

# Last lecture

## Probability Event (Ch 1.2)

- Axioms of Probability

## Counting the size of events (Ch 1.3)

- Independent events
- Dependent but countable

$$P(A) = \frac{|A|}{|\Omega|}$$
$$= \frac{|A_1|}{|\Omega_1|} \times \frac{|A_2|}{|\Omega_2|}$$

# Agenda

Probability with equally likely outcomes (Ch 1.4)

- Draw socks from the drawer
- Poker hands

// HW1

Random Variables (RV) (Ch 2.1)

- Definition
- Probability Mass Function (pmf)

Mean and Variance (Ch 2.2)

# Overcounting

# Permutation

- The *# of ways* to order  $n$  different items
- How many ways can you order letters  $A, B, C, D$ ?

$$4 \times 3 \times 2 \times 1$$

- $N$  letters  $\rightarrow N!$
- What if I want to order "A, B, C ... G" 7 letters, but only pick the first 4?  $7 \times 6 \times 5 \times 4 = \frac{7!}{3!} \rightarrow$  order 7 letters
- What if I want to order letters ILLINOIS?  $\rightarrow$  order of last 3 letters

ACBD	ETG	FGT
ACBD	EGT	GTF
ACBD	T-EG	GTE

# Principle of over-counting

- What if I want to order letters ILLINI?

6 Letters, 3 same "I", 2 same "N"

$$\Rightarrow \frac{6!}{3!2!}$$

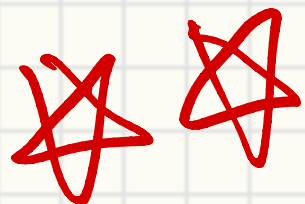
- For an integer  $K \geq 1$ , if each element of a set is counted  $K$  times, then the number of elements in the set is the total count divided by  $K$

# Combination

don't care picking order



- $\binom{n}{k}$  or  $C(n, k)$
- The # of ways to choose  $k$  out of  $n$  different items
- $\binom{n}{k} = \frac{n!}{k!(n-k)!}$  → order  $n$  items



Don't care first  $k$  item order →  $k!(n-k)!$  → Don't care last  $n-k$  items

- Draw 3 balls out of 5 balls without replacement

different

default

$$\binom{5}{3} = \frac{5!}{3!2!} = \frac{5 \times 4}{2!} = 10$$

ABC	ADE
ABD	BCD
ABE	BCE
ACD	BDE
ACE	CDE

# The Socks Problem

$$\{B_1, \dots, B_8, W_1, \dots, W_4\}$$

8B

4W

$$\{ \{W_1, W_2\}, \{W_3, W_4\} \}$$

I have 4 pairs of black socks and 2 pairs of white socks

$P(\text{Draw two socks, color is the same})?$

$$\frac{|A|}{|\Omega|} = \frac{|A_B| + |A_W|}{\binom{12}{2}} = \frac{\binom{8}{2} + \binom{4}{2}}{\binom{12}{2}}$$

$$\{B_1, B_2, B_3, W_1, B_4, W_2, B_5, W_3, B_6, W_4, B_7, W_4\}$$

Slido!

RRRR BB G  $\rightarrow \underline{4 \times 2 \times 1}$   
 $\left(\frac{6}{3}\right)$

A bag contains  $\{R, R, R, B, B, G\}$

What's the probability that I draw 3 balls all different colors?

No order

A.  $\frac{3 \times 2 \times 1}{6!}$

✓ B.  $\frac{3 \times 2 \times 1}{\binom{6}{3}}$

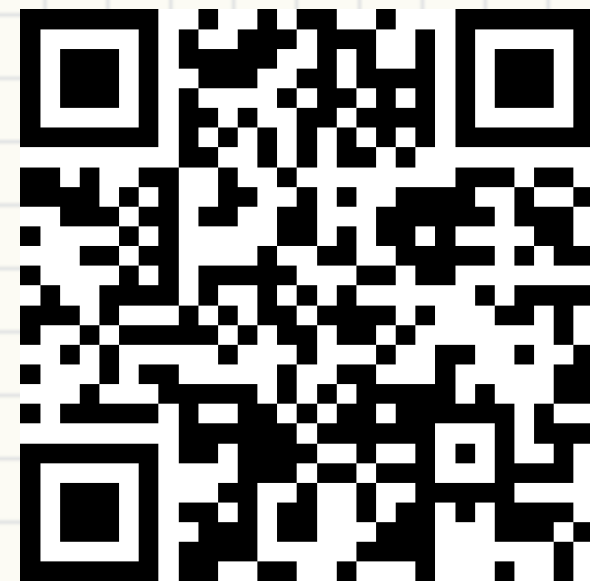
C.  $\frac{3 \times 2 \times 1}{6 \times 5 \times 4}$

