Last lecture

Probability Event (Ch 1.2)

- Axioms of Probability

Counting the size of events (Ch 1.3)

- Independent events
- Dependent but countable

$$P(A) = \frac{|A|}{|\Omega|}$$

$$= \frac{|A_1|}{|\Omega_1|} \times \frac{|A_2|}{|\Omega_2|}$$

Agenda

Probability with equally likely outcomes (Ch 1.4)

- Draw socks from the drawer
- Poker hands

// HW1

Random Variables (RV) (Ch 2.1)

- Definition
- Probability Mass Function (pmf)

Mean and Variance (Ch 2.2)

Overcounting

Permutation

- The # of ways to order n different items
- How many ways can you order letters A, B, C, D?

$4 \times 3 \times 2 \times 1$

- N letters -> $N!$
- What if I want to order "$A, B, C \ldots G$" 7 letters, but only pick the first 4?

$$7 \times 6 \times 5 \times 4 = \frac{7!}{3!}$$

$7!$ → order 7 letters

$3!$ → order of last 3 letters

- What if I want to order letters ILLINI?

ACBD EFG	FGE
ACBD EGF	GEF
ACBD FEG	GFE

Principle of over-counting

- What if I want to order letters ILLINI?

6 Letters, 3 same "I", 2 same "N"

$\Rightarrow \dfrac{6!}{3!\,2!}$

- For an integer $K \geq 1$, if each element of a set is counted K times, then the number of elements in the set is the total count divided by K

Combination

don't care picking order

- $\binom{n}{k}$ or $C(n,k)$
  - The # of ways to choose k out of n different items
  - $\binom{n}{k} = \dfrac{n!}{k!(n-k)!}$

$n! \rightarrow$ order n items

Don't care first k item order $\rightarrow k!$

$(n-k)! \rightarrow$ Don't care last n-k items

- Draw 3 balls out of 5 balls without replacement

different. default

$$\binom{5}{3} = \frac{5!}{3!\,2!} = \frac{5\times 4}{2!} = 10$$

ABC	ADE
ABD	BCD
ABE	BCE
ACD	BDE
ACE	CDE

The Socks Problem

$\{B_1 \ldots B_8 \ldots W_1 \ldots W_4\}$

8B 4W

I have 4 pairs of black socks and 2 pairs of white socks

$P(Draw\ two\ socks, color\ is\ the\ same)$?

$$\|\{W_1W_2,\ W_2W_4\}\|$$

$$\frac{\|A\|}{\|\Omega\|} = \frac{\|A_B\| + \|A_W\|}{\binom{12}{2}} = \frac{\binom{8}{2} + \binom{4}{2}}{\binom{12}{2}}$$

$$\|\{B_1B_2, B_1W_1, B_3W_4, B_8W_4 \ldots\}\|$$

Slido!

$RRRR\ BB\ G \Rightarrow \frac{4 \times 2 \times 1}{\binom{7}{3}}$

A bag contains $\{R, R, R, B, B, G\}$

What's the probability that I draw 3 balls all different colors?

No order.

A. $\frac{3\times2\times1}{6!}$

B. $\frac{3\times2\times1}{\binom{6}{3}}$

C. $\frac{3\times2\times1}{6\times5\times4}$

D. None of the above

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$\|\Omega\| = \binom{6}{3}$

$\|A\| = \|\|R\| = \|\{R_1, R_2, R_3\}\| = 3$

$\|\|B\| = 2$

$\times)\|\|G\| = 1$

$3 \times 2 \times 1$

Poker Problem

$\Omega_{card} =$ 4 suits × 13 numbers = 52 cards

Draw 5 cards out of 52 cards

FULL HOUSE = 3 same numbers, other 2 same numbers

e.g. 3 × Qs 2 As

$P(FULL\ HOUSE) =$

$$\frac{\|All\|}{\|\Omega\|} = \frac{\|A_{3kind}\| \times \|A_{2kind}\|}{\binom{52}{5}} = \frac{\overset{\text{suit}}{\binom{4}{3}}\overset{\#}{\binom{13}{1}} \times \overset{\text{suit}}{\binom{4}{2}}\overset{\#}{\binom{12}{1}}}{\binom{52}{5}}$$

FALSE

$$\|A\| \overset{?}{=} \|A_{2\,kind}\|\,\|A_{2\,kind}\|\,\underbrace{\|A_{ny\ previous\ kind}\|}$$

$$\binom{4}{2}\binom{13}{1}\binom{4}{2}\binom{12}{1} \times 4 \;/\; 2 \;/\; 3\ ?$$

ordered

$(2+2)$

♥A ♦A

♣K ♠K

♣A

2x ♣A ♠A = ♥A

Sample space with infinite cardinality → size of sample space / event.

(Not focus)

Interval probability space

- $\Omega = \{\omega: 0 \leq \omega \leq 1\}$ $P([a, b]) = b - a$
- $A = [0.2, 0.8]$

$\{\Omega, \mathcal{F}, P\}$

Probability of continuous intervals…

- $P(x = 0.234) =?$ 0
- Ask the right question!

$P\{0.2 \leq X \leq 0.3\} = 0.1$

Random Variable

Random Variable

$P\{X=0\} = \frac{1}{2} = \frac{3}{6}$

$P\{X=1\} = \frac{1}{2}$

Rolling a die $\Omega = \{1, 2, 3, 4, 5, 6\}$, Event "odd" $A = \{1, 3, 5\}$

- As if I put a "coated face" to the die $[1, 0, 1, 0, 1, 0]$
- Coated die X is a "Random Variable."

e.g. $H \to 1$

$T \to 0$.

Random Variable (RV.)

- A random variable is a real-value function. on Ω

- A random variable X is said to be discrete type if there's a finite/countable infinite set $\{u_1, u_2, \dots\}$ s.t. $P\{X \in \{u_1, u_2, \dots\}\} = 1$

$P\{X = \{0, 1\}\} = 1$

RV in real-world

- Heights of the classmates
- # of computers fixed in the college life
- Scores of 313 midterm

X_{mid1} X_{mid2} X_{final}

Why do we need RV?

- Compute complex events
  - What's my final letter grade at 313?

$P\{X_{final} > 85 \mid X_{mid1} = 80\}$

- Evaluate relationships between events
  - Is heights related with the # of computers fixed?