Last lecture

Probability Event (Ch 1.2)

- Axioms of Probability

Counting the size of events (Ch 1.3)

- Independent events
- Dependent but countable

Agenda

Probability with equally likely outcomes (Ch 1.4)

- Draw socks from the drawer
- Poker hands

Random Variables (RV) (Ch 2.1)

- Definition
- Probability Mass Function (pmf)

Mean and Variance (Ch 2.2)

Overcounting

Permutation

- The to order n different items
- How many ways can you order letters A, B, C, D?

- N letters ->
- What if I want to order "A, B, C ... G " 7 letters, but only pick the first 4?

- What if I want to order letters ILLINI?

Principle of over-counting

- What if I want to order letters ILLINI?

- For an integer $K \geq 1$, if each element of a set is counted K times, then the number of elements in the set is the total count divided by K

Combination

- $\binom{n}{k}$ or $C(n,k)$
  - The to choose k out of n different items
  - $\binom{n}{k} =$

- Draw 3 balls out of 5 balls replacement

The Socks Problem

I have 4 pairs of black socks and 2 pairs of white socks

$P(Draw\ two\ socks, color\ is\ the\ same)$?

Slido!

A bag contains $\{R, R, R, B, B, G\}$
What's the probability that I draw 3 balls all different colors?

A. $\frac{3\times2\times1}{6!}$

B. $\frac{3\times2\times1}{\binom{6}{3}}$

C. $\frac{3\times2\times1}{6\times5\times4}$

D. None of the above

#3626145

Poker Problem

$\Omega_{card} =$

Draw 5 cards out of 52 cards
FULL HOUSE = 3 same numbers, other 2 same numbers

$P(FULL\ HOUSE) =$

Sample space with infinite cardinality

Interval probability space

- $\Omega = \{\omega: 0 \leq \omega \leq 1\}$ $P([a,b]) = b - a$
- $A = [0.2, 0.8]$

Probability of continuous intervals…

- $P(x = 0.234) =?$
- Ask the right question!

Random Variable

Random Variable

Rolling a die $\Omega = \{1, 2, 3, 4, 5, 6\}$, Event "odd" $A = \{1, 3, 5\}$

- As if I put a " " to the die $[1, 0, 1, 0, 1, 0]$
- Coated die X is a " "

Random Variable ( )

- A random variable is a on Ω

- A random variable X is said to be if there's a finite/ countable infinite set $\{u_1, u_2, \dots\}$ s.t. $P\{X \in \{u_1, u_2, \dots\}\} = 1$

RV in real-world

- Heights of the classmates
- # of computers fixed in the college life
- Scores of 313 midterm

Why do we need RV?

- Compute complex events
  - What's my final letter grade at 313?

- Evaluate relationships between events
  - Is heights related with the # of computers fixed?

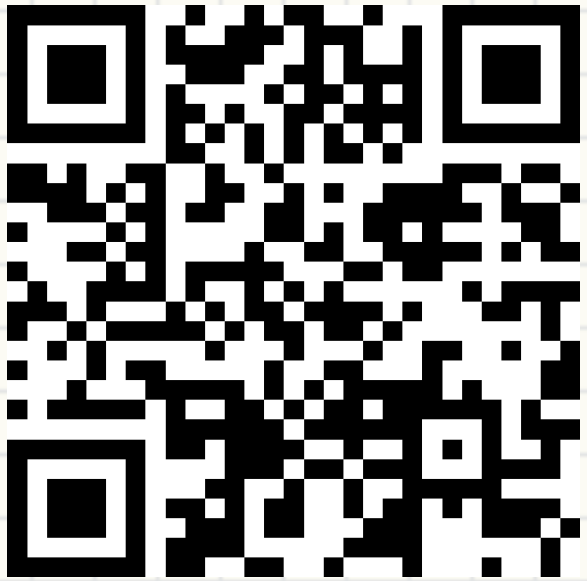
Probability Mass Function (PMF)

- $p_X(u) = P\{X = u\}$ for a discrete RV X
- $\sum_i p_X(u_i) = 1$
- Let X be the outcome of a fair die roll
  - $p_X(2) =$
- PMF can probabilities of all events determined by X

Slido!

Let’s create a custom die and plot the PMF

Please vote for your preferred number from 1-6



#3626145

Probability Mass Function (PMF)

- Let S be the sum of rolling two dice
  - $p_S =$

- Let M be the max number of rolling two dice
  - $p_M =$

Probability Mass Function (PMF)

- Let N be the # of toss until getting first tail

- Let M be the # of heads observed until getting first tail

Mean and Variance (Ch 2.2)

Mean

Do we need detailed p_{height}?

