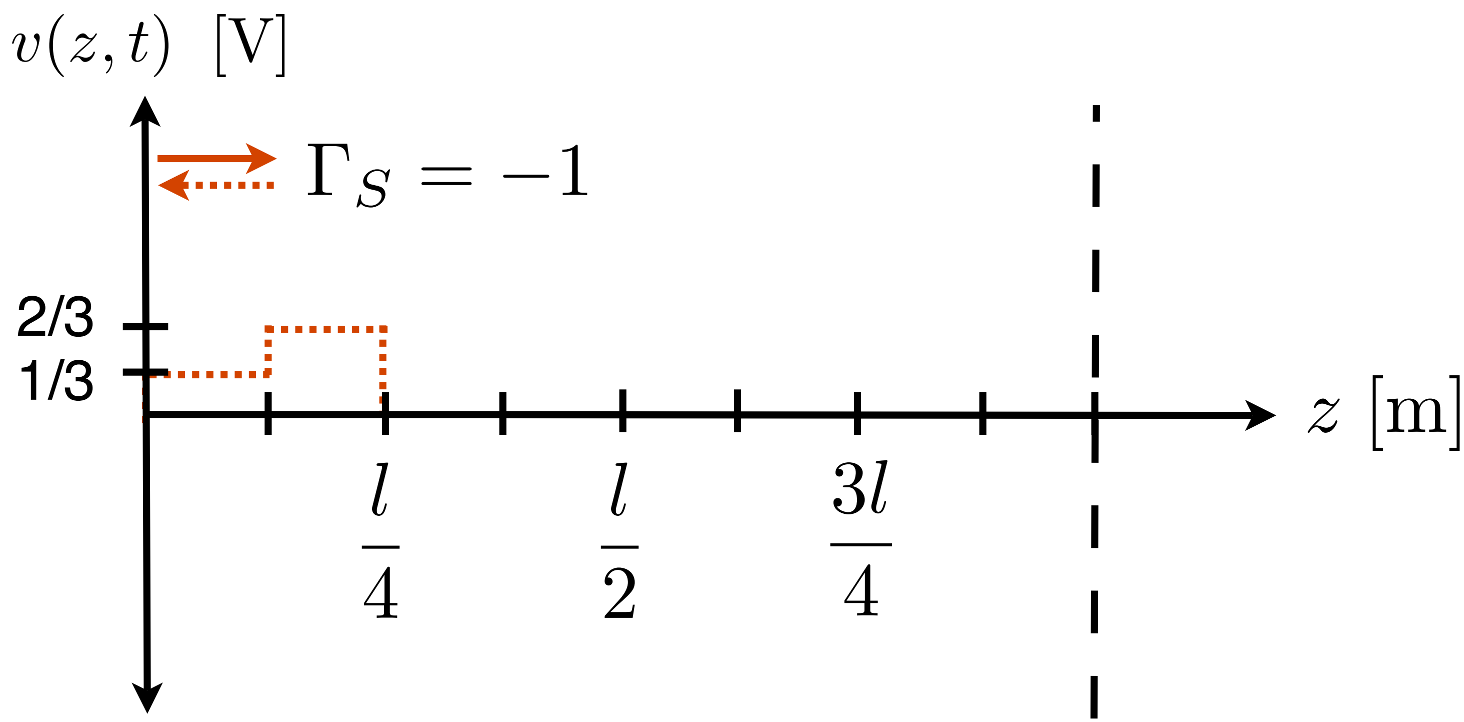
$n = 0$

EXAMPLE 1: $t = T + 2\ \mu\text{s} = 10\ \mu\text{s}$



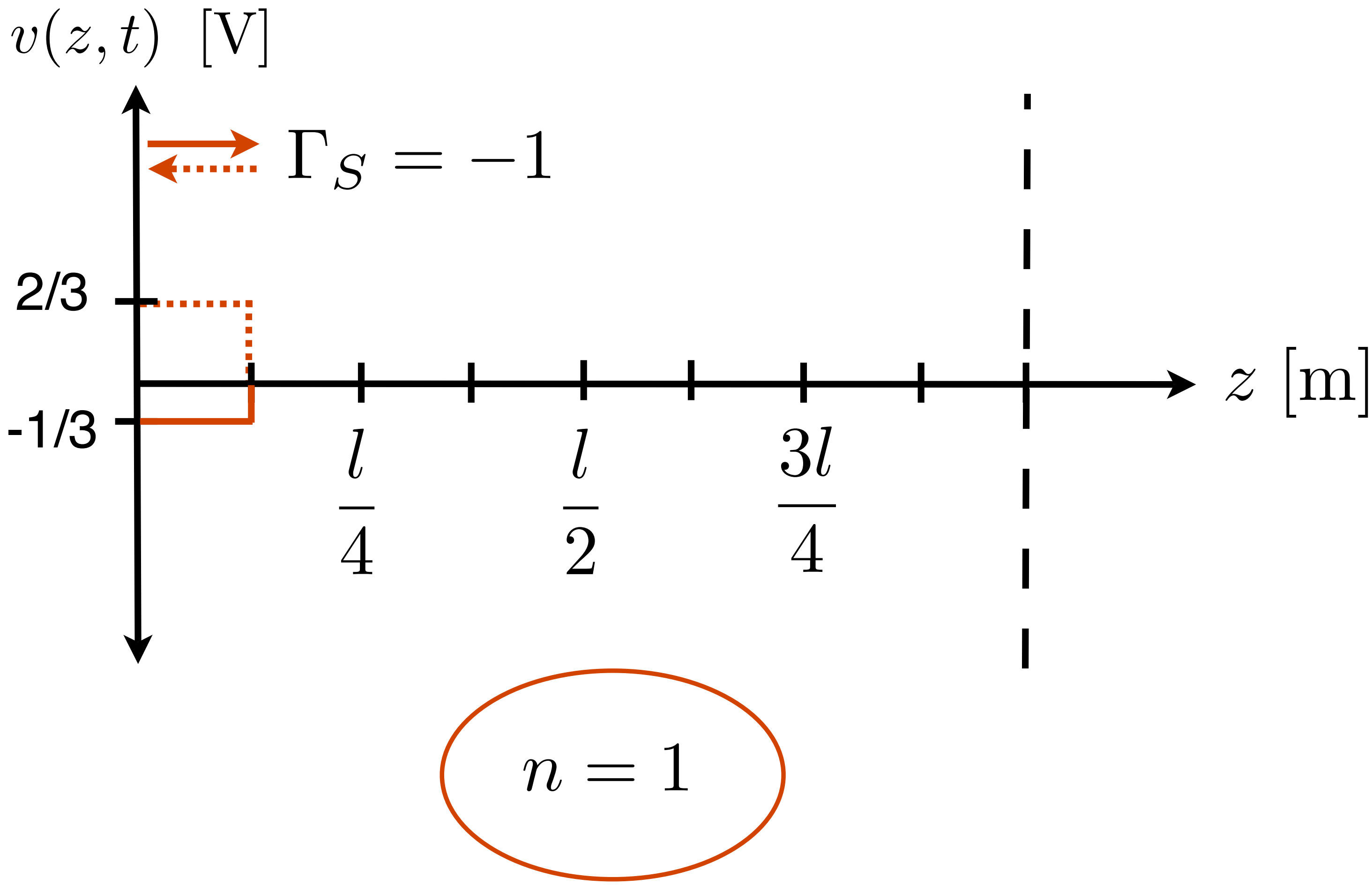
$n = 0$

