Last lecture

Random Variables (RV) (Ch 2.1)

- Probability Mass Function (pmf)
- Mean and Variance (Ch 2.2)

Agenda

Random Variables (RV) (Ch 2.1)

- Mean and Variance recap (Ch 2.2)

Conditional Probability (Ch 2.3)

- Motivation
- Examples
- Solver

Law of Total Probability (Ch 2.10)

Mean and Variance

Mean and Variance of linear RV functions

Always true:

By LOTUS:

1. $E[aX + b] =$
2. $E[X + Y] =$

By definition:

1. $Var(aX + b) =$
2. σ_{aX+b}

Detailed later - Requires the relationship between X, Y

1. $Var(X + Y)$
2. $E[(X + Y)^2]$

- All functions related to cross terms, e.g. $E[XY]$

Examples

$A \triangleq$ "Rolling a die twice and sum the results"
$B \triangleq$ "Roll the same die once, multiply the result by 2"

- $p_A(k) = p_B(k)$?

- $E[A] = E[B]$?

- $Var(A) = Var(B)$?

Standardized RV

For any RV X

- $\frac{X-\mu_X}{\sigma_X}$ is a standardized RV – Mean 0, variance 1

Conditional Probability (Ch 2.3)

Motivation

The probability of B happens given A happens

- $P($ pair of socks are same color $)$ given $S_1 = B$
- $P(I\ win\ Texas\ Hold'em)$ given $X = Ace + Ace$
- $P(I\ pass\ 313)$ given I skip HW1…

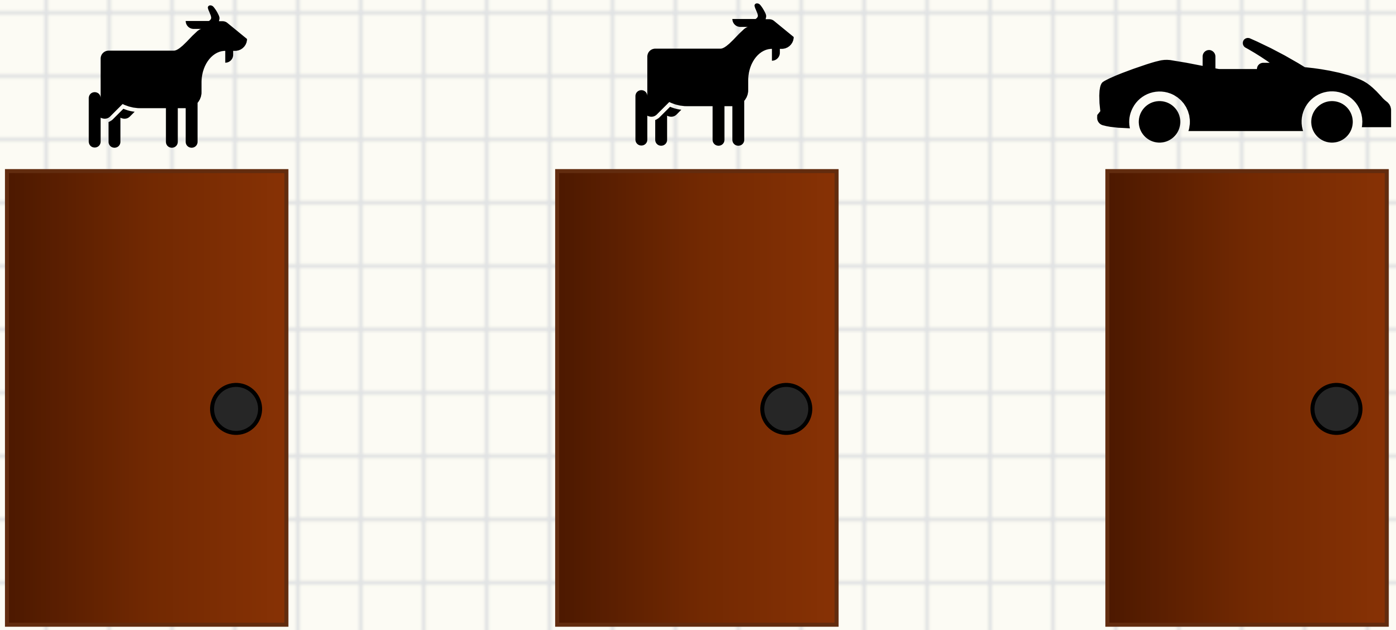
Why do we need conditional probability?

