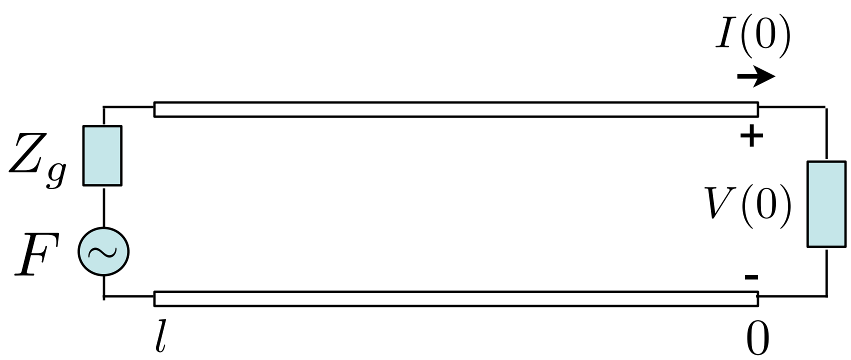
TL terminated by arbitrary load



$$Z_L = \frac{V(0)}{I(0)}$$

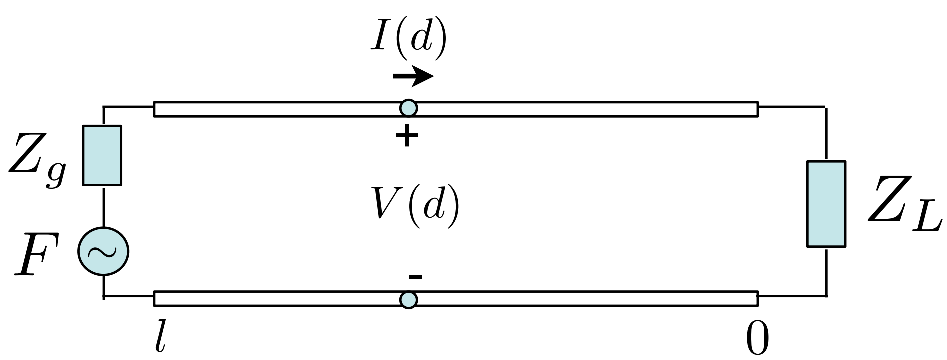
normalized load impedance:

$$z_L \equiv \frac{Z_L}{Z_0} = r_L + jx_L$$

load reflection coefficient:

$$\Gamma_L \equiv \Gamma(d=0) = \frac{V^-(0)}{V^+(0)} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{z_L - 1}{z_L + 1}$$

TL terminated by arbitrary load



normalized line impedance:

$$z(d) \equiv \frac{Z(d)}{Z_0} = \frac{V(d)}{Z_0 I(d)} = \frac{1+\Gamma(d)}{1-\Gamma(d)}$$

generalized reflection coefficient:

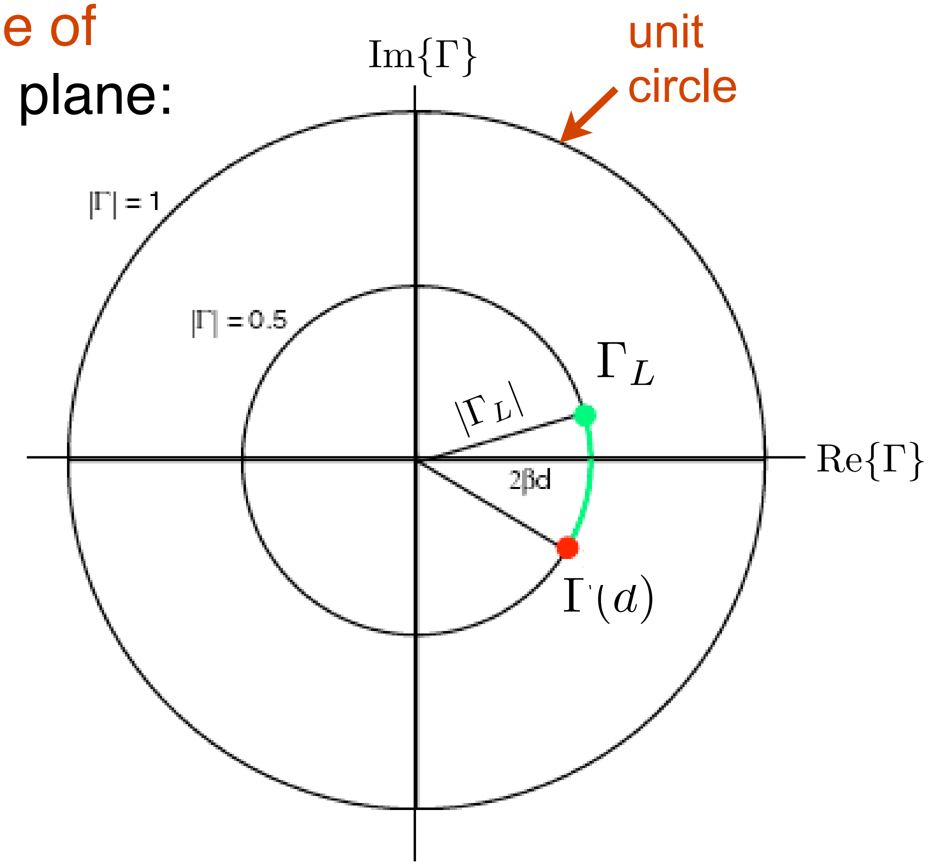
$$\Gamma(d) \equiv \frac{V^-(d)}{V^+(d)} = \Gamma_L e^{-j2\beta d} = \frac{z(d)-1}{z(d)+1}$$

Graphical representation of $\Gamma(d)$ transformations:

$$\Gamma(d) = \Gamma_L e^{-j2\beta d} \quad \text{where} \quad |\Gamma(d)| = |\Gamma_L| \leq 1$$

$$\angle\Gamma(d) = \angle\Gamma_L - 2\beta d$$

As d changes, we trace out a circle of constant radius on the complex Γ plane:



A full circle is traced out when

$$2\pi = 2\beta d = 2(2\pi/\lambda)d$$

$$d = \lambda/2$$

Increasing d (towards generator) corresponds to CLOCKWISE rotation around the circle

Visualize standing wave patterns in voltage and current:

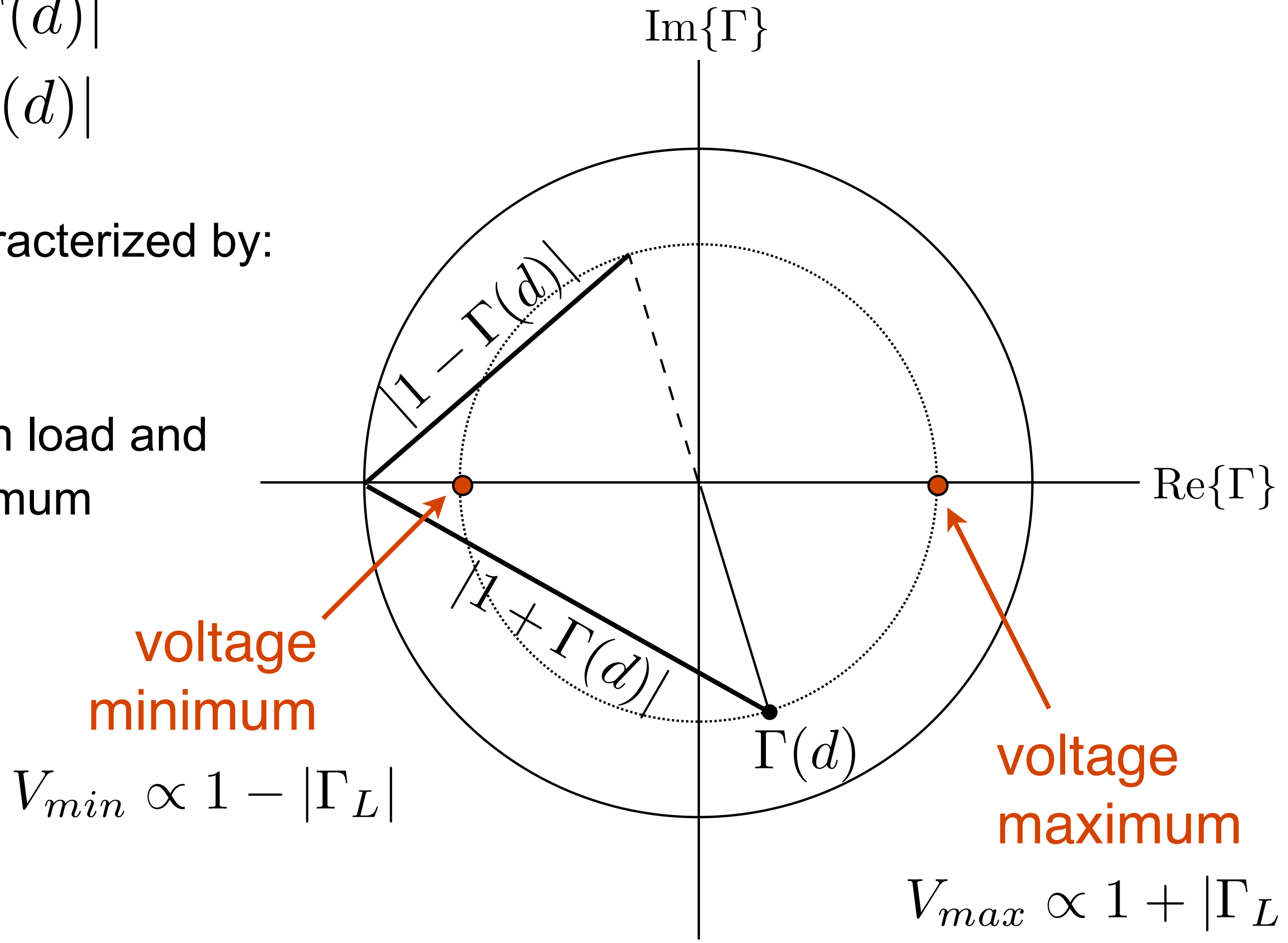
$$V(d) \propto |1 + \Gamma(d)|$$
$$I(d) \propto |1 - \Gamma(d)|$$

standing waves characterized by:

(1) SWR = V_{max}/V_{min}

(2) wavelength

(3) distance between load and first voltage minimum



Separate real and imaginary parts of Γ to find special cases:

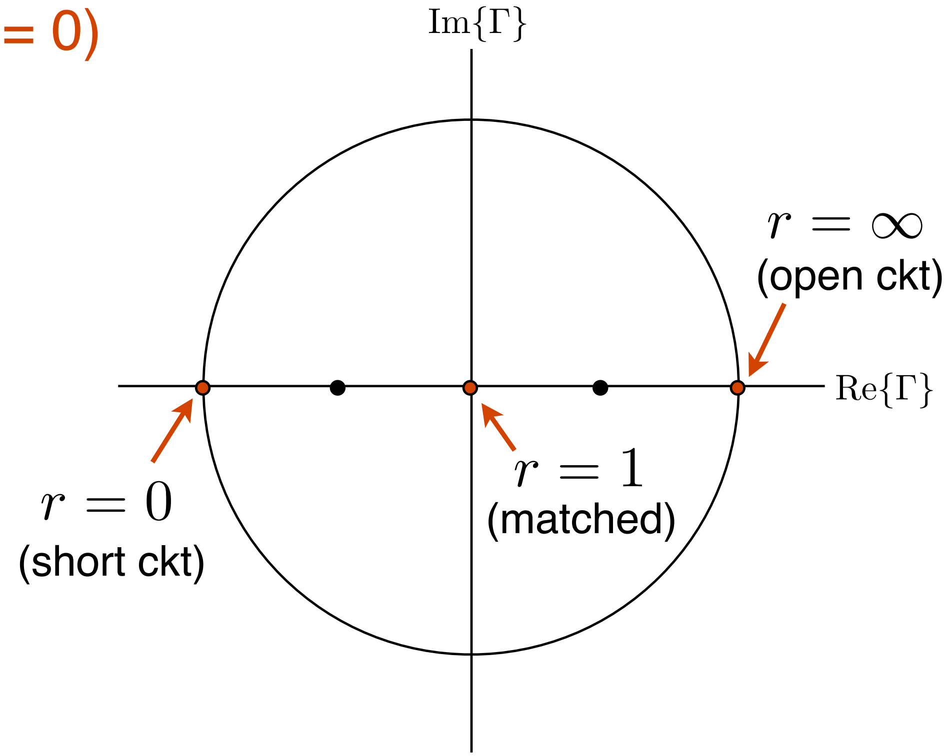
$$\Gamma(d) = \Gamma_r + j\Gamma_i = \frac{(r+jx)-1}{(r+jx)+1}$$

$$= \frac{r^2 - 1 + x^2}{(r+1)^2 + x^2} + j\frac{2x}{(r+1)^2 + x^2}$$

CASE: z(d) purely real (so x = 0)

$$\Gamma(d) = \frac{r-1}{r+1} + j0$$

r	Γ
0	-1
1/3	-1/2
1	0
3	1/2
∞	1



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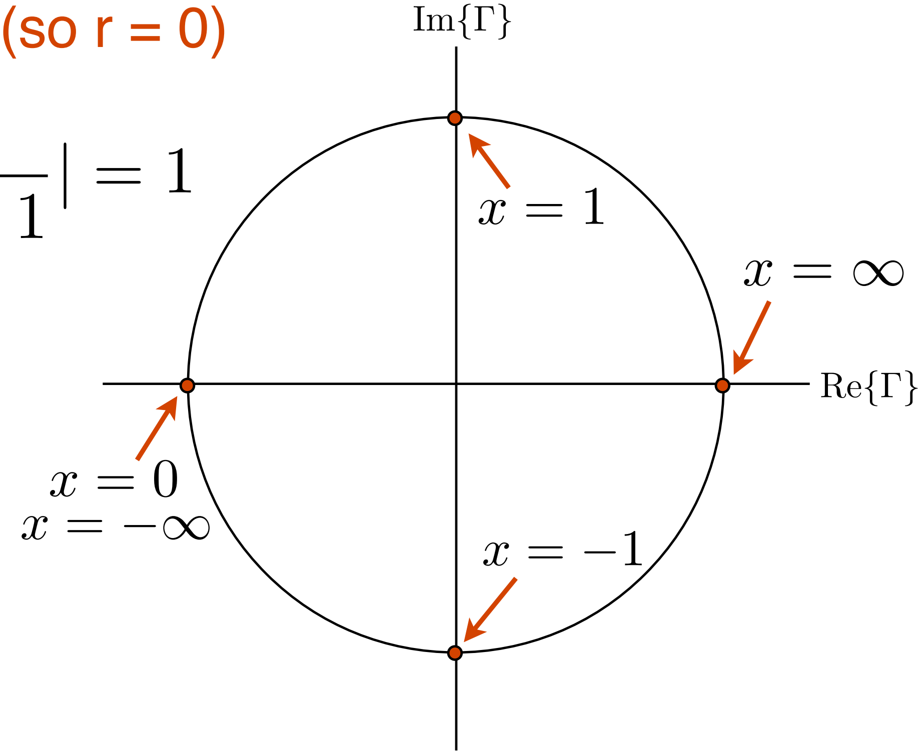
$$= \frac{r^2 - 1 + x^2}{(r+1)^2 + x^2} + j\frac{2x}{(r+1)^2 + x^2}$$