EXAMPLE 1: $t = 2 \times T = 16\ \mu s$

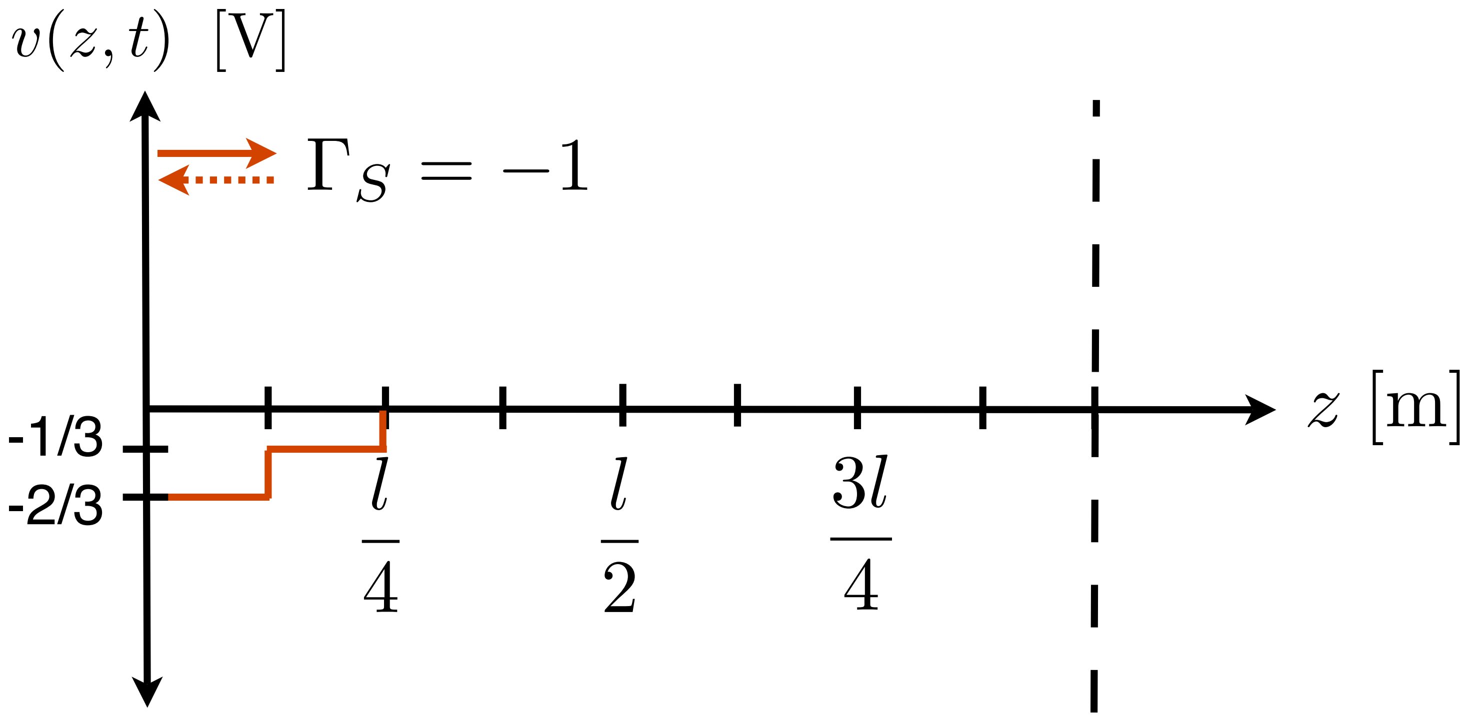


$n = 0$

EXAMPLE 1:



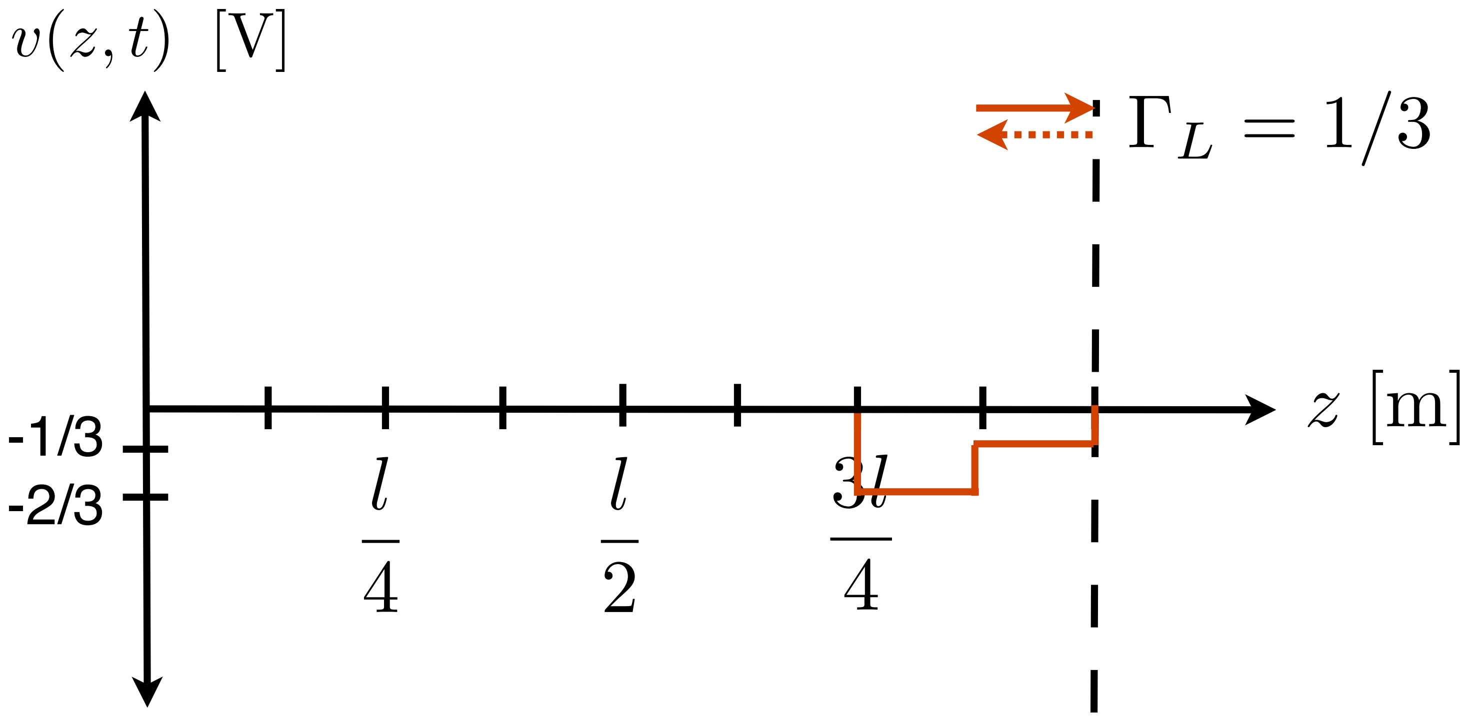
EXAMPLE 1:



