Last lecture

Probability with equally likely outcomes (Ch 1.4)

- Draw socks from the drawer
- Poker hands

Random Variables (RV) (Ch 2.1)

- Definition

Agenda

Random Variables (RV) (Ch 2.1)

- Probability Mass Function (pmf)
- Mean and Variance (Ch 2.2)

Conditional Probability (Ch 2.3)

- Motivation
- Examples
- Solver

Law of Total Probability (Ch 2.10)

Probability Mass Function (PMF)

Probability Mass Function (PMF)

- $p_X(u) = P\{X = u\}$ for a discrete RV X
- $\sum_i p_X(u_i) = 1$
- Let X be the outcome of a fair die roll
  - $p_X(2) = \frac{1}{6}$
- PMF can determine probabilities of all events determined by X

Slido!

Let’s create a custom die and plot the PMF

Please vote for your preferred number from 1-6

#2887579

Probability Mass Function (PMF)

- Let S be the sum of rolling two dice
  - $p_S =$

- Let M be the max number of rolling two dice
  - $p_M =$

Probability Mass Function (PMF)

- Let N be the # of toss until getting first tail

- Let M be the # of heads observed until getting first tail

Mean and Variance (Ch 2.2)

Mean

Do we need detailed p_{height}?

- In many cases, we just need mean μ_X instead of p_X
- $\mu_X = E[X] = \sum_i a_i p_X(a_i)$

- X is the number for a die roll
- $Y = 2X$
- $Z = |X - 3|$

Function of RV - LOTUS

X is RV uniformly sampled from $\{-1, 0, 1, 2, 3\}$

- $p_X(x) =$

$Y = X^2$

- $\mu_y = E[Y] =$

Function of RV - LOTUS

X is RV uniformly sampled from $\{-1, 0, 1, 2, 3\}$, $Y = X^2$
But we can also compute $E[Y]$ from p_X!

Mean of RV function $g(X)$ is

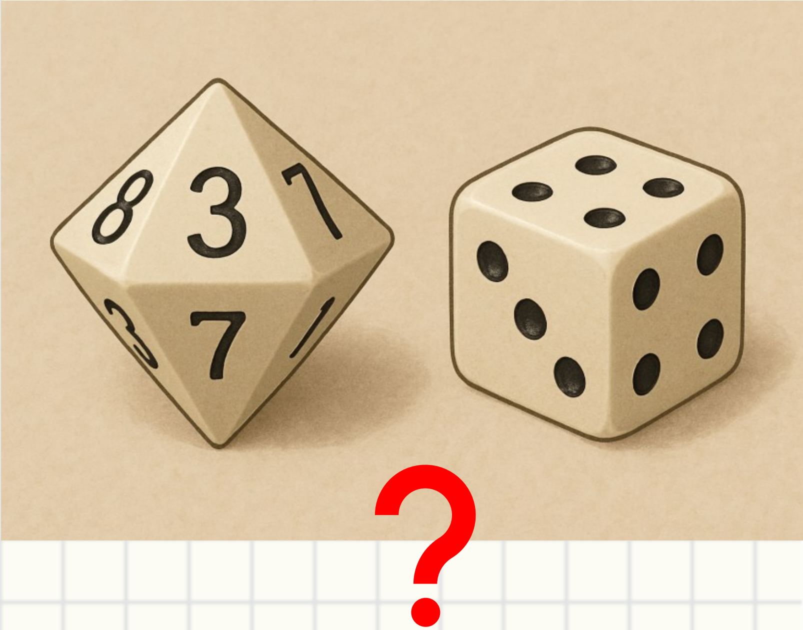
$$E[g(X)] =$$

Law of the unconscious statistician (LOTUS)

LOTUS examples

X is rolling a D6, Y is rolling a D8 (8-sided die)

$E[XY] = ?$



LOTUS examples

Math magic trick

- Pick a number from 1-10
- Multiply it by 3
- Subtract it by 2
- Divided it by 6 and keep the remainder
- Plus 2
- Is it 3 or 6?