CASE: z(d) purely imaginary (so r = 0)

$$|\Gamma(d)| = \left|\frac{x^2-1}{x^2+1} + j\frac{2x}{x^2+1}\right| = 1$$

x	$\angle\Gamma$
0	π
1	$\pi/2$
∞	0
-1	$-\pi/2$
$-\infty$	$-\pi$



Separate real and imaginary parts of z to find general contours:

$$z = r + jx = \frac{1 + (\Gamma_r + j\Gamma_i)}{1 - (\Gamma_r + j\Gamma_i)} = \frac{1 - \Gamma_r^2 - \Gamma_i^2 + j2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

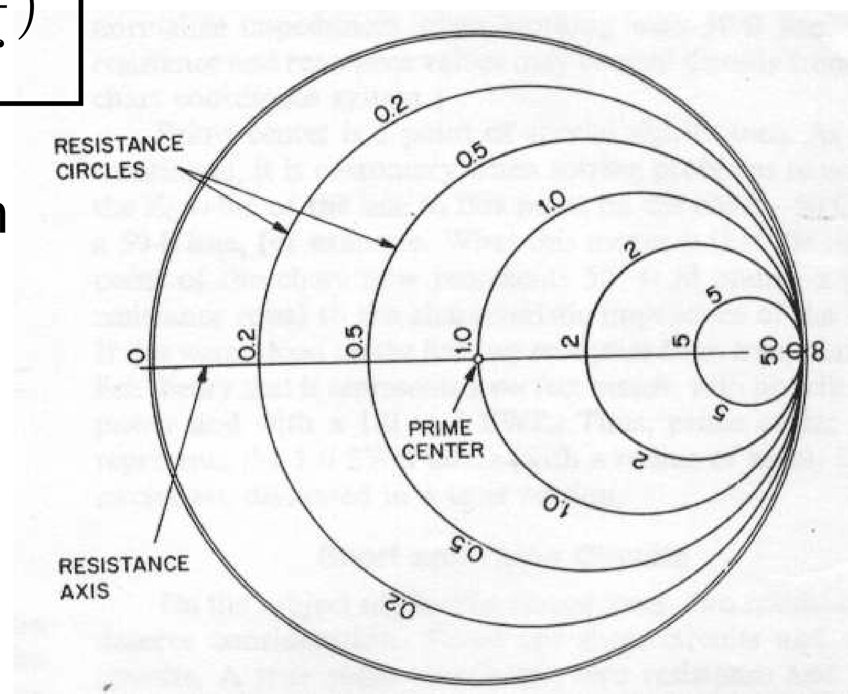
contours of constant r? equate real parts of above expression

$$\boxed{(\Gamma_r - \frac{r}{1+r})^2 + \Gamma_i^2 = (\frac{1}{1+r})^2}$$

yields the equation of a circle with

center at: $(\Gamma_r, \Gamma_i) = (\frac{r}{1+r}, 0)$

radius of: $\frac{1}{1+r}$



Separate real and imaginary parts of z to find general contours:

$$z = r + jx = \frac{1 + (\Gamma_r + j\Gamma_i)}{1 - (\Gamma_r + j\Gamma_i)} = \frac{1 - \Gamma_r^2 - \Gamma_i^2 + j2\Gamma_i}{(1 - \Gamma_r)^2 + \Gamma_i^2}$$

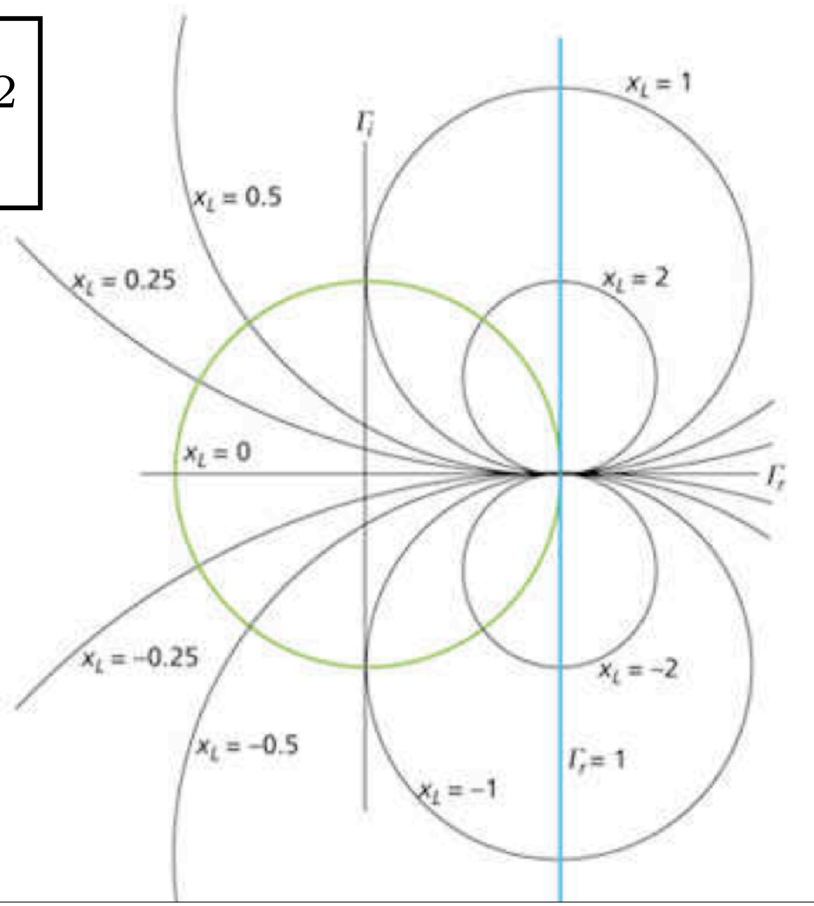
contours of constant x? equate imag. parts of above expression

