Last lecture

Joint PDF (Ch 4.3)

- Definition
- Examples
  - Uniform distribution
  - Conditional distribution

Independent RV (Ch 4.4)

- From event to RV - CDF
- Check using PDF

Agenda

Independent RV (Ch 4.4)

- Product Set
- Examples

Sums of joint RVs (Ch 4.5)

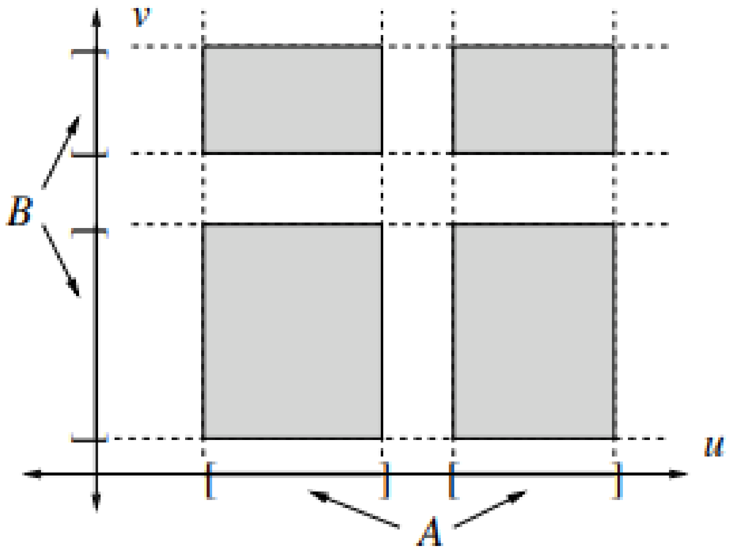
- Motivation
- Examples

More examples on joint RVs (Ch 4.6)

- Max of two RVs
- Buffon's needle problems

Independent RV

Product Set



Let A, B denote a finite union of intervals

- $|A|$ denotes the total length of $|A|$

The product set $A \times B = \{(u, v): u \in A, v \in B\}$

- The total area $|A \times B| = |A| \times |B|$

Swap property: $S \in \mathbb{R}^2$ has the swap property if

- For any pair of points $(a, b), (c, d) \in S$, (a, d) and (c, b) also in S

Proposition - $S \in \mathbb{R}^2$, S is a product set if and only if it has the swap property

Properties of independent

- If X, Y are independent and jointly continuous type RVs, then support of $f_{X,Y}$ is a product set

- Support X, Y are uniformly distributed over set $S \in \mathbb{R}^2$, then X and Y are independent iif S is a product set

Examples

Decide whether the if X and Y are independent if

- $f_{X,Y}(u,v) = Cu^2v^2$ for $u,v > 0$ and $u + v \leq 1$; 0 else
- $f_{X,Y}(u,v) = u + v$ for $u,v \in [0,1]$; 0 else
- $f_{X,Y}(u,v) = 9u^2v^2$ for $u,v \in [0,1]$; 0 else

Slido

X and Y are independent based if $f_{XY}(u, v)$ is...

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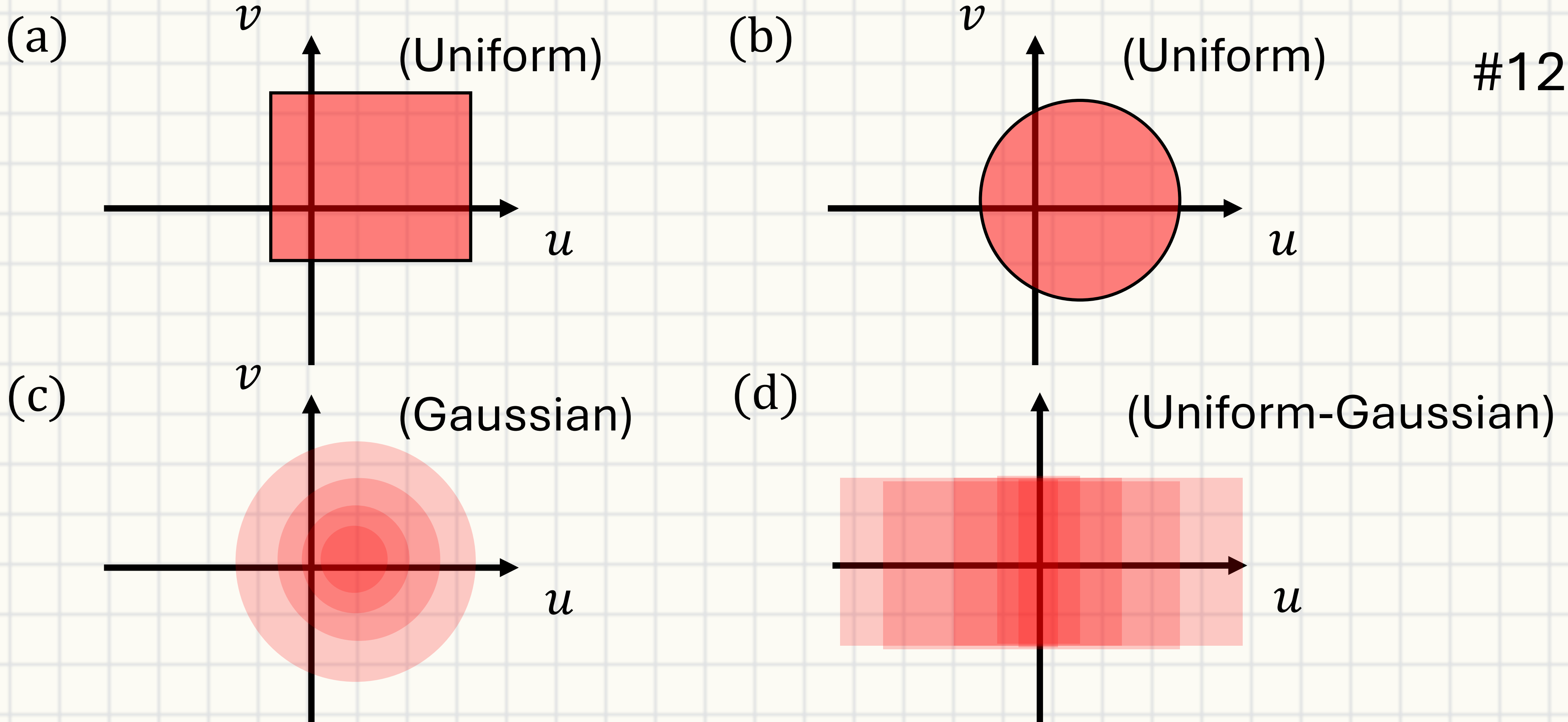


