Last lecture

Probability Event (Ch 1.2)

- Experiments, outcomes, trials
- Sample space Ω, events, complement

Roll D6 $\{1, 2, 3 \dots 6\}$

$A_{even} = \{2, 4, 6\}$

$A^c = \Omega \setminus A$

$= \{1, 3, 5\} = B_{odd}$

- Karnaugh Map

	A	A^c
B		
B^c		

- De Morgan's Law

$(A \cup B)^c = A^c B^c$

Corrections.

HW0 will not be graded.

HW1 will be released Jan 23

Due next Friday.

Agenda

Probability Event (Ch 1.2)

- Axioms of Probability

Counting the size of events (Ch 1.3)

- Independent events
- Dependent but countable

Probability with equally likely outcomes (Ch 1.4)

- Draw socks from the drawer
- Poker hands

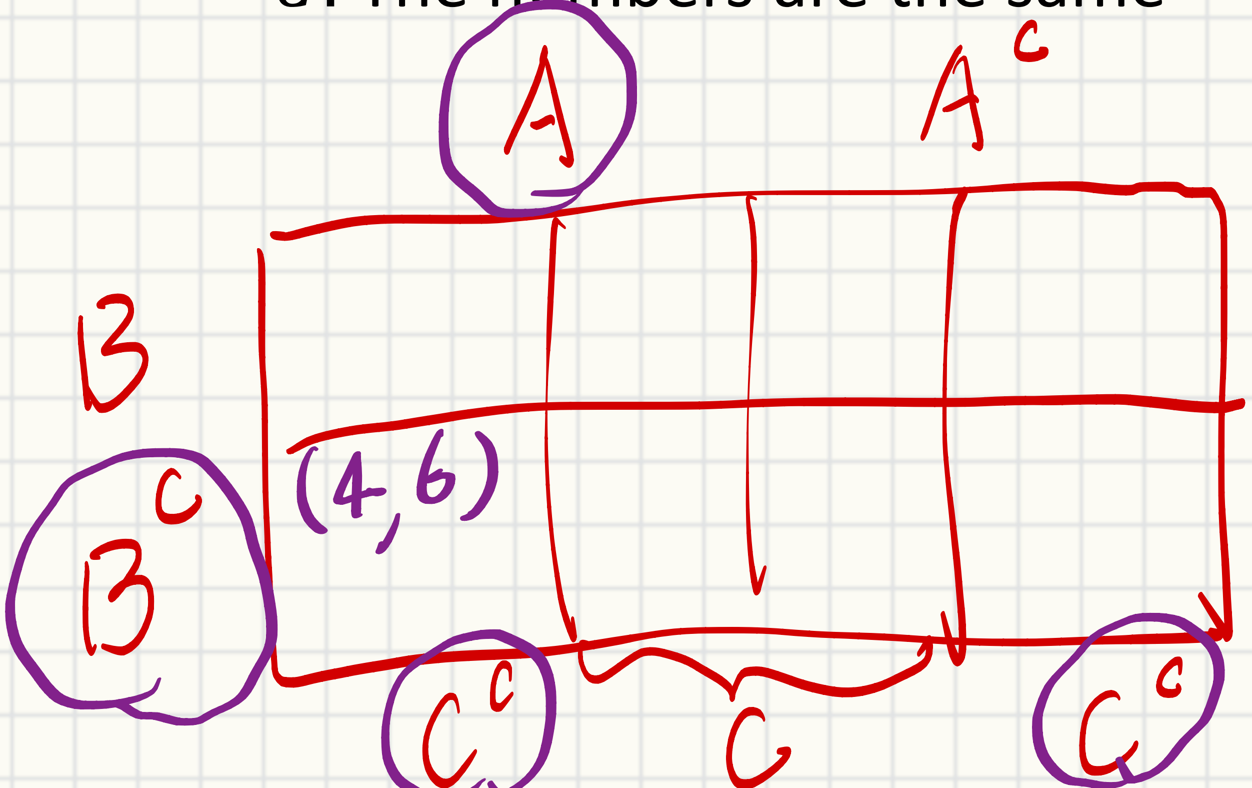
Karnaugh Map Recap

(Ex. 1.4.2) Roll two fair dice

- A: Sum is **even**
- B: Sum is **multiple of 3**
- C: The numbers are the same

$2^3 = 8$ cells

(4,6)



Karnaugh Map Recap

(Ex. 1.4.2) Roll two fair dice

- A: Sum is **even**
- B: Sum is **multiple of 3**
- C: The numbers are the same

	B^c		B	
	C^c	C	C	C^c
A^c	14,16,23,25 32,34,41,43, 52,56,61,65			12,21,36,45, 54,63
A	13,26,31,35, 46,53,62,64	11,22,44,55	33,66	15,24,42,51

Karnaugh Map on Probability

- Inclusion / Exclusion of events
- Example:
  - $P(A) = 0.3$, $P(B) = 0.4$, $P(A^c \cup B^c) = 0.8$, what is $P(A^c B^c)$?