$$(\Gamma_r - 1)^2 + (\Gamma_i - \frac{1}{x})^2 = (\frac{1}{x})^2$$

yields the equation of a circle with

center at: $(\Gamma_r, \Gamma_i) = (1, \frac{1}{x})$

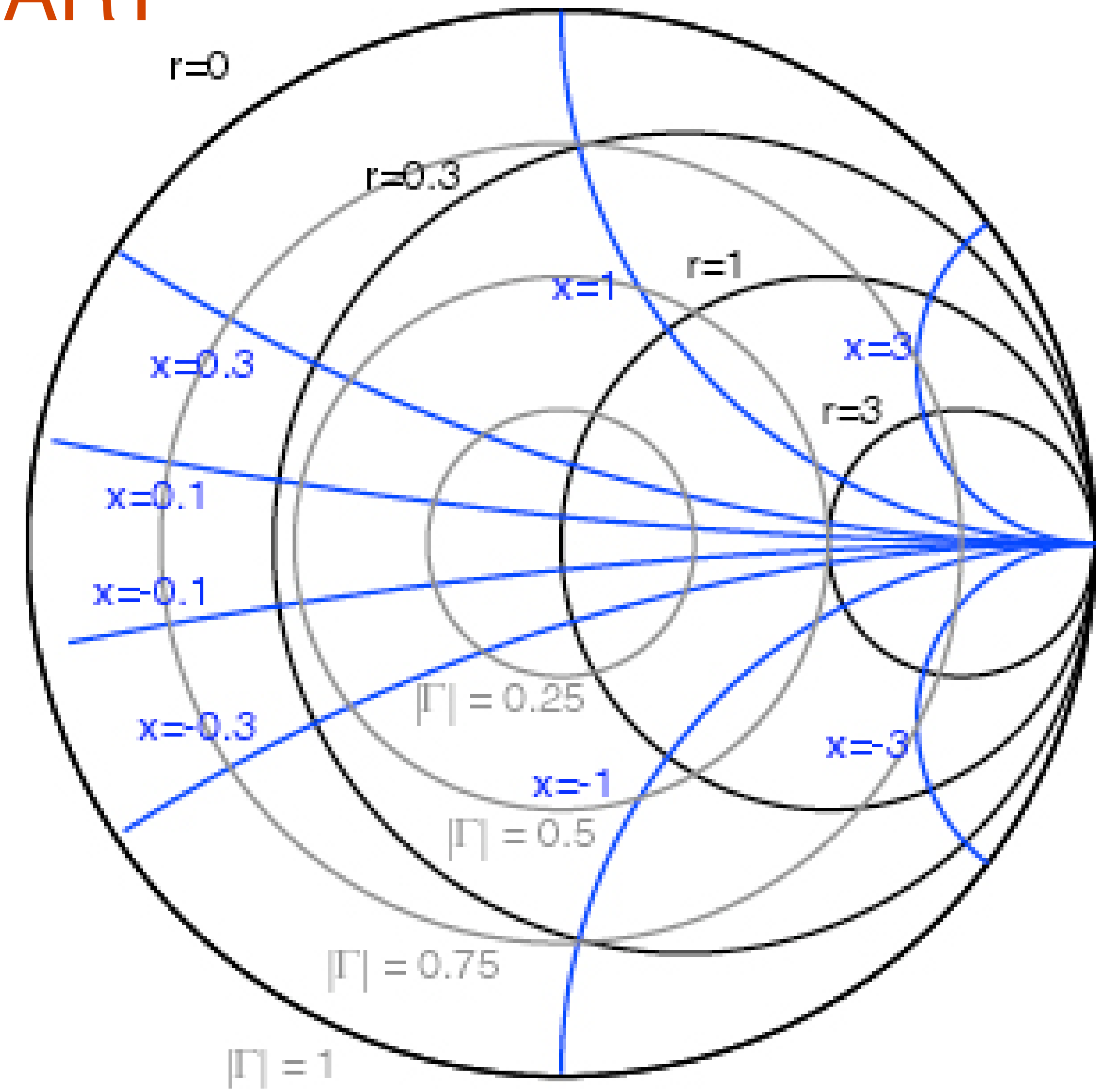
radius of: $\frac{1}{x}$

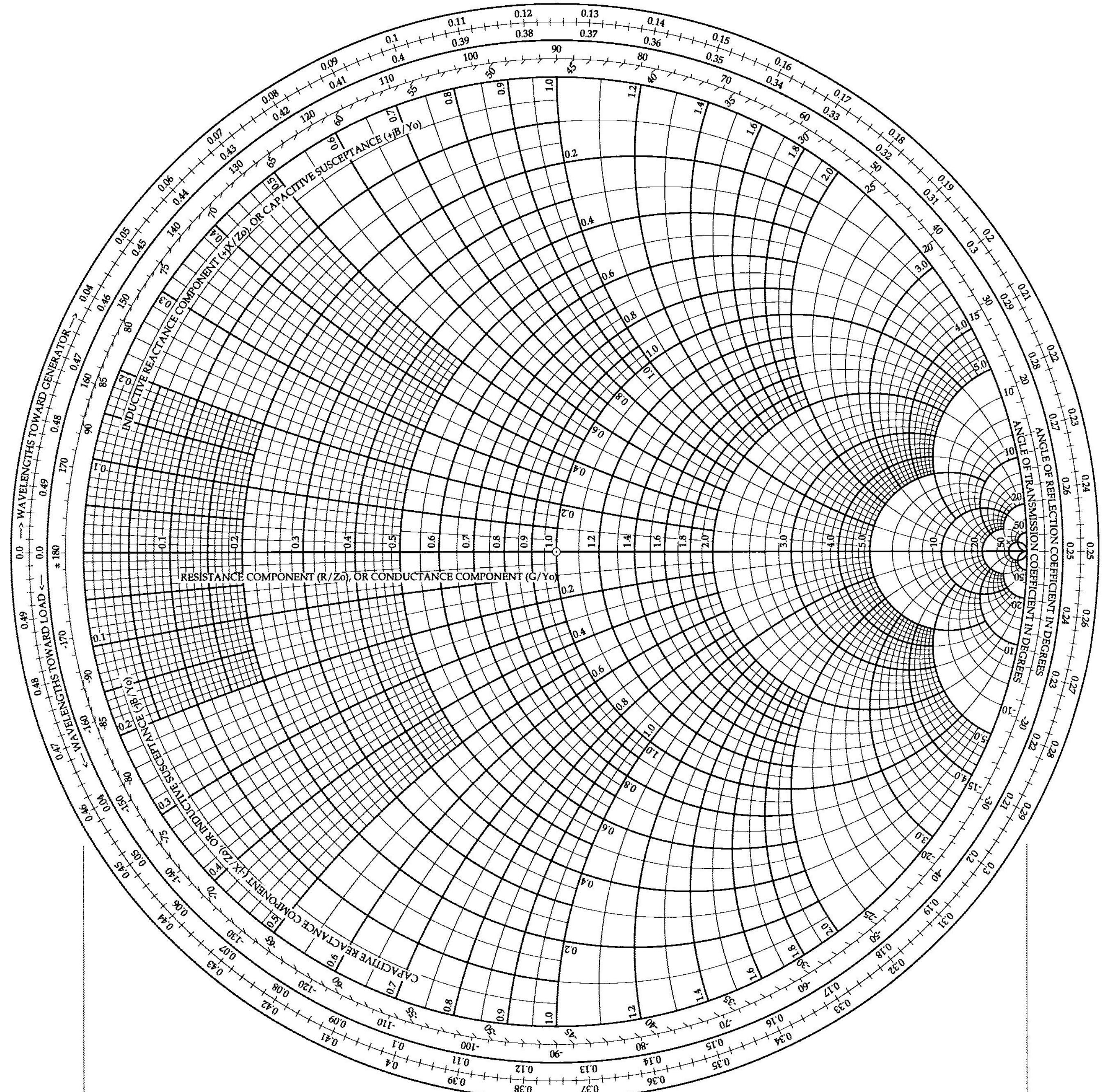


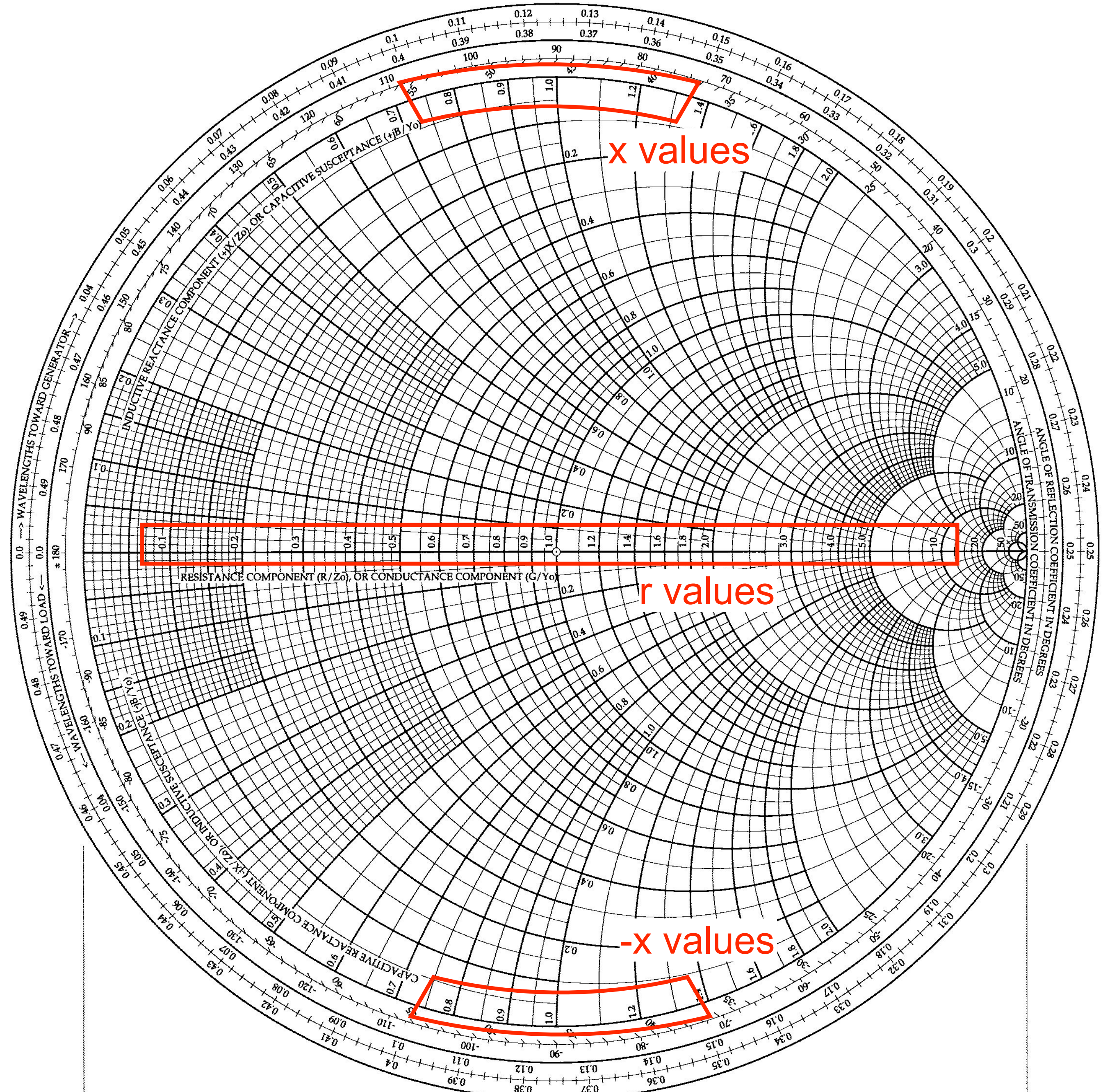
THE SMITH CHART

- Superimposes constant r and x circles inside unit circle on the complex Γ plane

- By drawing the SWR circle on the chart, we can quickly relate $\Gamma(d)$ with the unique line impedance z(d) = r + jx at that point




RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
INDUCTIVE REACTANCE COMPONENT (+jX/Zo), OR CAPACITIVE SUSCEPTANCE (+jB/Yo)
CAPACITIVE REACTANCE COMPONENT (-jX/Zo), OR INDUCTIVE SUSCEPTANCE (-jB/Yo)
—> WAVELENGTHS TOWARD GENERATOR —>
<— WAVELENGTHS TOWARD LOAD <—
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES


x values
r values
-x values
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
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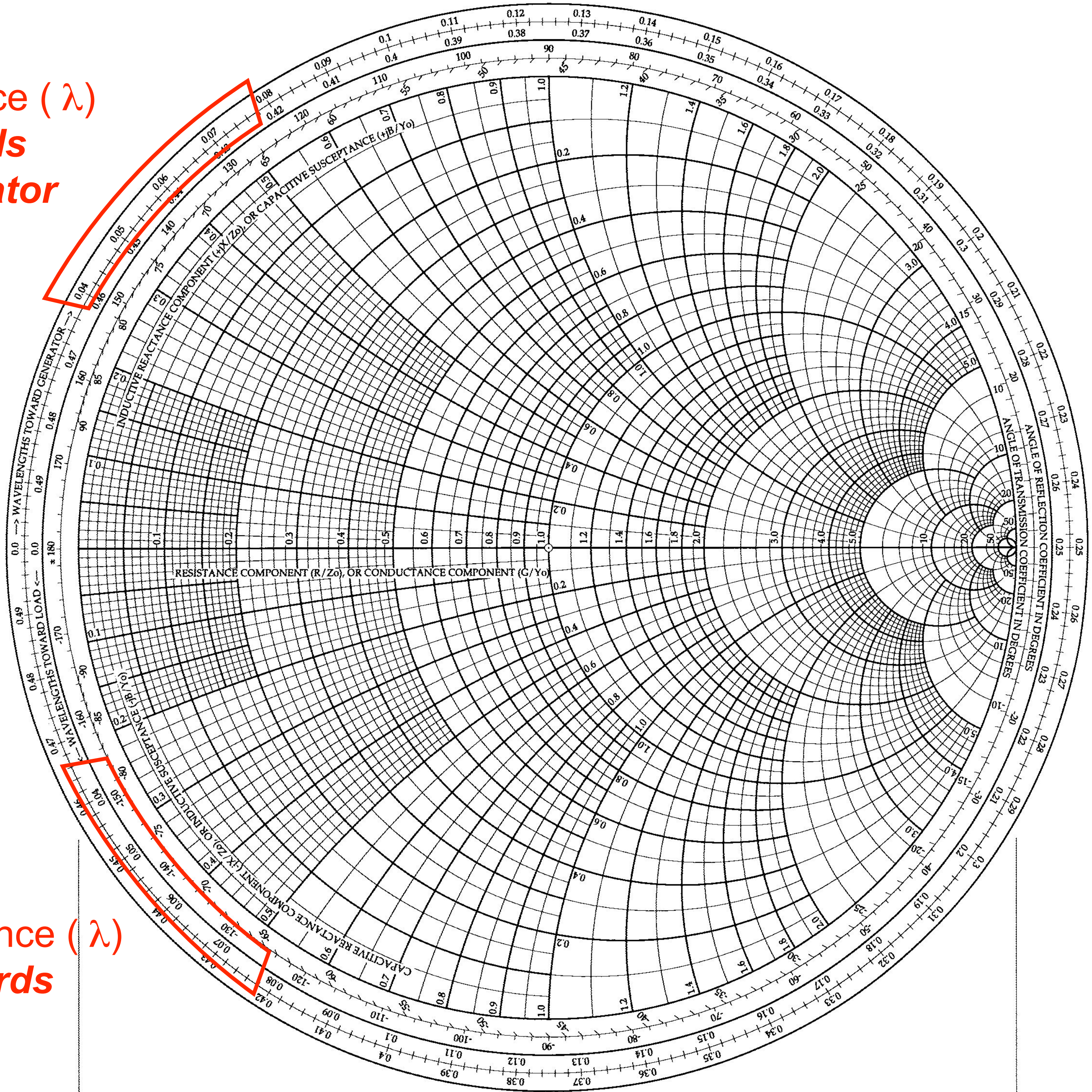
Distance (λ)
towards generator