Variance and Standard Deviation

Do you want your salary to be
$p_X(10K) = 1$ or
$p_Y(0) = 0.09, p_Y(1000K) = 0.01$

What if it's bonus?
$p_X(1K) = 1$ or
$p_Y(0) = 0.09, p_Y(100K) = 0.01$

Variance and Standard Deviation

Mean is important… but not complete enough

- Variance $Var(X)$ is how PMF spreads apart from μ_X

- $Var(X) \triangleq E[(X - \mu_x)^2] = E[X^2] - (E[X])^2$

Variance and Standard Deviation

Standard deviation $\sigma_X \triangleq \sqrt{Var(X)}$; $Var(X) = \sigma_X^2$

σ_X is of the same unit as X

$Var(X + c) =$

$Var(aX + c) =$

Note: $2X \neq X + X$

Standardized RV

For any RV X

- $\frac{X-\mu_X}{\sigma_X}$ is a standardized RV – Mean 0, variance 1

Conditional Probability (Ch 2.3)

Motivation

The probability of B happens given A happens

- $P($ pair of socks are same color $)$ given $S_1 = B$
- $P(I\ win\ Texas\ Hold'em)$ given $X = Ace + Ace$
- $P(I\ pass\ 313)$ given I skip HW1…

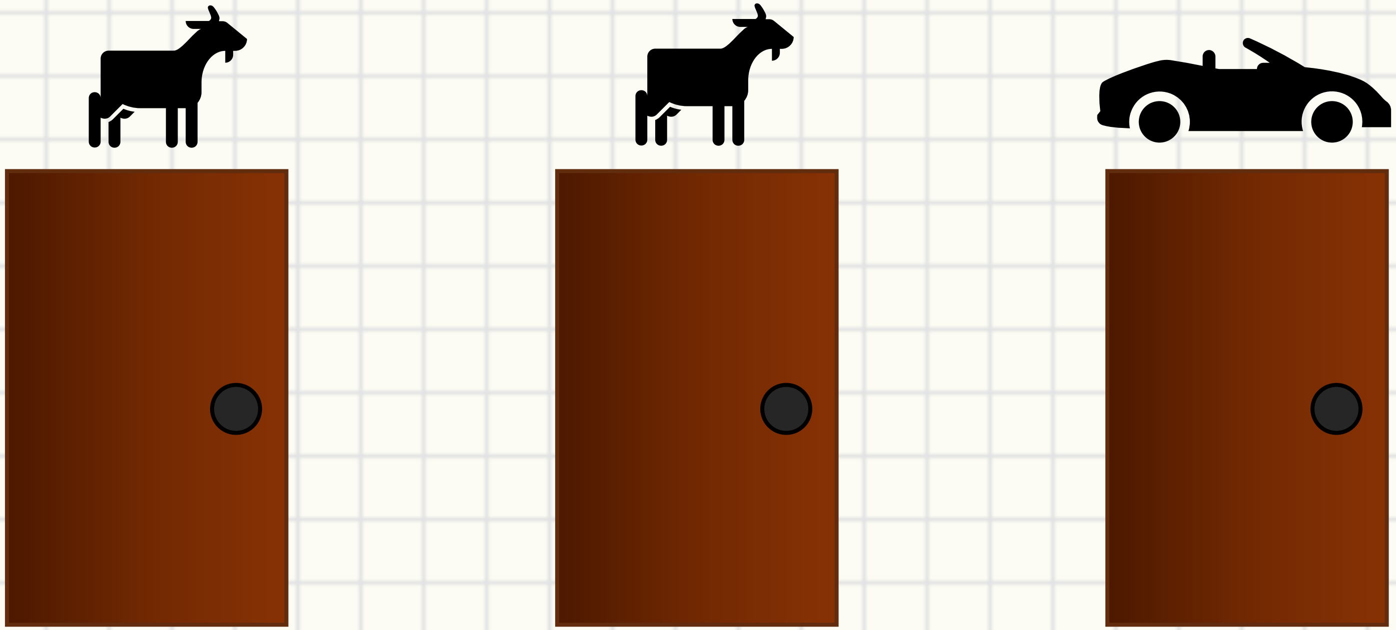
Why do we need conditional probability?