$1-0.8$

	A	A^c
B	0.2	0.2
B^c	0.1	

$0.4-0.2$

$0.3-0.2$

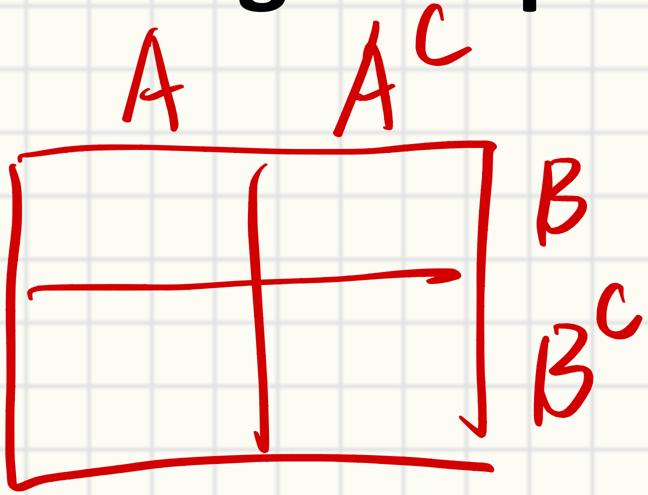
$A^c \cup B^c = (A \cap B)^c$

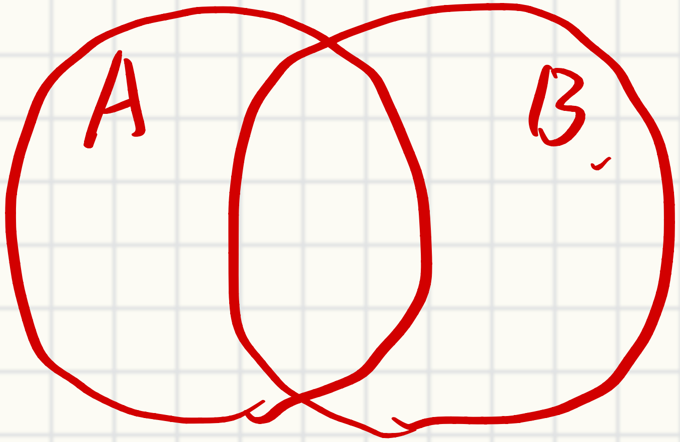
$1-0.2-0.2-0.1 = \boxed{0.5}$

$\square$

Karnaugh Map vs. Venn Diagram

Not on exam





$\Rightarrow$ Emphasize intersection
(Positive event)

Axioms of Probability

Probability Space

- An experiment can be modeled by a triplet $\{\Omega, \mathcal{F}, P\}$
  - Ω: sample space
  - $\mathcal{F}$: Event sets / Set of subsets (events)
  - P: Probability of events

Axioms on both $\mathcal{F}$ and P

$P = \{p(f_1)\ p(f_2) \ldots\}$

$\mathcal{F} = \{f_1, f_2 \ldots\}$

Event Axioms on $\mathcal{F}$

- Some interpretation of "Set", $\mathcal{F}$ is " Set of events"

- Axiom E.1 $\Omega \in \mathcal{F}$ is an event
- Axiom E.2 If $A \in \mathcal{F}$, then $A^c \in \mathcal{F}$
- Axiom E.3 If $A \in \mathcal{F}$ and $B \in \mathcal{F}$, then $A \cup B \in \mathcal{F}$

Try to proof these:

- e.4 $\emptyset \in \mathcal{F}$
- e.5 If $A \in \mathcal{F}$ and $B \in \mathcal{F}$, then $AB \in \mathcal{F}$