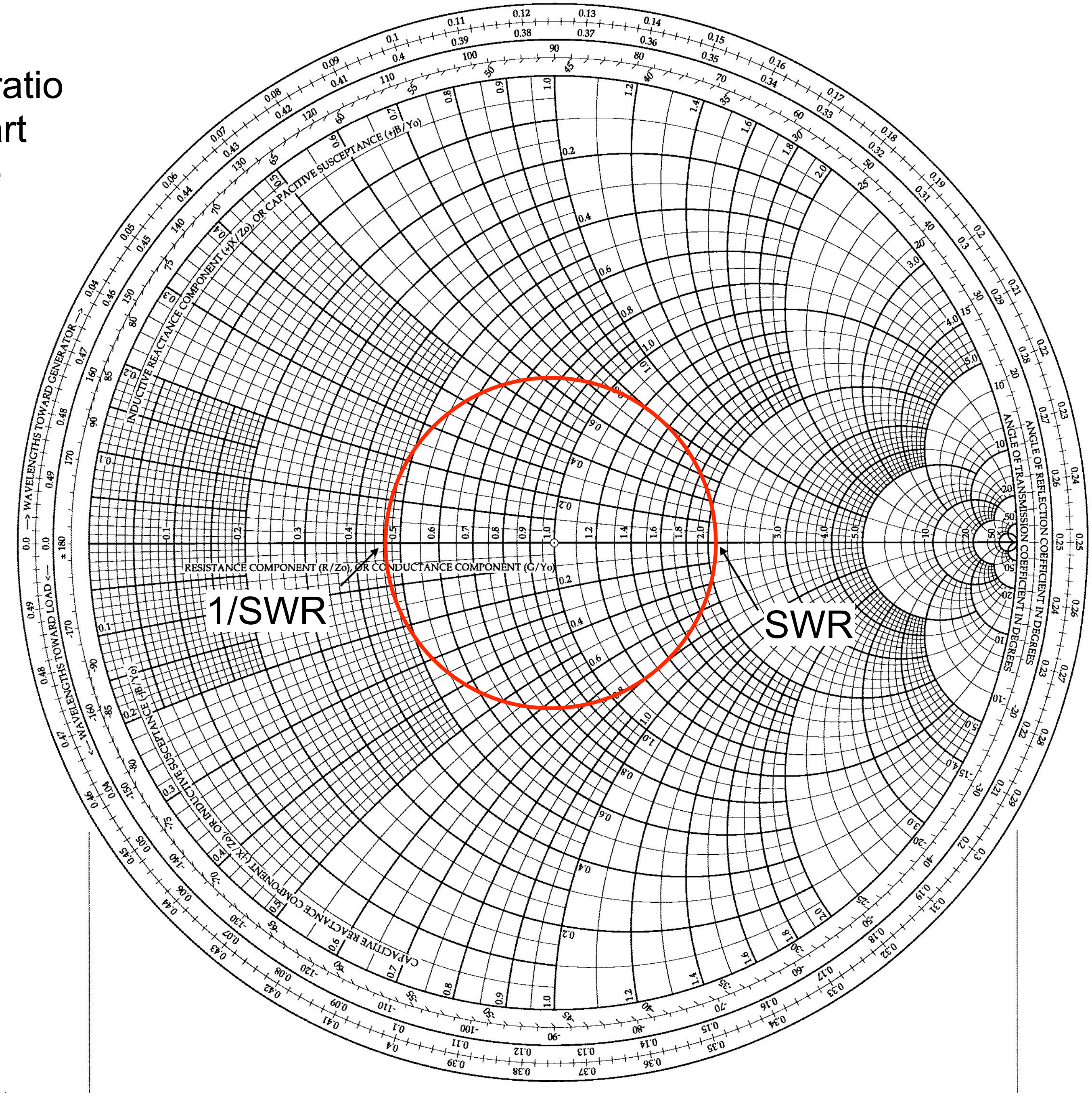


Distance (λ)
towards load

The standing wave ratio is read off of the chart by noting the r value where a constant Γ circle intersects the Γ_r axis

