Last lecture

Probability Event (Ch 1.2)

- Experiments, outcomes, trials
- Sample space Ω, events, complement
- Karnaugh Map
- De Morgan's Law

Agenda

Probability Event (Ch 1.2)

- Axioms of Probability

Counting the size of events (Ch 1.3)

- Independent events
- Dependent but countable

Probability with equally likely outcomes (Ch 1.4)

- Draw socks from the drawer
- Poker hands

Karnaugh Map Recap

(Ex. 1.4.2) Roll two fair dice

- A: Sum is **even**
- B: Sum is **multiple of 3**
- C: The numbers are the same

Karnaugh Map Recap

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	B^c		B	
	C^c	C	C	C^c
A^c	*14,16,23,25 32,34,41,43, 52,56,61,65*			*12,21,36,45, 54,63*
A	*13,26,31,35, 46,53,62,64*	*11,22,44,55*	*33,66*	*15,24,42,51*

Karnaugh Map on Probability

- / of events
- Example:
  - $P(A) = 0.3, P(B) = 0.4, P(A^c \cup B^c) = 0.8$, what is $P(A^c B^c)$?

Karnaugh Map vs. Venn Diagram

Not on exam

Axioms of Probability

Probability Space

- An experiment can be modeled by a triplet
  - Ω:
  - $\mathcal{F}$:
  - P:

Axioms on both $\mathcal{F}$ and P

Event Axioms

- Some interpretation of “Set”, $\mathcal{F}$ is “   ”

- Axiom E.1 $\Omega \in \mathcal{F}$ is an event
- Axiom E.2 If $A \in \mathcal{F}$, then $A^c \in \mathcal{F}$
- Axiom E.3 If $A \in \mathcal{F}$and $B \in \mathcal{F}$, then $A \cup B \in \mathcal{F}$

Try to proof these:

- e.4 $\emptyset \in \mathcal{F}$
- e.5 If $A \in \mathcal{F}$and $B \in \mathcal{F}$, then $AB \in \mathcal{F}$

Probability Axioms

- Axiom P.1 $\forall A \in \mathcal{F}, P(A) \leq 1$
- Axiom P.2 If $A, B \in \mathcal{F}$ and A, B are mutually exclusive then $P(A \cup B) = P(A) + P(B)$
- Axiom P.3 $P(\Omega) = 1$

Try to proof these:

- p.4 $P(A^c) = 1 - P(A)$
- p.8 $P(A \cup B) = P(A) + P(B) - P(AB)$

- More on textbook p.9

Slido!

Select the correct ones

A. $(A^c B) \cup (AB^c) \cup (AB) = \Omega$

B. If $A \subset B$, $(A^c B)(AB^c) = \emptyset$

C. If A, B are disjoint, $A^c = (AB^c) \cup (A^c B)$

D. If A, B, C are mutually exclusive, at most 3 entries in Karnaugh map is none-empty

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Counting the size of events

How large is A and Ω

- If events contain outcomes
  - $P(A) =$

- But how large is $||A||$ and $||\Omega||$?
- Independent experiments
  - Toss a coin and roll a die
  - Roll a die twice

- Dependent
  - Bin of balls $\Omega = \{\text{Red}, \text{Red}, \text{Red}, \text{Green}, \text{Green}\}$
  - Draw two balls
  - Pokers

Independent experiments

- If we toss a coin and roll a die
- $\Omega_c =$ $\Omega_d =$
- $\Omega =$
- $\left||\Omega|\right| =$
- $\left||A|\right| =$
- Independent events $P(AB) =$

Principle of counting

- If there are m ways to select one variables, and n ways to select the other.
- If these two variables are selected .
- Then there are ways to make the pair of selections

Dependent experiments

- What if the first draw affects the second one?
- Example:
  - I have 4 pairs of black socks and 2 pairs of white socks
  - $P(Draw\ two\ socks, color\ is\ the\ same)$?

  - $\Omega_1 =$
  - $A =$

- We need a tool – /

Overcounting

Permutation

- The   to order n different items
- How many ways can you order letters A, B, C, D?

- N letters ->
- What if I want to order "A, B, C … G " 7 letters, but only pick the first 4?

- What if I want to order letters ILLINI?

Principle of over-counting

- What if I want to order letters ILLINI?

- For an integer $K \geq 1$, if each element of a set is counted K times, then the number of elements in the set is the total count divided by K

Combination

- $\binom{n}{k}$ or $C(n,k)$
  - The to choose k out of n different items
  - $\binom{n}{k} =$

  - Draw 3 balls out of 5 balls replacement

The Socks Problem

I have 4 pairs of black socks and 2 pairs of white socks

$P(Draw\ two\ socks, color\ is\ the\ same)$?

Slido!

A bag contains $\{R, R, R, B, B, G\}$
What's the probability that I draw 3 balls all different colors?

A. $\frac{3\times2\times1}{6!}$

B. $\frac{3\times2\times1}{\binom{6}{3}}$

C. $\frac{3\times2\times1}{6\times5\times4}$

D. None of the above

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Poker Problem

$\Omega_{card} =$

Draw 5 cards out of 52 cards
FULL HOUSE = 3 same numbers, other 2 same numbers

$P(FULL\ HOUSE) =$

Sample space with infinite cardinality

Interval probability space

- $\Omega = \{\omega: 0 \leq \omega \leq 1\}$ $P([a, b]) = b - a$
- $A = [0.2, 0.8]$

And others…