$n = 1$

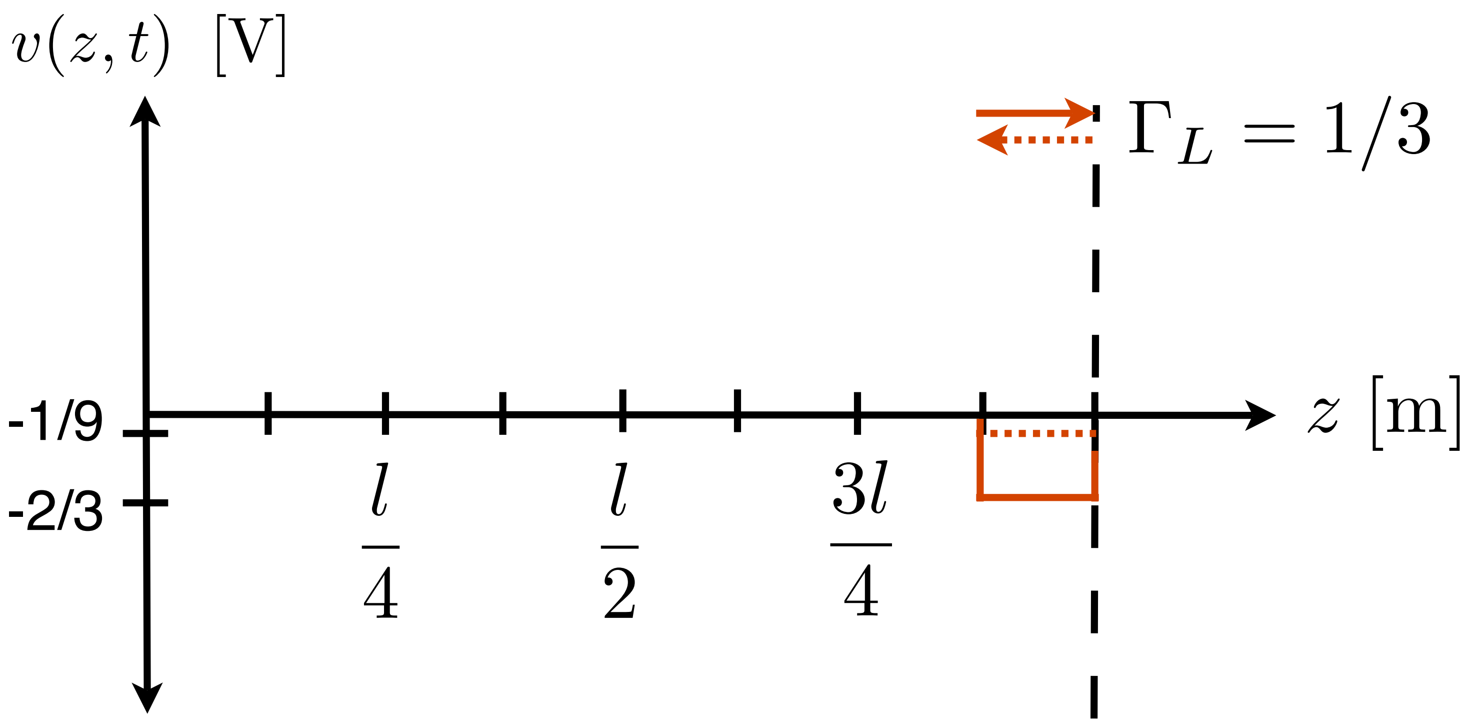
EXAMPLE 1:

$$t = 3\,T = 24\ \mu\text{s}$$



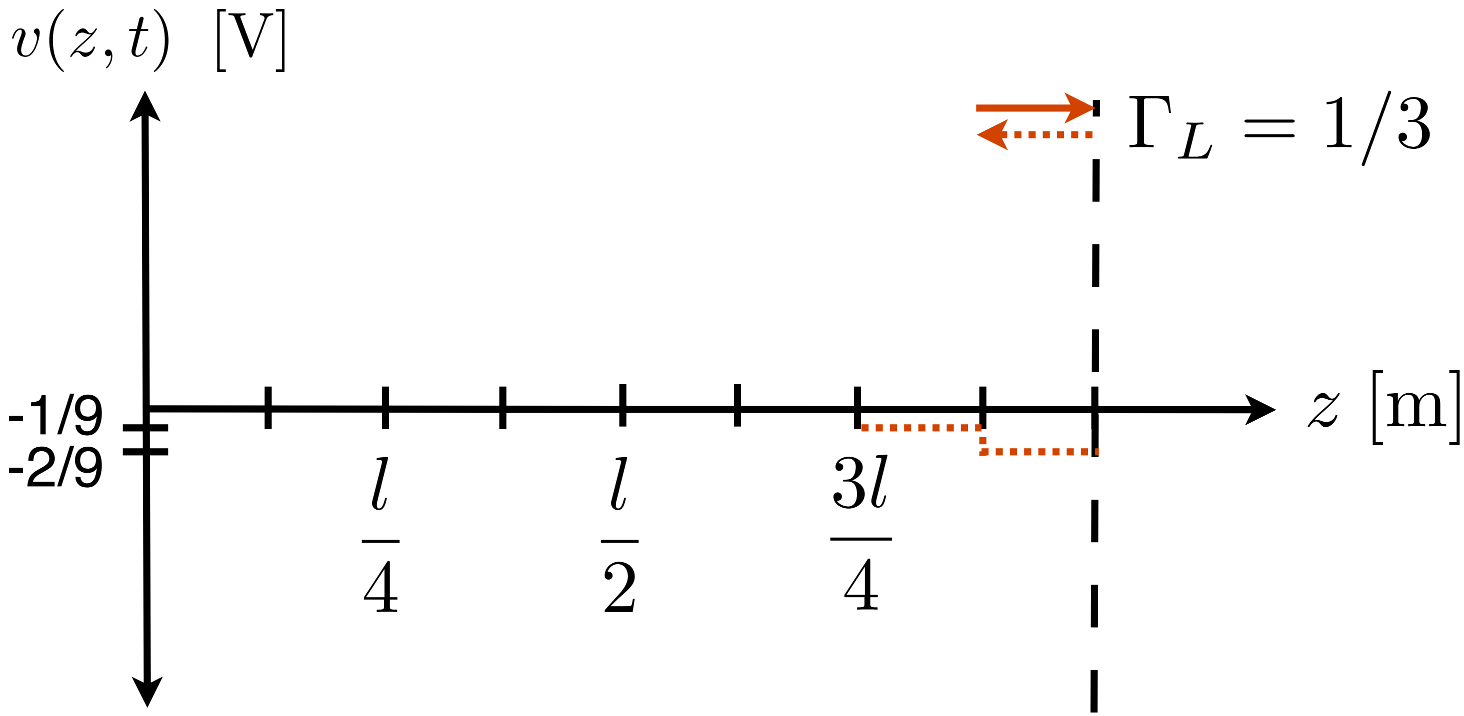
$$n = 1$$

EXAMPLE 1:



$n = 1$

EXAMPLE 1:



$$n = 1$$

EXAMPLE 1:

in this example:

$$v^+(z=0,t) = \sum_n (-\frac{1}{3})^n f(t-16n)$$

$$v^-(z=0,t) = \frac{1}{3}\sum_n (-\frac{1}{3})^n f(t-16(n+1))$$

so that

$$v(z,t) = \sum_n (-\frac{1}{3})^n \left[f(t-16n-\frac{z}{c}) + \cdots\right.$$
$$\left.\cdots + \frac{1}{3} f(t-16(n+1)+\frac{z}{c})\right]$$

EXAMPLE 1:

voltage waveform:

$$v(z,t) = \sum_n (-\frac{1}{3})^n \left[f(t - 16n - \frac{z}{c}) + \cdots \right.$$

$$\left. \cdots + \frac{1}{3} f(t - 16(n+1) + \frac{z}{c})\right]$$

vs. current waveform:

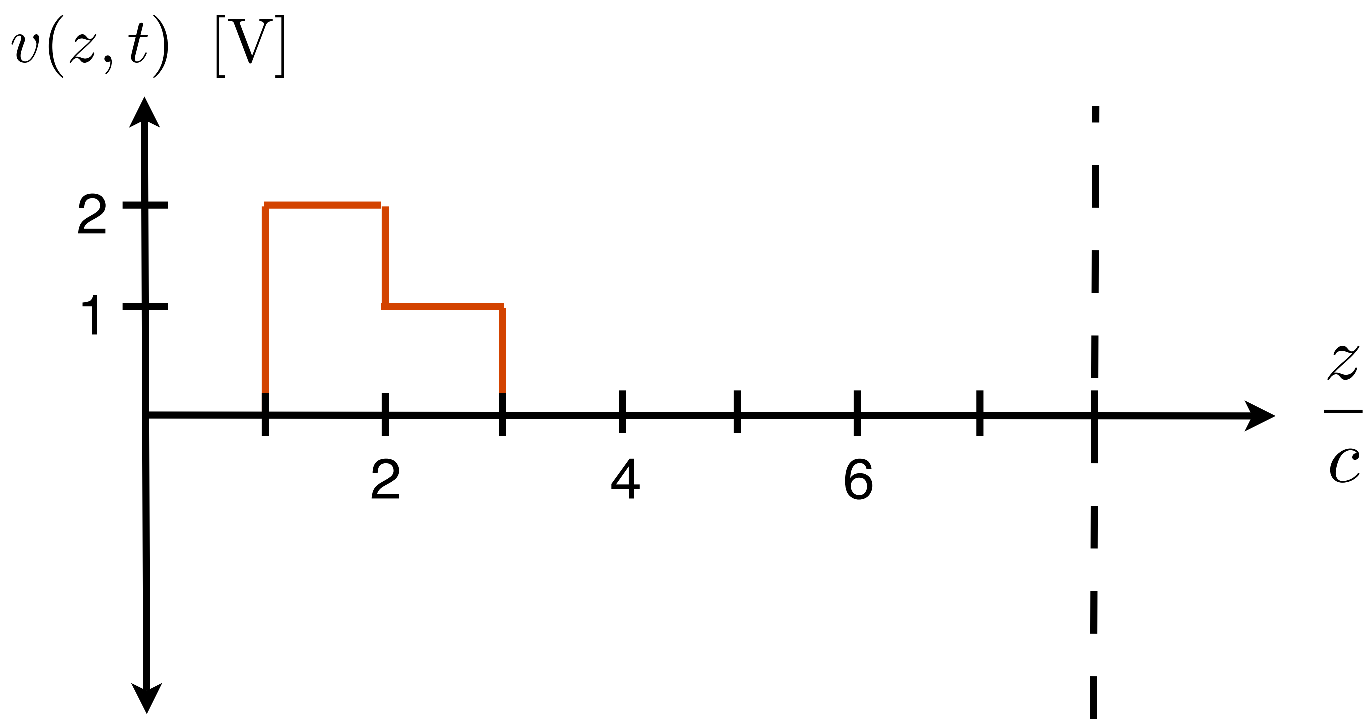
$$i(z,t) = \frac{1}{50} \sum_n (-\frac{1}{3})^n \left[f(t - 16n - \frac{z}{c}) \cdots \right.$$

$$\left. \cdots - \frac{1}{3} f(t - 16(n+1) + \frac{z}{c})\right]$$

EXAMPLE 1:

? what is voltage waveform at: $t = 3\ \mu\text{s} \quad (n = 0)$

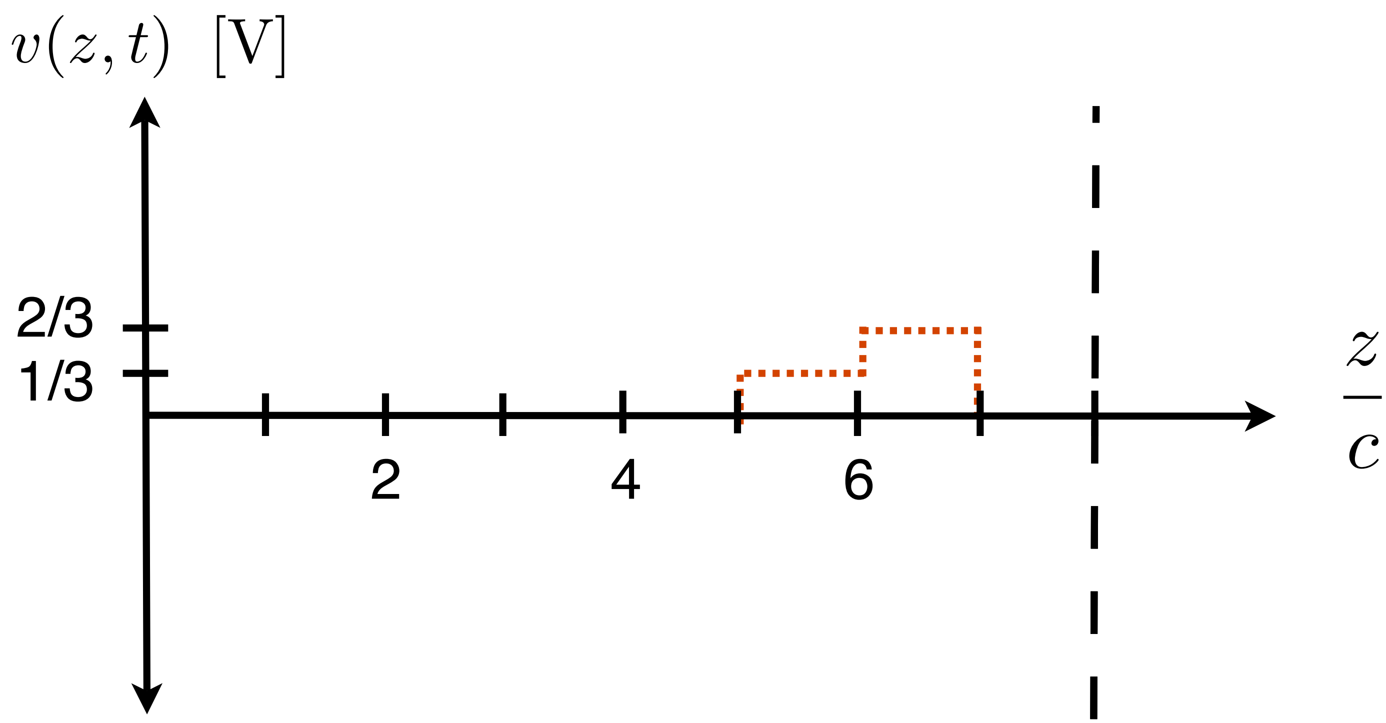
$$v(z, t = 3) = 2\ \text{rect}(\frac{3}{2} - \frac{z}{c}) + \text{rect}(\frac{5}{2} - \frac{z}{c})$$