- Analyze the relationship between two events
- Find the optimal solution to make an event probable

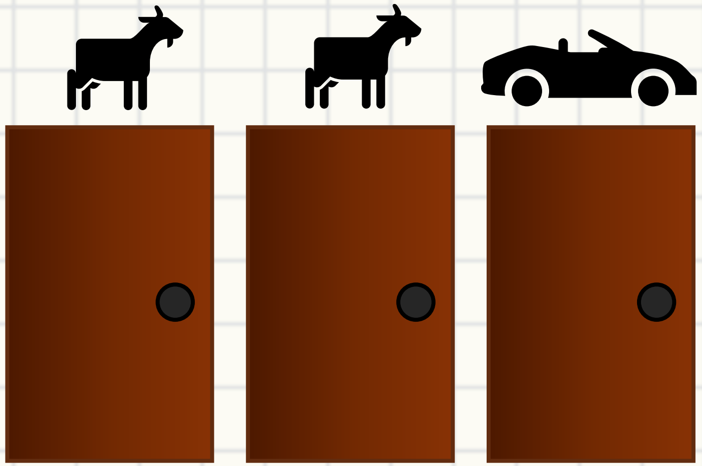
Examples

3 doors (Monty Hall) problem

- 3 closed doors – 1 leads to a car, the others lead to goats
- After you choose one, the host will open a "Goat" door
- Should you change the door?



Examples



- Let X_i denotes i-th choice
- Never change (NC)
  - $P(W|NC) = P(W|X_2 = X_1)$
$= P(X_1 = Car)$

- Change (C)
  - $P(W|C) = P(W|X_2 \neq X_1)$
$= P(X_1 = Goat)$

- What if there are 4 doors… 2 cars and 2 goats?

Conditional Probability

$$P(B|A) = \begin{cases} \\ \end{cases}$$

Roll two dice, $A =$ sum is 6; $B =$ numbers are not equal

$P(B) = ?$ $P(B|A) = ?$ $P(B^c|A) = ?$

Facts of conditional probability

- $P(B|A) > 0$
- $P(B|A) + P(B^c|A) = 1$
- $P(\Omega|A) = 1$
- $P(AB) = P(A|B)P(B)$
- $P(ABC) = P(A|BC)P(B|C)P(C)$

Unfair Games

Alice and Bob are playing a game

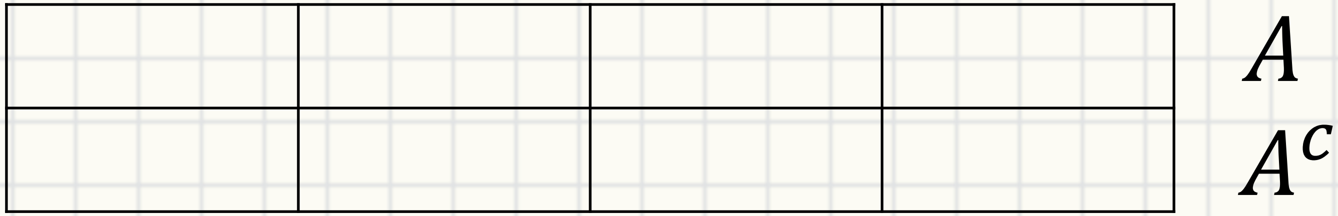
- They both play one of "O" or "X" simultaneously
- Alice earns the profits as follows

- If Alice wants to maximize her profits, $P_A(O) =$

	O	X
O	-1	2
X	2	-3

Law of total probability

Law of total probability



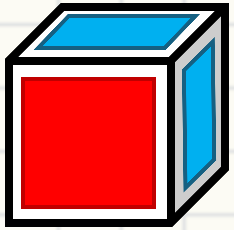
- Case-by-case discussion law…
- $P(A)$ is the summed of “Partitioned conditional probability”

- $P(A) = \sum_i P(A|E_i)P(E_i)$

Law of total probability

There are 3 dice A, B, C in the bag

- $A = [R \times 1; B \times 5]$
- $B = [R \times 2; B \times 4]$
- $C = [R \times 3; B \times 3]$



Draw one die and roll many times

- $P(R_1)$
- $P(R_2|R_1)$

Bayes Formula

Conditional probability + Law of total probability

- How do we get $P(B|A)$ from $P(A|B)$?

- $P(B|A) =$

- $P(E_i|A) =$

Disease problems

Assume there is a disease A, and the corresponding test T

- What do the followings mean?
- $P(T|A) = 0.9$
- $P(T|A^c) = 0.05$
- $P(A) = 0.01$

- $P(A|T) =$

Disease problems

According to CDC survey on smoker

- 18% of adults are smokers
- 15% of women are smokers
- Population = 50% men + 50% women

- What fraction of adult smokers are women

Disease problems

According to CDC survey on smoker vs. lung cancer

- 15% of women are smokers
- Compared to nonsmokers, women who smoker are 13 times likely to get lung cancer

- If I pick a female lung cancer patient, how likely she is a smoker?