1) $SWR = Z_{max}/Z_0$
$= z_{max}$
$= r_{max}$

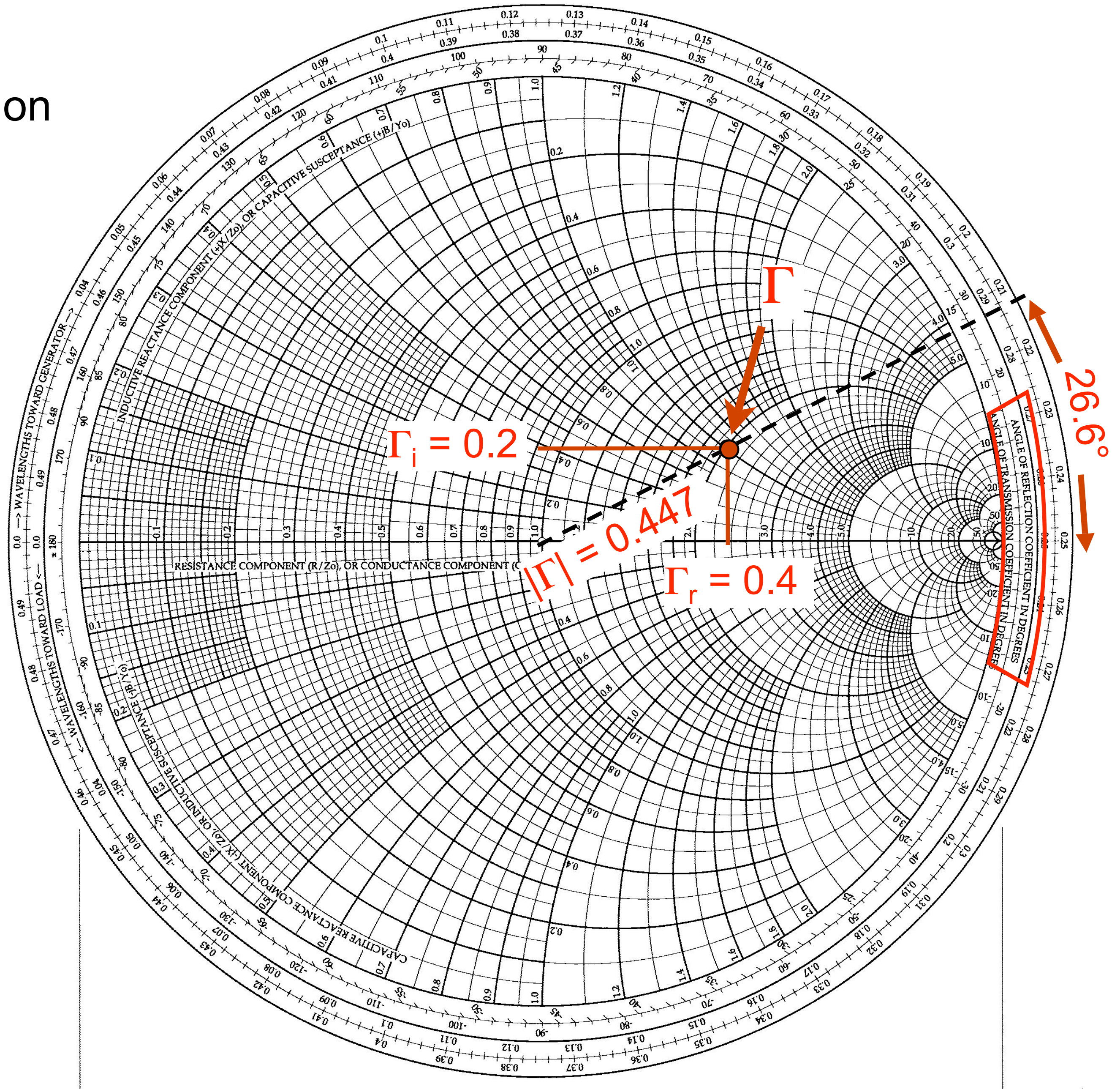
2) $SWR = Z_0/Z_{min}$
$= 1/z_{min}$
$= 1/r_{min}$



EXAMPLE:

If the effective reflection coefficient on a piece of 50 Ω line is $\Gamma=0.4+j0.2$, what is the corresponding line impedance at that point ?

1) Find Γ on the Smith Chart



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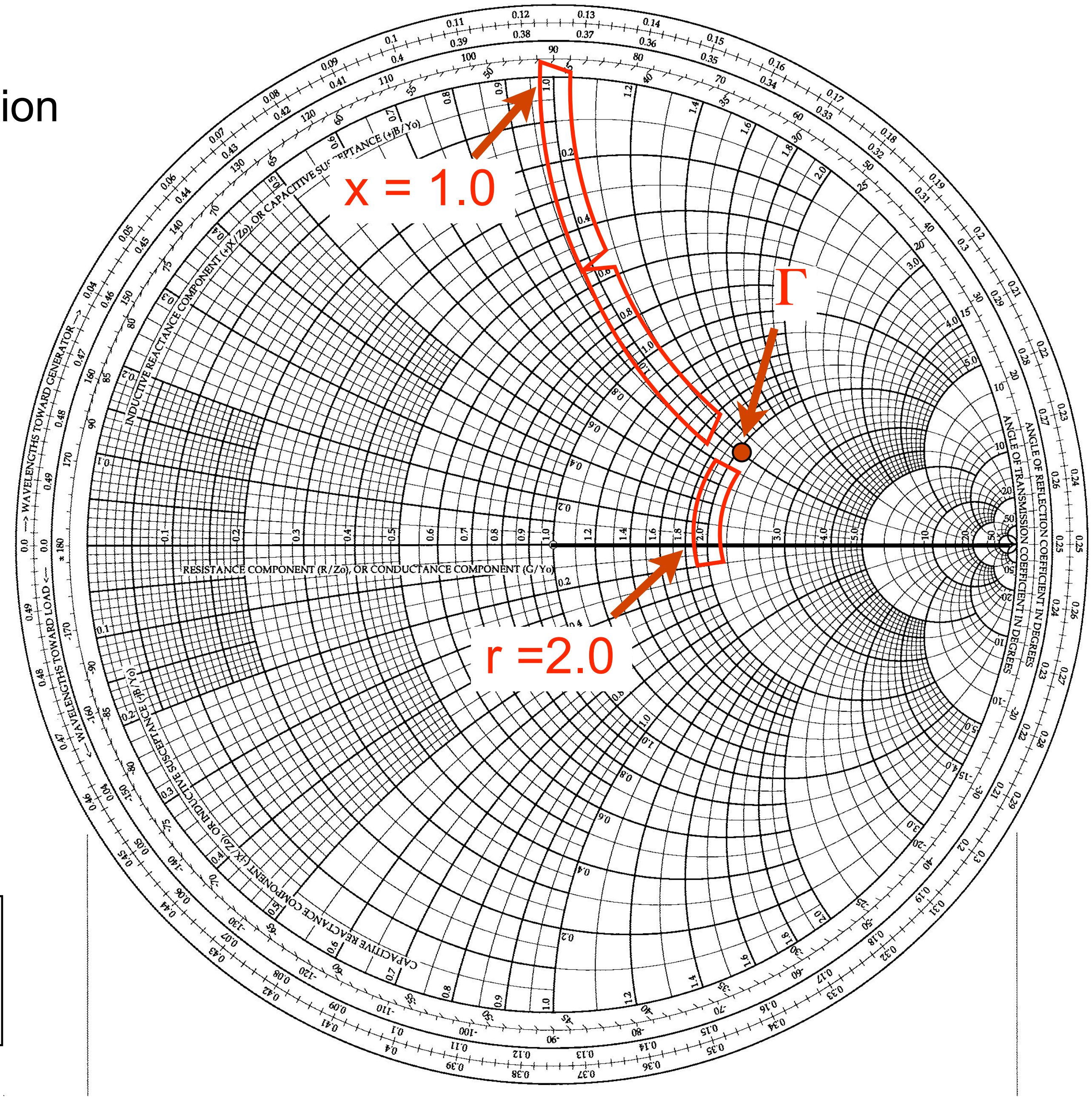