Sums of joint RVs

Motivation

Recall, we learnt if X and Y are independent

- $E[X+Y] = E[X] + E[Y]$
- $\sigma_{X+Y} = \sigma_X + \sigma_Y$

What if we know p_{XY} or f_{XY}?

- $E[X+Y] = E[X] + E[Y]$ still holds
- What's the sum of your midterm #1 and midterm #2

Sums of Discrete RVs

Let $S = X + Y$

- $p_S(k) =$

If X and Y are independent, $p_{XY}(j, k - j) = p_X(j)p_Y(k - j)$

- $p_S(k) =$
- Denoted as

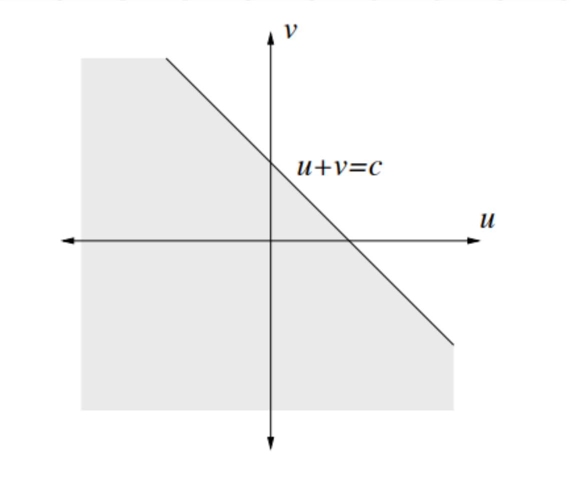
Example

Let $X = Bi(n,p)$ and $Y = Bi(m,p)$. $S = X + Y$. Find $p_S(k)$ if X and Y are independent

- Intuitively, $X + Y$ equals "toss a p Head coin times"