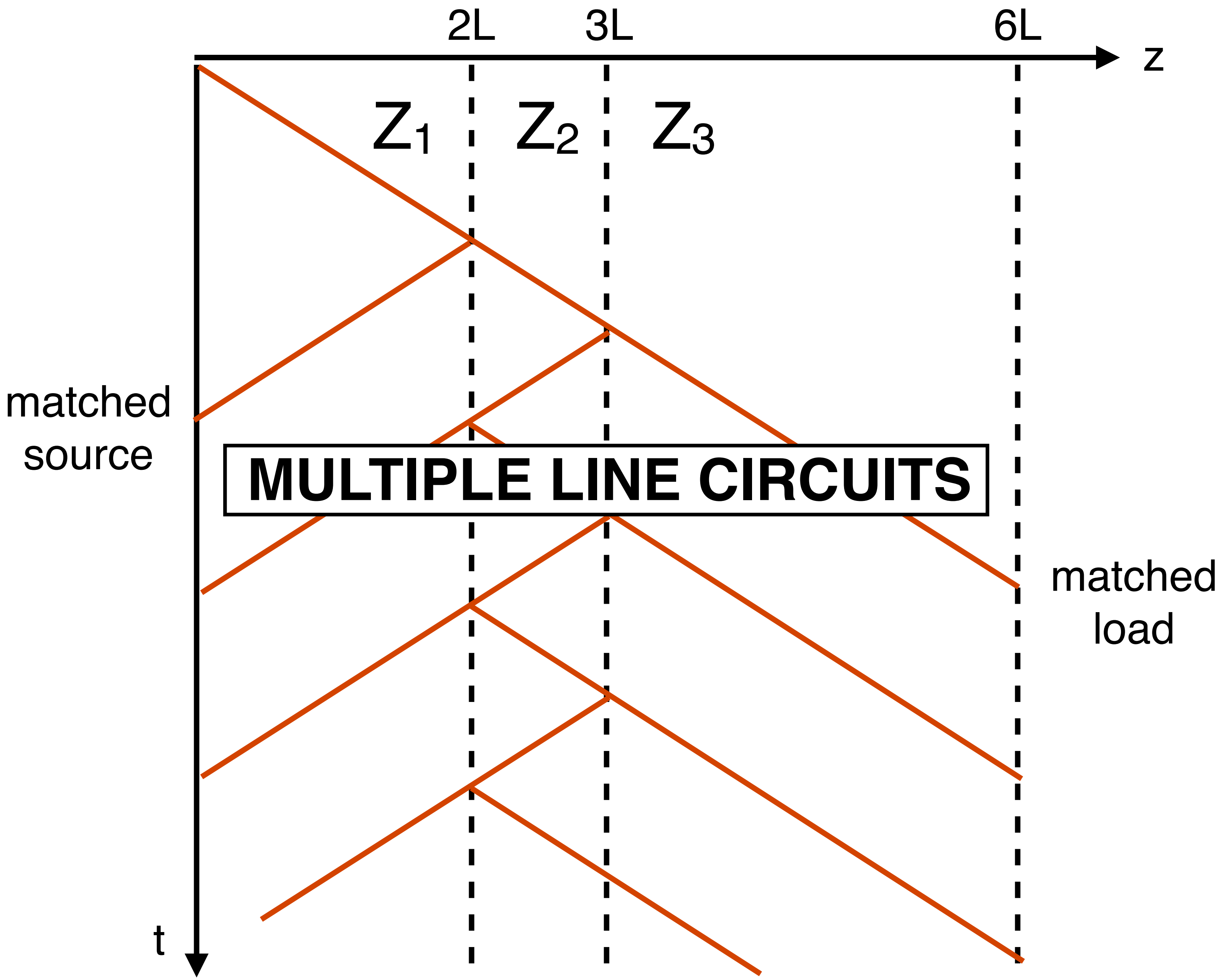
EXAMPLE 1:

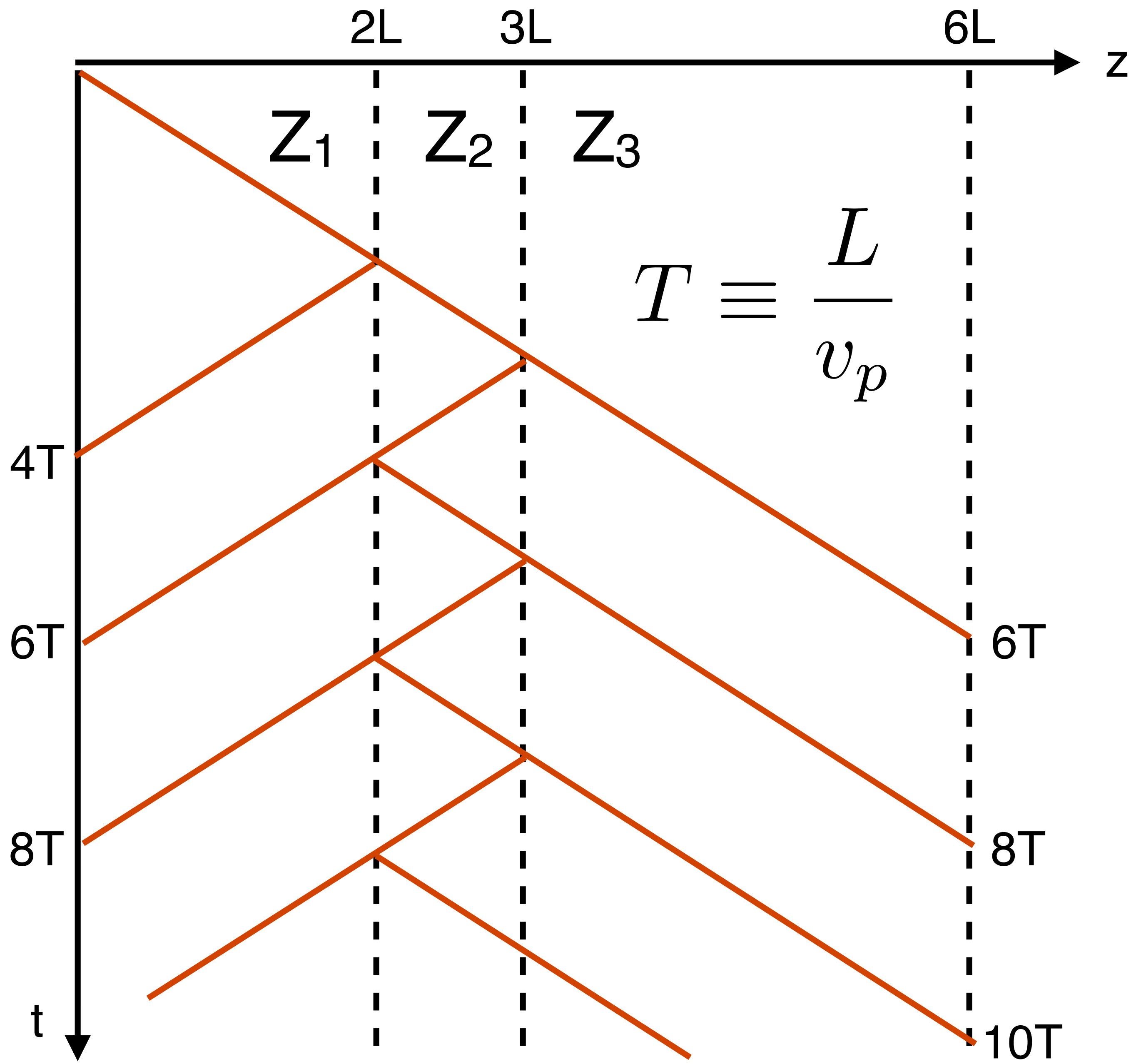
? what is voltage waveform at: $t = 11\ \mu s \quad (n = 0)$

$$v(z, t = 11) = \frac{2}{3}\,\mathrm{rect}(-6.5 + \frac{z}{c})) + \frac{1}{3}\,\mathrm{rect}(-5.5 + \frac{z}{c}))$$