D. None of the above

$||\Omega|| = \binom{6}{3}$

$||A|| = ||R|| = ||\{R_1, R_2, R_3\}|| = 3$   
 $||B|| = 2$

$\cancel{||G|| = 1}$   
 $\underline{3 \times 2 \times 1}$



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# Poker Problem

$$\Omega_{card} = 4 \text{ suits} \times 13 \text{ numbers} = 52 \text{ cards}$$

Draw 5 cards out of 52 cards

FULL HOUSE = 3 same numbers, other 2 same numbers

e.g.  $3 \times Qs$   $2 As$

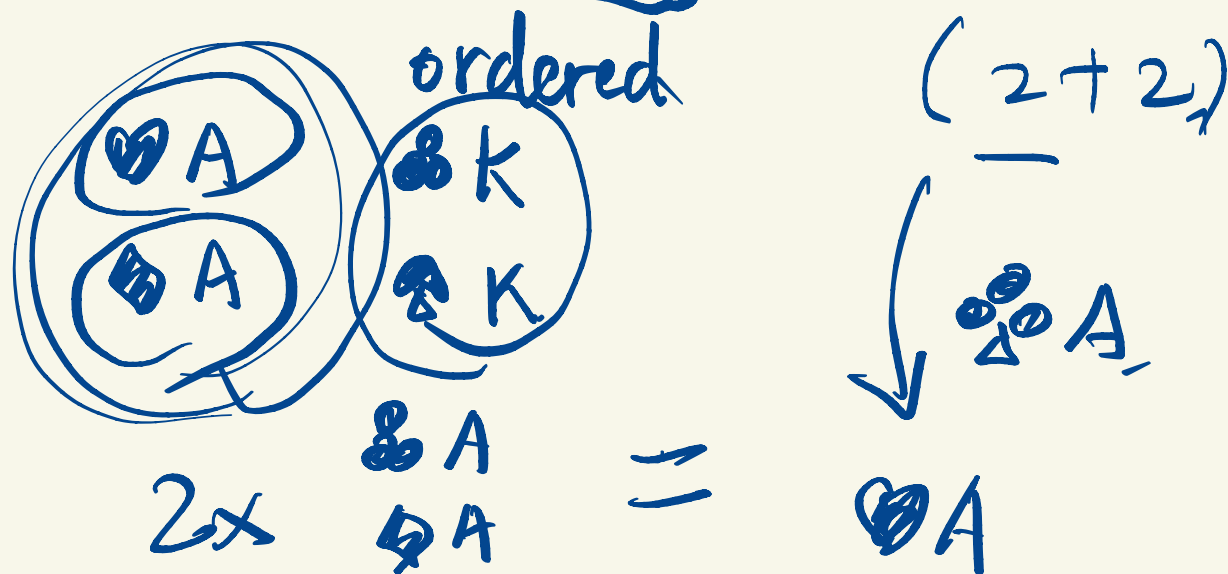
$$P(\text{FULL HOUSE}) =$$

$$\frac{||A||}{||\Omega||} = \frac{||A_{3\text{kind}}|| \times ||A_{2\text{kind}}||}{\binom{52}{5}} = \frac{\overset{\text{suit, \#}}{\binom{4}{3}} \binom{13}{1} \times \overset{\text{suit \#}}{\binom{4}{2}} \binom{12}{1}}{\binom{52}{5}}$$

FALSE

$||A|| \stackrel{?}{=} ||A_{2\text{kind}}|| ||A_{2\text{kind}}|| ||\text{Any previous kind}||$

$\binom{4}{2} \binom{13}{1} \binom{4}{2} \binom{12}{1} \times 4 / 2 / 3 ?$



# Sample space with infinite cardinality

→ size of sample space / event.  
(Not focus)

Interval probability space

- $\Omega = \{\omega: 0 \leq \omega \leq 1\}$   $P([a, b]) = b - a$
- $A = [0.2, 0.8]$

$\{\Omega, \mathcal{F}, P\}$

Probability of continuous intervals...

- $P(x = \underline{0.234}) = ?$  **0**
- Ask the right question!

$$P\{0.2 \leq X \leq 0.3\} = 0.1$$

# Random Variable

# Random Variable

$$P\{X=0\} = \frac{1}{2} = \frac{3}{6}$$
$$P\{X=1\} = \frac{1}{2}$$

Rolling a die  $\Omega = \{1, 2, 3, 4, 5, 6\}$ , Event "odd"  $A = \{1, 3, 5\}$

- As if I put a "coated face" to the die  $[1, 0, 1, 0, 1, 0]$
- Coated die  $X$  is a "Random Variable".

Random Variable (RV.)

e.g.  $H \rightarrow 1$   
 $T \rightarrow 0$

- A random variable is a  $\quad$  on  $\Omega$   
 $\text{real-value function.}$

- A random variable  $X$  is said to be  $\text{discrete type}$  if there's a finite/countable infinite set  $\{u_1, u_2, \dots\}$  s.t.

$$P\{X \in \{u_1, u_2, \dots\}\} = 1$$

$$P\{X \in \{0, 1\}\} = 1$$

# RV in real-world

- Heights of the classmates
- # of computers fixed in the college life
- Scores of 313 midterm

Why do we need RV?

- Compute complex events
  - What's my final letter grade at 313?

$X_{mid1}$   $X_{mid2}$   $X_{final}$

$$P\{X_{final} > 85 \mid X_{mid1} = 80\}$$

- Evaluate relationships between events
  - Is heights related with the # of computers fixed?