- Verify with formula
- $p_S(k) = \sum_{j=0}^{k} p_X(j) p_Y(k-j)$

Sums of Continuous RVs



Let $S = X + Y$

- $F_S(c) = P\{S \leq c\} =$

- $f_S(c) = \frac{dF_S(c)}{dc} =$

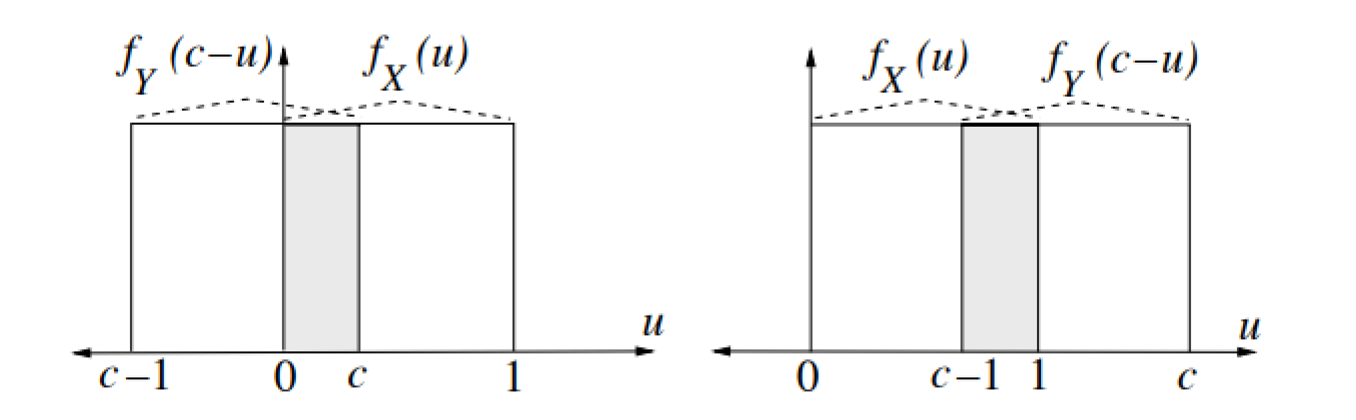
If X and Y are independent

- $f_S(c) =$

Examples

Suppose X and Y are independent, $X, Y \sim Uniform[0,1]$. Find the pdf of $S = X + Y$

- $f_S = f_X * f_Y$
- What is $f_Y(c-u)$?



- If $0 < c \le 1,\ f_X * f_Y =$
- If $1 < c \le 2,\ f_X * f_Y =$

Notes on Gaussian

Assume $X \sim N(0, \sigma_1^2)$, $Y \sim N(0, \sigma_1^2)$

- Sum of two Gaussian of same mean
  - Mean keeps the same
  - $\sigma_S^2 = \sigma_X^2 + \sigma_Y^2$

  - Tedious proof in textbook formula (4.20)
  - But high-level idea… Approximate by Binomials…

More examples on joint RVs

Max of two RVs

Let $W = \max(X, Y)$

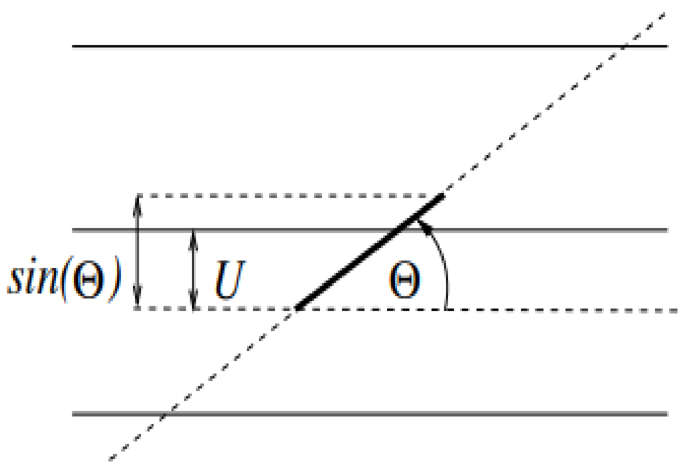
- $F_W(t) = P\{W \leq t\} =$

- $f_W(t) = \frac{dF_W(t)}{dt} =$

Abstract – on $P\{W \in (t, t+h]\} = f_W(t)h + o(h)$

- Case (a): $Y \leq t, X \in (t, t+h]$
- Case (b): $X \leq t, Y \in (t, t+h]$
- Case (c): $X \in (t, t+h], Y \in (t, t+h]$

Buffon's needle problem

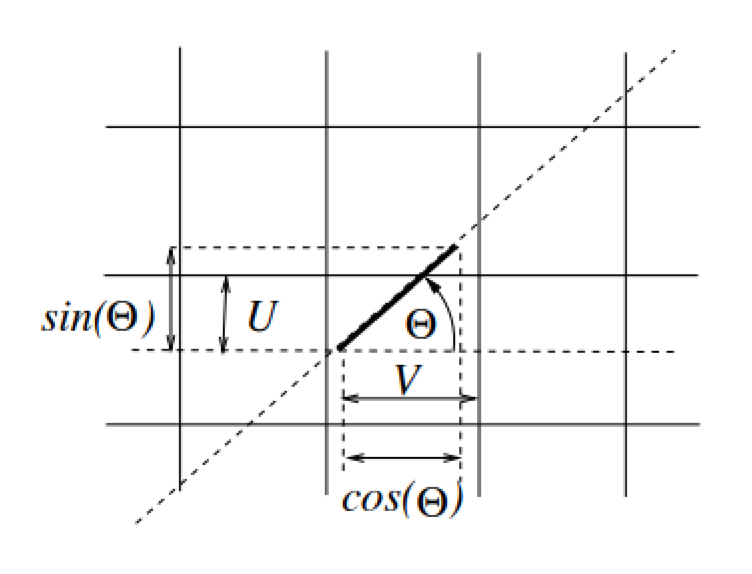


- Draw many parallel horizontal lines
  - Space 1 inch between two lines
  - Throw a needle of 1 inch length on the plane
  - Find P{ "The needle intersect with a line" }

Define U = "distance from the needle lower end to the first line above

Buffon's needle problem (2)

- What if there are "horizontal" and "vertical" lines?



Let M_h denotes "missing horizontal lines"
M_v denotes "missing vertical lines"