- Analyze the relationship between two events
- Find the optimal solution to make an event probable

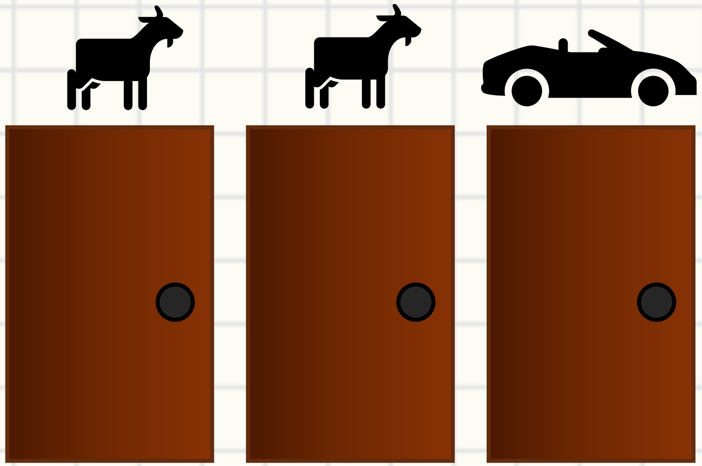
Examples

3 doors (Monty Hall) problem

- 3 closed doors – 1 leads to a car, the others lead to goats
- After you choose one, the host will open a “Goat” door
- Should you change the door?



Examples



- Never change (NC)
  - $P(W|NC) = P(W|X_2 = X_1)$
  $= P(X_1 = Car)$
- Change (C)
  - $P(W|C) = P(W|X_2 \neq X_1)$
  $= P(X_1 = Goat)$
- What if there are 4 doors… 2 cars and 2 goats?

Conditional Probability

$$P(B|A) = \begin{cases} \\ \end{cases}$$

Roll two dice, $A =$ sum is 6; $B =$ numbers are not equal

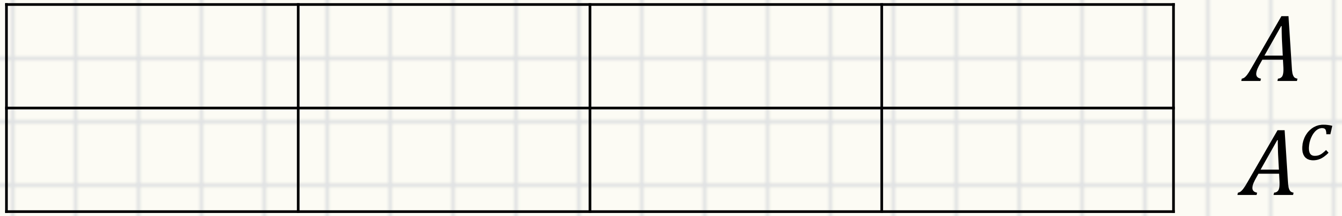
$P(B) = ?$ $P(B|A) = ?$ $P(B^c|A) = ?$

Facts of conditional probability

- $P(B|A) > 0$
- $P(B|A) + P(B^c|A) = 1$
- $P(\Omega|A) = 1$
- $P(AB) = P(A|B)P(B)$
- $P(ABC) = P(A|BC)P(B|C)P(C)$

Law of total probability

Law of total probability



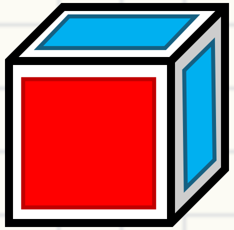
- Case-by-case discussion law…
- $P(A)$ is the summed of “Partitioned conditional probability”

- $P(A) = \sum_i P(A|E_i)P(E_i)$

Law of total probability

There are 3 dice A, B, C in the bag

- $A = [R \times 1; B \times 5]$
- $B = [R \times 2; B \times 4]$
- $C = [R \times 3; B \times 3]$



Draw one die and roll many times

- $P(R_1)$

- $P(R_2|R_1)$