E.1 $\Omega \in \mathcal{F}$

E.2 if $\Omega \in \mathcal{F} \Rightarrow \boxed{\Omega^c \in \mathcal{F}}$

$\boxed{\phi \in \mathcal{F}}$

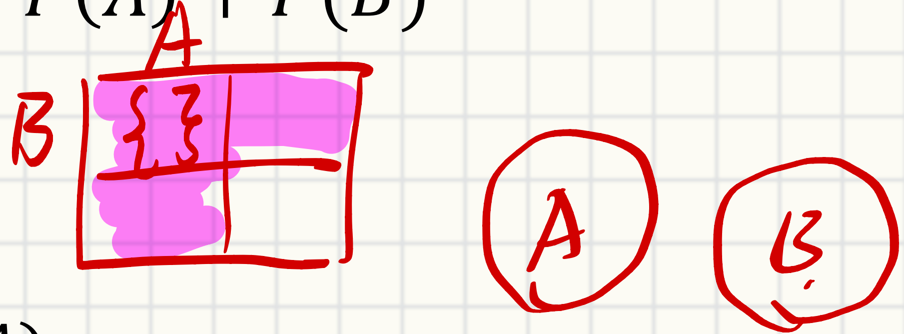
E.2 $\Downarrow$ $\Downarrow$

$\underbrace{A^c \in \mathcal{F} \quad B^c \in \mathcal{F}}_{E.3}$

$A^c \cup B^c \in \mathcal{F} \overset{E.2}{\Rightarrow} AB \in \mathcal{F}$

Probability Axioms

- Axiom P.1 $\forall A \in \mathcal{F}, P(A) \leq 1$
- Axiom P.2 If $A, B \in \mathcal{F}$ and A, B are mutually exclusive then $P(A \cup B) = P(A) + P(B)$
- Axiom P.3 $P(\Omega) = 1$



Try to proof these:

- p.4 $P(A^c) = 1 - P(A)$
- p.8 $P(A \cup B) = P(A) + P(B) - P(AB)$

- More on textbook p.9

A^c and A are mutually exclusive

Slido!

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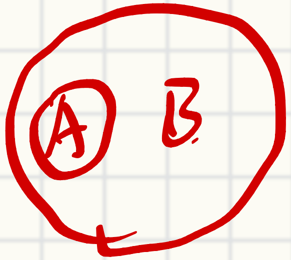
Select the correct ones

$\Omega - A^cB^c$

- ✗ A. $(A^cB) \cup (AB^c) \cup (AB) = \Omega$ ✗
- ✓ B. If $A \subset B$, $(A^cB)(AB^c) = \emptyset$

  B $\emptyset$
- ✗ C. If A, B are disjoint, $A^c = (AB^c) \cup (A^cB)$

  $\rightarrow A^cB^c$
- ✗ D. If A, B, C are mutually exclusive, at most ~~3~~ 4 entries in Karnaugh map is none-empty



	A	A^c
B	$\{\}$	
B^c		

$ABC^c = \{\}$ $ABC = \{\}$

AB^cC^c A^cBC^c A^cB^cC $A^cB^cC^c$

Counting the size of events

How large is A and Ω

- If events contain equally probable outcomes
  - $P(A) = \dfrac{||A||}{||\Omega||}$
- But how large is $||A||$ and $||\Omega||$?
- Independent experiments
  - Toss a coin and roll a die
  - Roll a die twice
- Dependent
  - Bin of balls $\Omega = \{\text{Red}, \text{Red}, \text{Red}, \text{Green}, \text{Green}\}$
  - Draw two balls
  - Pokers

Independent experiments

A = even # die

- If we toss a coin and roll a die
- $\Omega_c = \{H, T\}$ $\Omega_d = \{1, 2, \ldots, 6\}$
- $\Omega = \{H1, H2, \ldots, H6, T1, T2, \ldots, T6\}$
- $||\Omega|| = ||\Omega_c|| \times ||\Omega_d|| = 2 \times 6 = 12$
- $||A|| = ||\{H2, H4, H6, T2, T4, T6\}|| = ||\Omega_c|| \times ||A_d|| = 2 \times 3 = 6$
- Independent events $P(AB) = P(A)P(B)$

Principle of counting

- If there are m ways to select one variables, and n ways to select the other.
- If these two variables are selected independently .
- Then there are $m \times n$ ways to make the pair of selections

Dependent experiments

- What if the first draw affects the second one?
- Example:
  - I have 4 pairs of black socks and 2 pairs of white socks
  - $P(Draw\ two\ socks, color\ is\ the\ same)$? ↳ ignore L/R foot
  - $\Omega_1 = \{B1, B2 \ldots B8, W1, W2 \ldots W4\}$
  - $A = \{B1B2, B1B3 \ldots B7B8, W1W2, W1W3 \ldots W3W4\}$

$P = \frac{||A||}{||\Omega||}$

- We need a tool – Combination / permutation

Overcounting

Permutation

- The # of ways to order n different items $4 \times 3 \times 2 \times 1$
- How many ways can you order letters A, B, C, D?

$E = \{ABCD, ABDC \ldots\}$ $\|E\| = ?$ $\square\square\square\square$

- N letters -> $N!$ factorial $= N \times (N-1) \ldots \times 1$ $= 4!$
- What if I want to order "$A, B, C \ldots G$" 7 letters, but only pick the first 4?

$\boxed{7}\boxed{6}\boxed{5}\boxed{4} = 7 \times 6 \times 5 \times 4 = \frac{7!}{3!}$

- What if I want to order letters ILLINI?

# of L $\frac{6!}{2!\,3!}$ # of I

$I^1 L^2 L^3 I^4 N^5 I^6$

$I^1 L^3 L^2 I^4 N^5 I^6$