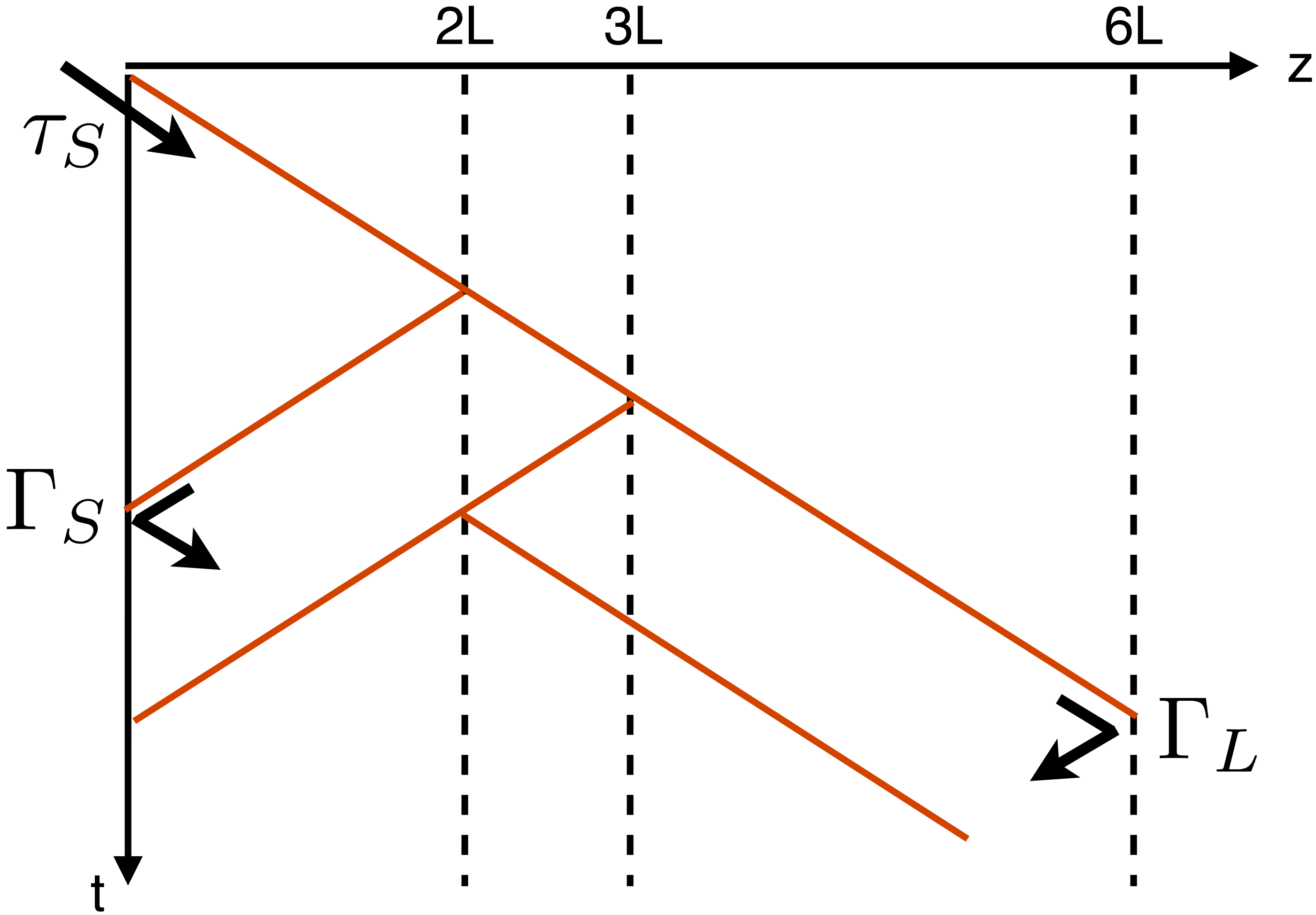


MULTIPLE LINE CIRCUITS

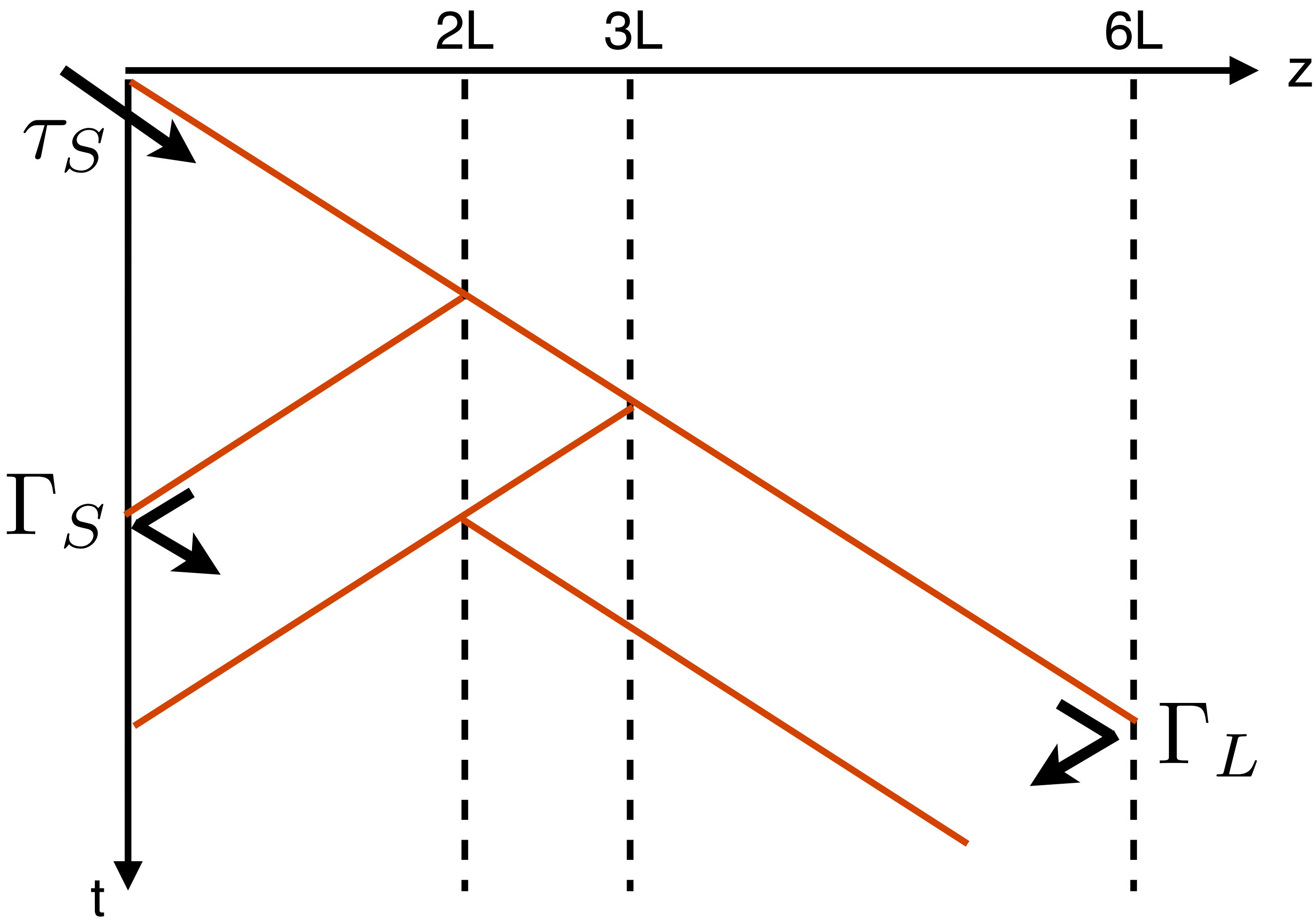




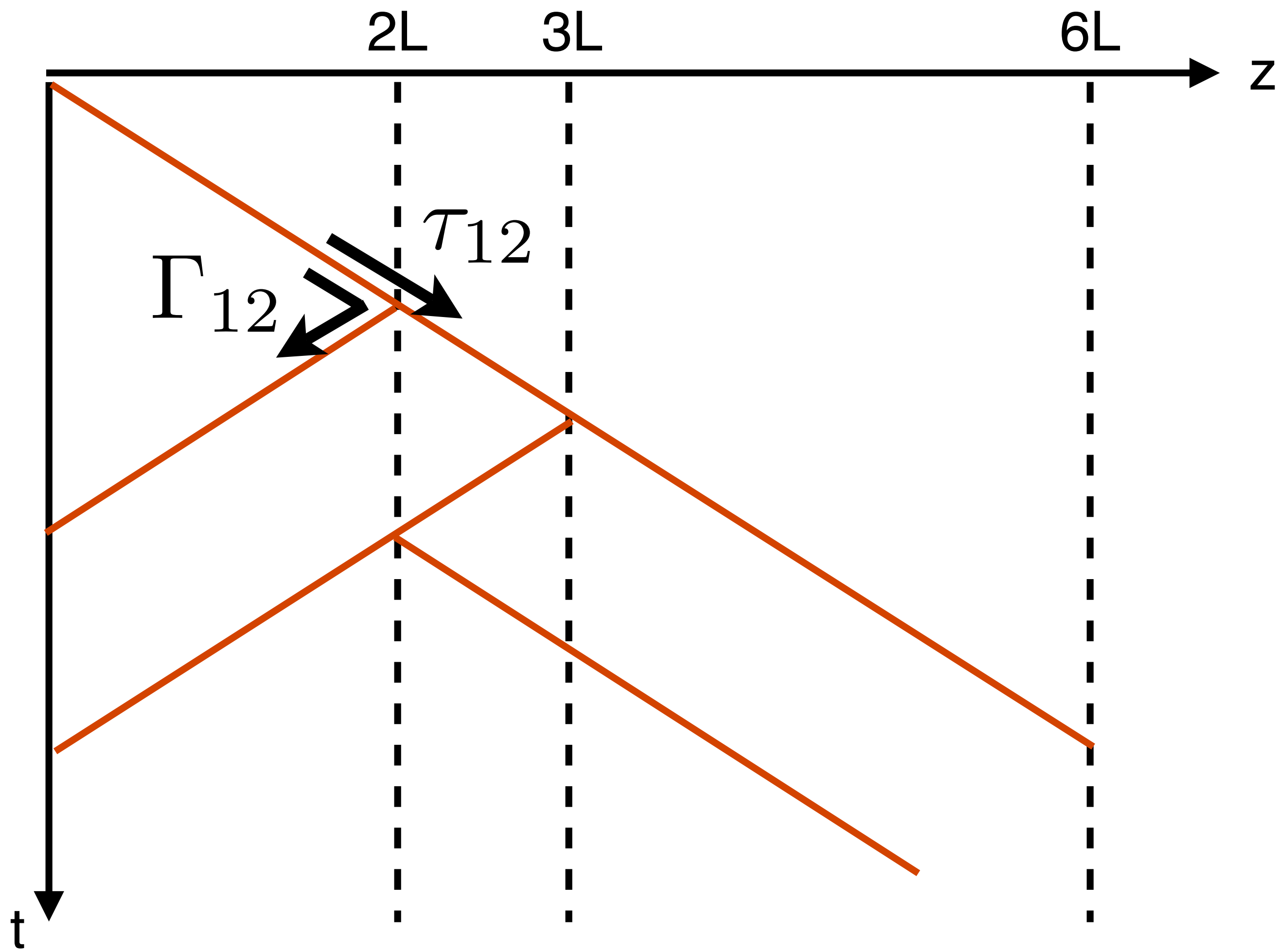
2L
3L
6L
z
Z1
Z2
Z3
$T \equiv \frac{L}{v_p}$
4T
6T
8T
t
6T
8T
10T