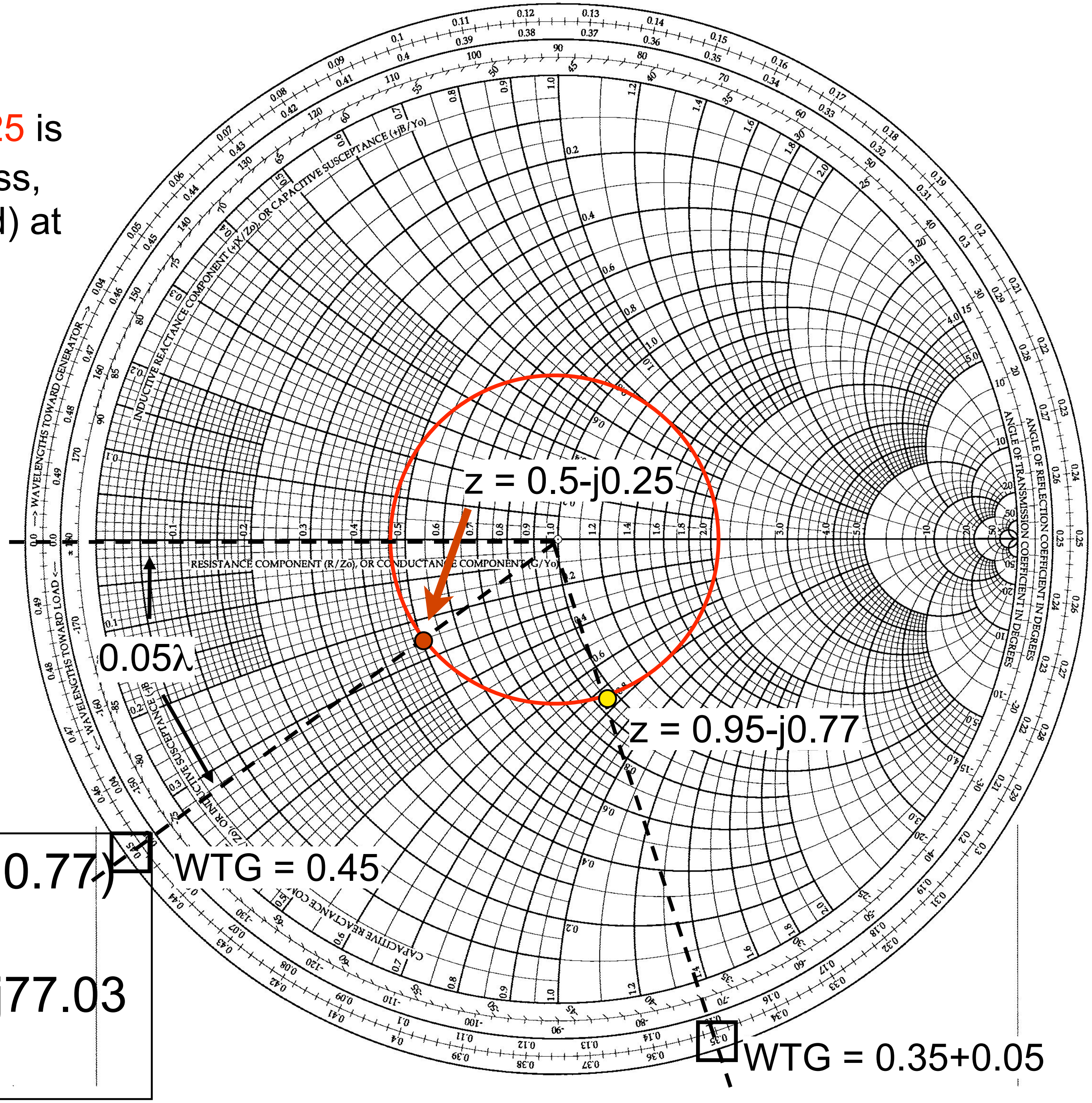
1) Find Γ on the Smith Chart
2) Read r and x off of chart
3) Use Z_0 to re-normalize

$$Z = 50\ (2.0 + j1.0)$$
$$= 100.0 + j\ 50.0\ \Omega$$

EXAMPLE:

A load with Z_L=50-j25 is attached to a lossless, 100 Ω T-L. Find Z(d) at d = 0.4λ

1) Normalize Z_L
   z_L = 0.5-j0.25
2) Find z_L on the Smith Chart
3) Rotate along constant Γ by 0.4λ
4) Read off new values of z
4) Use Z_0 to re-normalize



Z(d) = 100 (.95-j0.77)
= 95-j77 Ω

$Z(d)_{calc}$ = 95.29-j77.03

%error ~ 0.2%

Mapping between impedance and admittance:

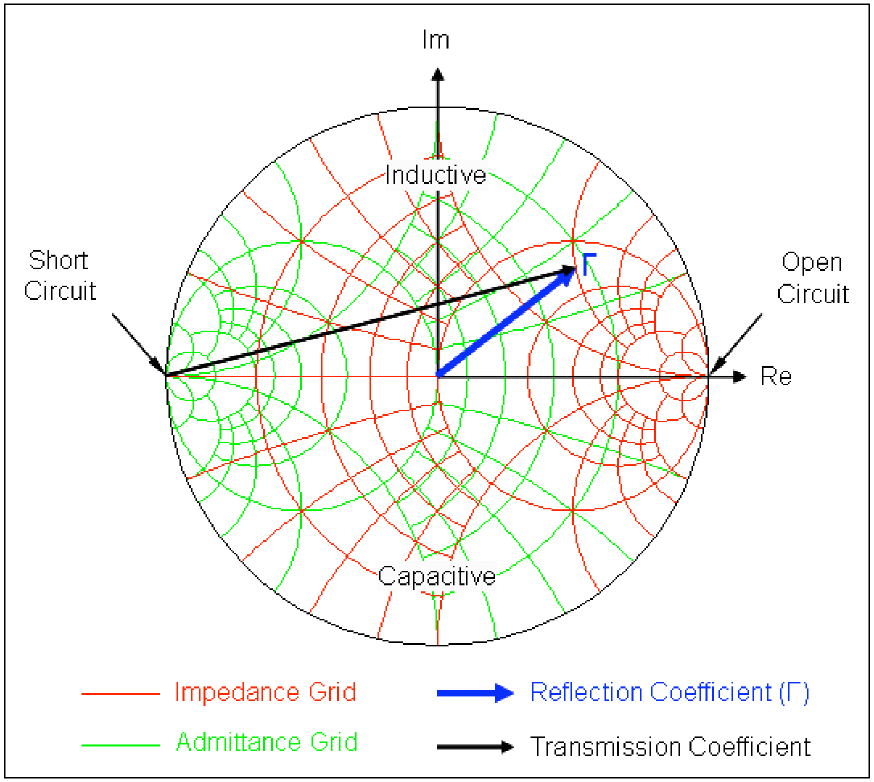
$$y(d) = \frac{1}{z(d)} = \frac{1-\Gamma(d)}{1+\Gamma(d)} = \frac{1-\Gamma_L e^{-j2\beta d}}{1+\Gamma_L e^{-j2\beta d}}$$

$$= z(d \pm \frac{\lambda}{4})$$

since: $e^{-j2\beta(d \pm \frac{\lambda}{4})} = e^{-j2\beta d} e^{\mp j2\beta(\frac{\lambda}{4})}$

$$= e^{-j2\beta d} e^{\mp \pi}$$

$$= -e^{-j2\beta d}$$



admittance grid is just the impedance grid reflected around the $\Gamma_r = 0$ axis

-or- can just read off values for y(d) = g(d) + j b(d) as

$$g(d) = r(d \pm \frac{\lambda}{4})$$

$$b(d) = x(d \pm \frac{\lambda}{4})$$

We can transform z into y by rotating z half way around a constant Γ circle

Given Z = 95+j20 on a 50 Ω line, find Y

1) Find z
   $z = 1.9+j0.4$
2) Draw Γ circle
3) Draw line through origin
4) Find intersection with Γ circle
5) Read off y
   $y = 0.5-j0.1$
6) Renormalize y
   $Y = y/Z_0$
   $= 10-j2$ mS
   $Y_{calc} = 10.1-j2.12$ mS