

Last lecture

Random Variables (RV) (Ch 2.1)

- Mean and Variance recap (Ch 2.2)

Conditional Probability (Ch 2.3)

- Motivation
- Examples
- Solver

Agenda

Law of Total Probability (Ch 2.10)

- Bayes formula

Independent Events/ RVs (Ch 2.4)

- Definition
- Motivation
- Examples and Facts

Distributions

- Bernoulli (Ch 2.4.3)
- Binomial (Ch 2.4.4)

Law of total probability

Law of total probability

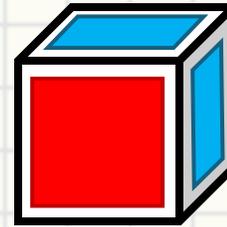
				A
				A^c

- Case-by-case discussion law...
- $P(A)$ is the summed of “Partitioned conditional probability”
- $P(A) = \sum_i P(A|E_i)P(E_i)$

Law of total probability

There are 3 dice A, B, C in the bag

- $A = [R \times 1; B \times 5]$
- $B = [R \times 2; B \times 4]$
- $C = [R \times 3; B \times 3]$



Draw one die and roll many times

- $P(R_1)$
- $P(R_2|R_1)$

Bayes Formula

Conditional probability + Law of total probability

- How do we get $P(B|A)$ from $P(A|B)$?

- $P(B|A) =$

- $P(E_i|A) =$

Disease problems

Assume there is a disease A , and the corresponding test T

- What do the followings mean?
- $P(T|A) = 0.9$
- $P(T|A^c) = 0.05$
- $P(A) = 0.01$

- $P(A|T) =$

Disease problems

According to CDC survey on smoker

- 18% of adults are smokers
- 15% of women are smokers
- Population = 50% men + 50% women

- What fraction of adult smokers are women

Disease problems

According to CDC survey on smoker vs. lung cancer

- 15% of women are smokers
- Compared to nonsmokers, women who smoke are 13 times likely to get lung cancer
- If I pick a female lung cancer patient, how likely she is a smoker?

Independent Events/ RVs

Definition

A and B are events, they are if

- $P(B|A) = P(B)$ or
- $P(AB) = P(A)P(B)$

Facts

- $P(B|A) = P(B)$ implies $P(A|B) = P(A)$
- If $P(A) = 0$, B is independent of A

Definition

A and B are events, they are independent if

- $P(B|A) = P(B)$ or
- $P(AB) = P(A)P(B)$

RVs X and Y are independent if A and B are independent for any $X \in A$ and $Y \in B$

Motivation

Independent is a common but strong property

- $P(AB) = P(A)P(B)$ the pmf
 - Compute the pmf easily
 - Will skip Ch 2.3 affect only my HW2 and Midterm 1?
 - If I join this club, will it affect my GPA?
- Decide the model complexity
 - What really affects the results?
 - What do I need to ask when reviewing a loan request?
 - What input data do I need to predict the defect?

Examples

Physically independence – Toss a coin and roll a die (N, X)

- $A \triangleq \{N = H\}$
- $B \triangleq \{X = 6\}$

Probabilistic independence

- $A \triangleq X \text{ is even}$
- $B \triangleq \{X \equiv 0 \pmod{3}\}$

Examples

Physically independence – Toss a coin and roll a die (N, X)

- $A \triangleq \{N = H\}$
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Probabilistic independence

- $A \triangleq X \text{ is even}$
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Slido

Choose “independent” RVs/ Events

- A. Pick X from 52 playing card,
{ X is RED} vs. { X is prime}
- B. { X is even} vs. { X is prime}
- C. Pick Y from 365 days,
{ Y is rainy at Champaign} vs. { Y is a holiday}
- D. {Any midterms on Y } vs. { Y is a holiday}



#3409457

Terms and Facts

- A, B, C are pairwise independent if $(A, B), (B, C), (A, C)$ are mutually independent
 - Toss a fair coin twice
 - $A \triangleq \{\text{First coin is Head}\}$
 - $B \triangleq \{\text{second coin is Head}\}$
 - $C \triangleq \{\text{toss results are the same}\}$