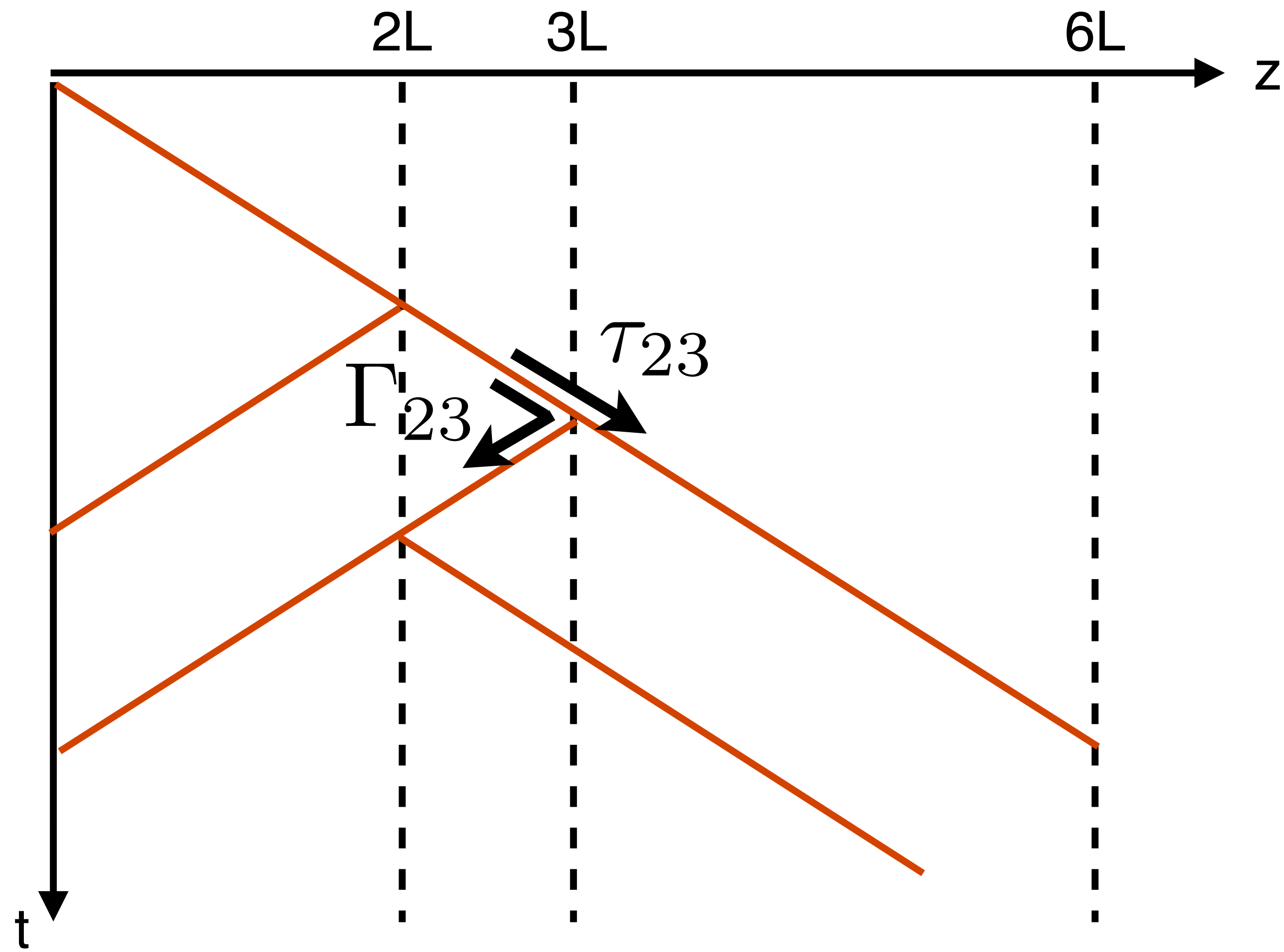
$$\tau_S = \frac{Z_1}{R_S + Z_1}$$



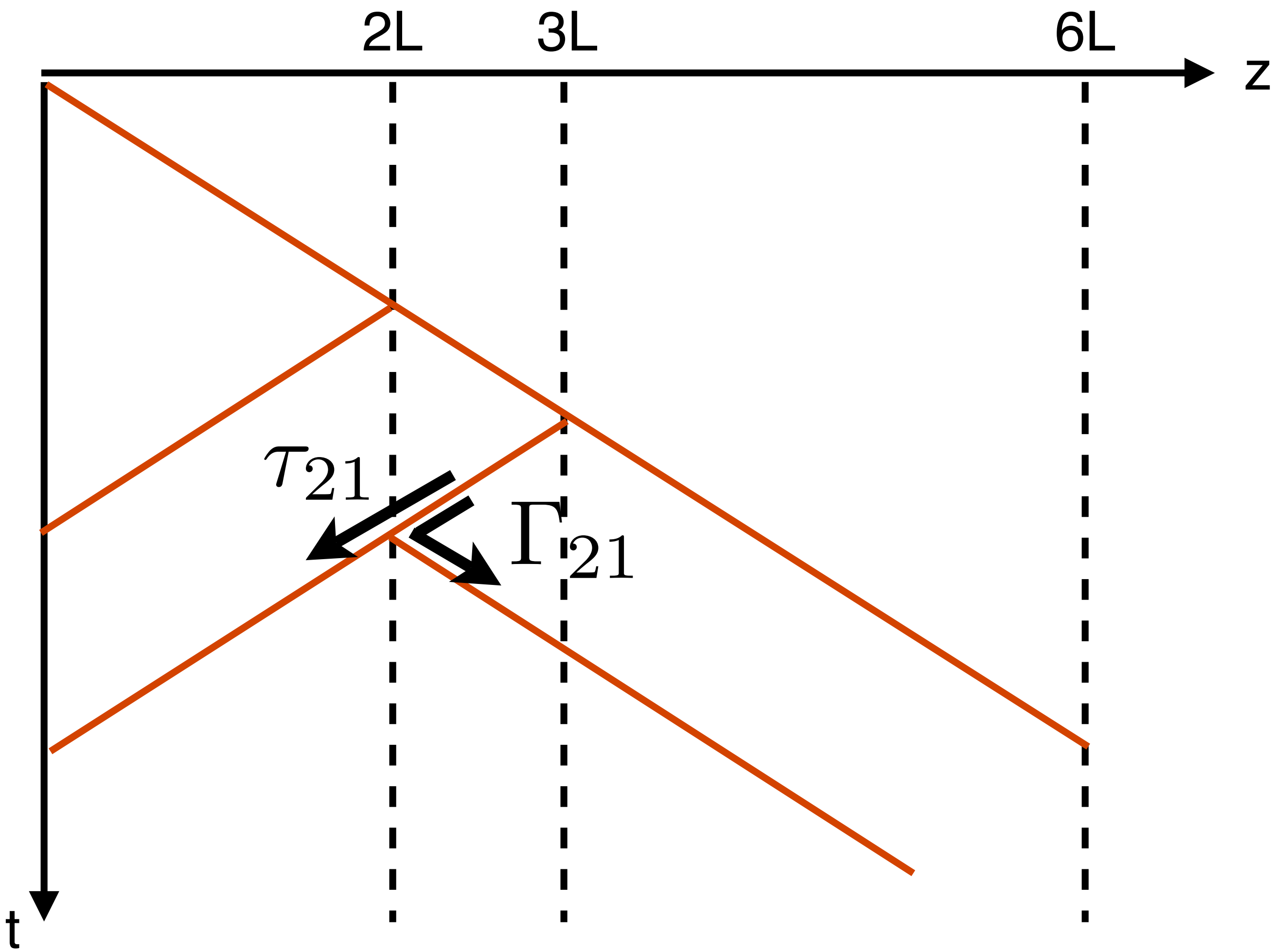
$$\Gamma_S = \frac{R_S - Z_1}{R_S + Z_1} \qquad \Gamma_L = \frac{R_L - Z_3}{R_L + Z_3}$$



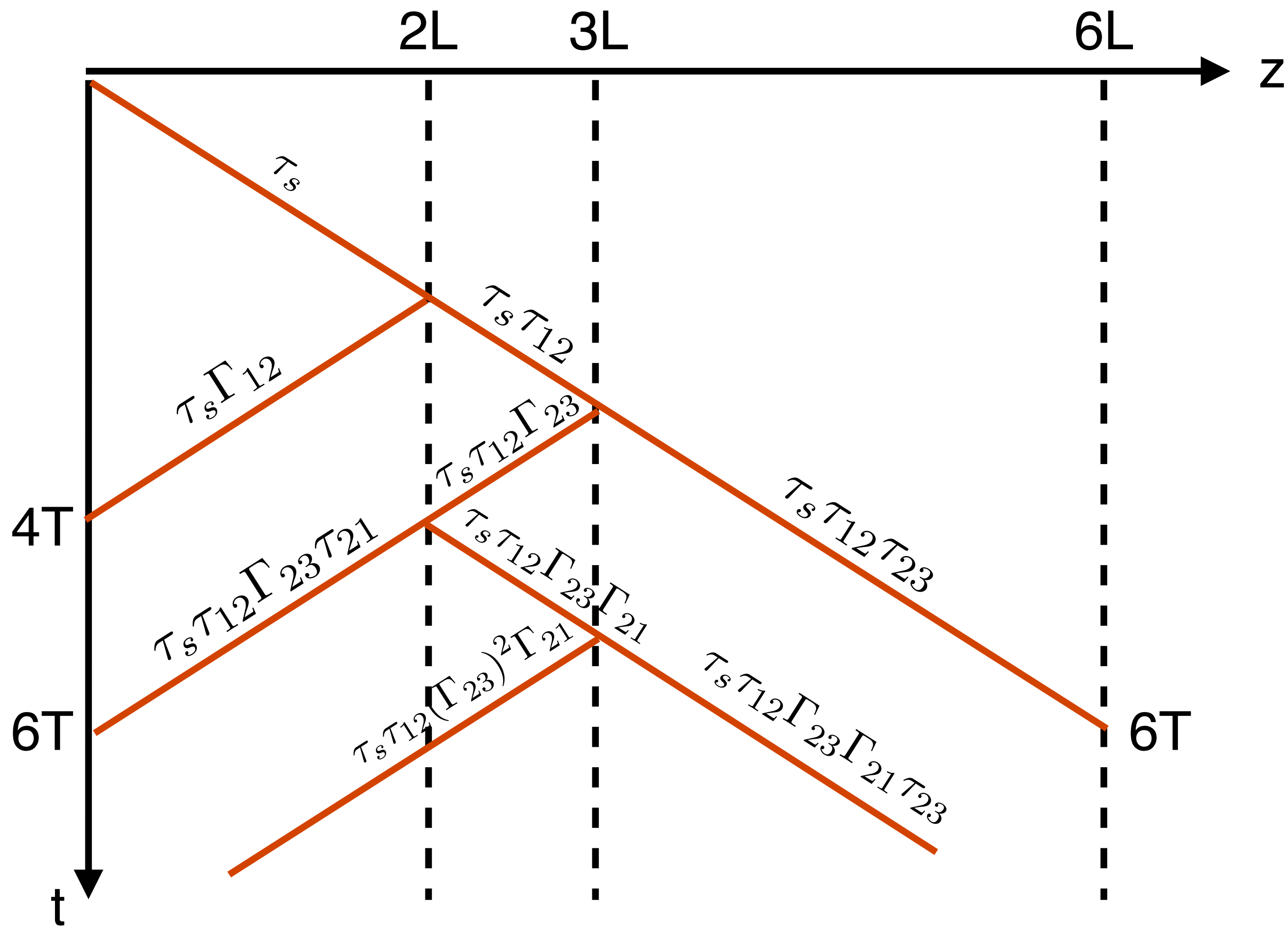
$$\Gamma_{12} = \frac{Z_2 - Z_1}{Z_2 + Z_1} \qquad \tau_{12} = 1 + \Gamma_{12}$$



$$\Gamma_{23} = \frac{Z_3 - Z_2}{Z_3 + Z_2} \qquad \tau_{23} = 1 + \Gamma_{23}$$



$$\Gamma_{21} = \frac{Z_1 - Z_2}{Z_1 + Z_2} = -\Gamma_{12}$$



$$v_{23}(t) = \tau_s\tau_{12}f(t-3T) + \tau_s\tau_{12}\Gamma_{23}f(t-3T) + \cdots$$

$$\cdots + \tau_s\tau_{12}\Gamma_{23}\Gamma_{21}f(t-5T) + \tau_s\tau_{12}(\Gamma_{23})^2\Gamma_{21}f(t-5T)$$