- In many cases, we just need mean μ_X instead of p_X
- $\mu_X = E[X] = \sum_i a_i p_X(a_i)$

- X is the number for a die roll
- $Y = 2X$
- $Z = |X - 3|$

Function of RV - LOTUS

X is RV uniformly sampled from $\{-1, 0, 1, 2, 3\}$

- $p_X(x) =$

$Y = X^2$

- $\mu_y = E[Y] =$

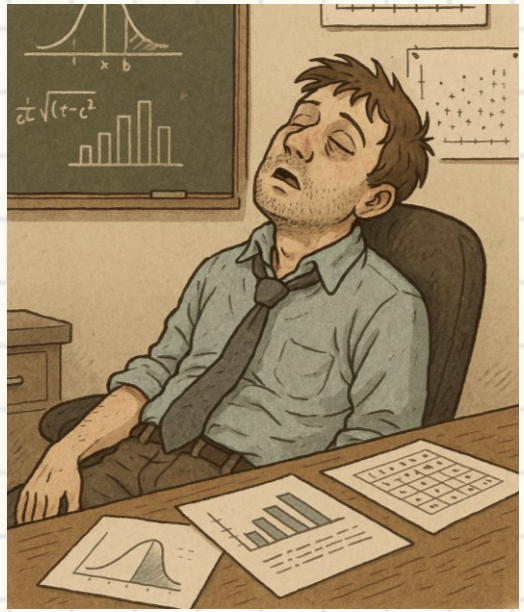
Function of RV - LOTUS

X is RV uniformly sampled from $\{-1, 0, 1, 2, 3\}$, $Y = X^2$
But we can also compute $E[Y]$ from p_X!

Mean of RV function $g(X)$ is

$$E[g(X)] =$$

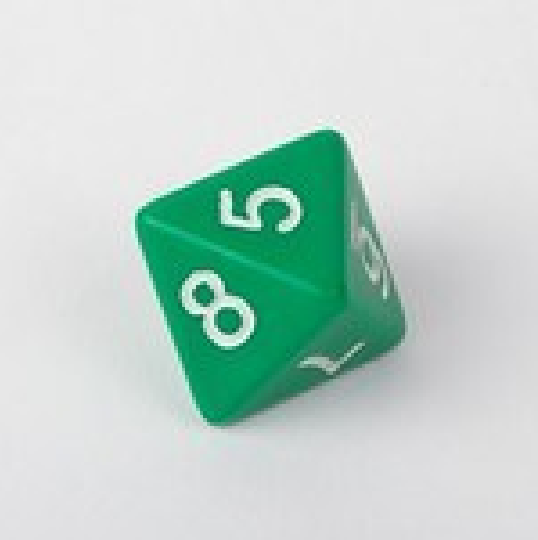
Law of the unconscious statistician (LOTUS)

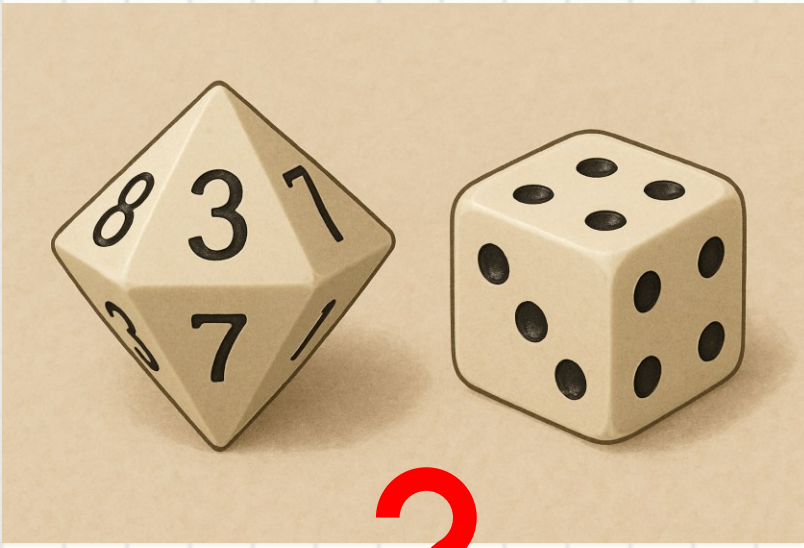


LOTUS examples

X is rolling a D6, Y is rolling a D8 (8-sided die)

$E[XY] = ?$





?

LOTUS examples

Math magic trick

- Pick a number from 1-10
- Multiply it by 3
- Subtract it by 2
- Divided it by 6 and keep the remainder
- Plus 2
- Is it 3 or 6?