Terms and Facts

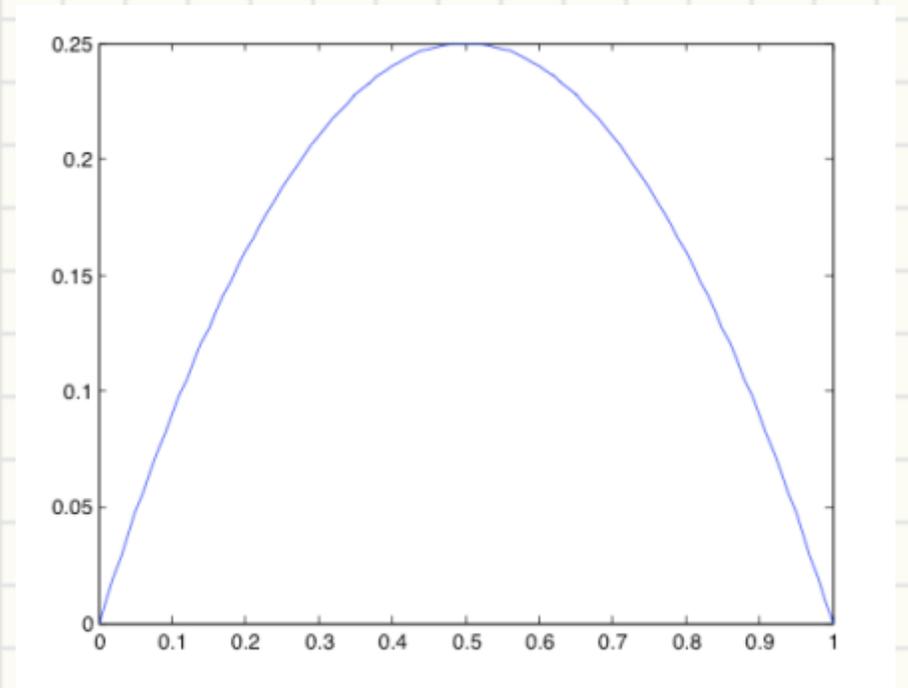
- A, B, C are independent they are pairwise independent and $P(ABC) = P(A)P(B)P(C)$
- A_1, A_2, \dots, A_i are independent if
$$P(A_{i_1} A_{i_2} \dots A_{i_k}) = P(A_{i_1})P(A_{i_2}) \dots P(A_{i_k})$$

Distributions

Bernoulli Distribution

X is Bernoulli distribution with parameter p if

- $P\{X = 1\} = p$ and $P\{X = 0\} = 1 - p$
- “Toss a (unfair) coin with p probability Head”
- Only two possible outcomes, pmf contains two bins
- $E[X] =$
- $E[X^2] =$
- $\sigma_x^2 =$

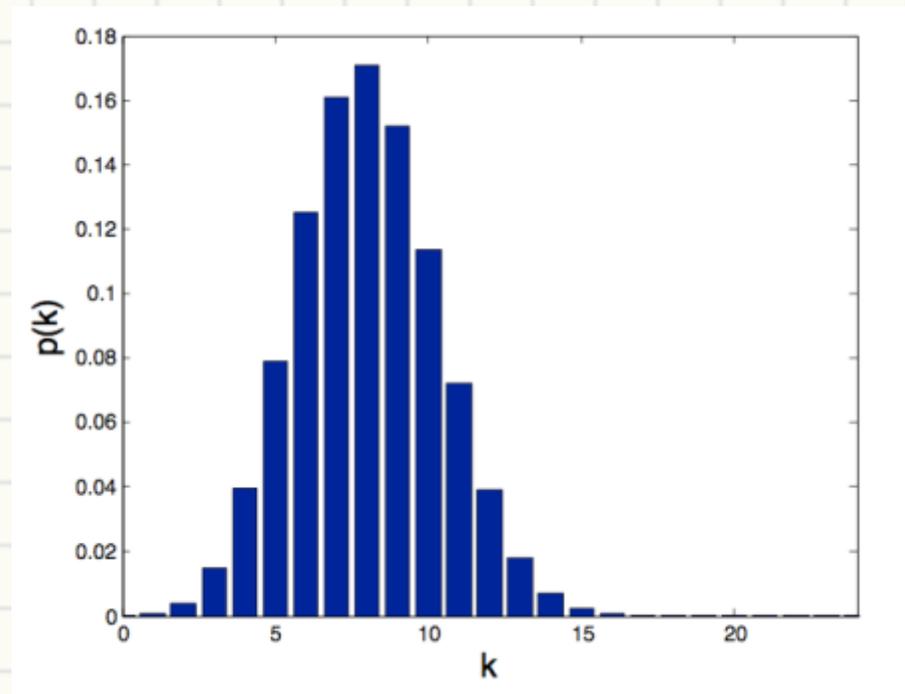


Binomial Distribution

X is binomial distribution with parameter (n, p) if

- X is sum of n Bernoulli trials with parameter p
- Draw the unfair coin n times and count the Head

- $p_X(k) =$



Binomial Distribution

$$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

Proof - $\sum_{k=0}^n p_X(k) = 1$

- $(1+x)^n = \sum_{k=0}^n x^k$
- $x = \frac{p}{1-p}$

Binomial mean

$$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$E[X] = \sum_{k=0}^n k \binom{n}{k} p^k (1-p)^{n-k}$$

$$= \sum_{k=1}^n k \frac{n!}{(n-k)!k!} p^k (1-p)^{n-k}$$

$$= np \sum_{k=1}^n \frac{(n-1)!}{(n-k)!(k-1)!} p^{k-1} (1-p)^{n-k}$$

$$= np \sum_{m=0}^{n-1} \frac{n-1!}{(n-1-m)!m!} p^m (1-p)^{n-1-m}$$

$$= np \sum_{m=0}^j \binom{j}{m} p^m (1-p)^{j-m}$$