Last lecture

Maximum Likelihood Estimation (MLE) (Ch 2.8)

- Not focus

Markov and Chebychev inequalities (Ch 2.9)

- Markov inequality
- Chebychev inequality
- Confidence interval

Binary Hypothesis Testing (Ch 2.11)

- Definition
- Likelihood table
- Maximum likelihood decision rule

Agenda

Binary Hypothesis Testing (Ch 2.11)

- Likelihood Ratio Test (LRT)
- Maximum likelihood decision rule
- Maximum A Posteriori (MAP) decision rule
- Examples

Union Bound/ Reliability (Ch 2.12)

Hypothesis Testing

Maximum Likelihood Table

- Table showing likelihood of two hypotheses $P(X|H_i)$

	$X = 0$	$X = 1$	$X = 2$	$X = 3$
H_1	0	0.1	0.3	0.6
H_0	0.4	0.3	0.2	0.1

- Decision rule can be shown on the table by underscore each column

False alarm and missing

	$X = 0$	$X = 1$	$X = 2$	$X = 3$
H_1	0	<u>0.1</u>	<u>0.3</u>	<u>0.6</u>
H_0	<u>0.4</u>	0.3	0.2	0.1

- $P_{false\ alarm} =$
- $P_{miss} =$

	$X = 0$	$X = 1$	$X = 2$	$X = 3$
H_1	0	0.1	<u>0.3</u>	<u>0.6</u>
H_0	<u>0.4</u>	<u>0.3</u>	0.2	0.1

Maximum Likelihood (ML) decision rule

Pick whichever is higher per column!

	$X = 0$	$X = 1$	$X = 2$	$X = 3$
H_1	$p_1(0) = 0$	0.1	0.3	0.6
H_0	0.4	0.3	0.2	0.1

Likelihood Ratio Test (LRT) $\Lambda(k) = \frac{p_1(k)}{p_0(k)}$

A LRT with threshold τ:

Maximum Likelihood (ML) decision rule

Pick whichever is higher per column!

	$X = 0$	$X = 1$	$X = 2$	$X = 3$
H_1	0	0.1	<u>0.3</u>	<u>0.6</u>
H_0	<u>0.4</u>	<u>0.3</u>	0.2	0.1

What's the problem?

Slidos – Midterm Review and Early Feedback

Midterm Review

- Next Thursday class time
- Vote for the contents!

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Early Feedback

- Suggest improvement
- Anonymous by default

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Maximum A Posteriori (MAP) decision rule

Let's pick joint probability $P(H, X)$ instead of $P(X|H)$

- Pick the higher $P(H, X)$ per column
- Lowest total error rate p_e

- But how do we get the joint probability?
- Recall conditional probability

$$P(A, B) =$$

$$P(H_1, X = k) =$$

$P(H, X)$	$X = 0$	$X = 1$	$X = 2$	$X = 3$
H_1	0	0.02	0.06	0.12
H_0	0.32	0.24	0.16	0.08

Prior and Posterior

$$P(H_1, X = k) = P(X = k|H_1)P(H_1)$$

- $P(H_1) \triangleq \pi_1$: "   ", probability assumed before observation
- $P(H_0) \triangleq \pi_0$
- Bayes' rule revisited (Param/Hypothesis vs. Observation)

$$P(H|X) = \frac{P(X|H)P(H)}{P(X)}$$

Terminologies

Exam Safe

$$P(\theta|X) = \frac{P(X|\theta)P(\theta)}{P(X)}$$

Likelihood	$P(X\|\theta)$	Tractable, well defined	Toss $p = 0.3$ coin
Posterior	$P(\theta\|X)$	Typical goal	$x_{1:3} = \{H, T, T\}, p = ?$
Prior	$P(\theta)$	Domain knowledge	# coins $p = 0.3$ in world
Evidence	$P(X)$	Approx. with large data	# H in world

Likelihood table to joint probability

Assume $\pi_1 = P(H_1) = 0.2, \pi_0 =$

- Decide on joint probability is same as posterior probability

- MAP rule = LRT rule with $\tau =$

$P(X\|H)$	$X = 0$	$X = 1$	$X = 2$	$X = 3$
H_1	0	0.1	0.3	0.6
H_0	0.4	0.3	0.2	0.1

$P(H, X)$	$X = 0$	$X = 1$	$X = 2$	$X = 3$
H_1				
H_0				

Example

X: Draw a coin from the bag and toss it 5 times

- Likelihood table
- Joint probability table
- Describe ML and MAP rule, compute
  - p_{false_alarm}
  - p_{miss}
  - p_e

$H_1: p = \frac{2}{3}$ coin

$H_0: p = \frac{1}{2}$ coin

Example

$H_1 : p = \frac{2}{3}$ coin

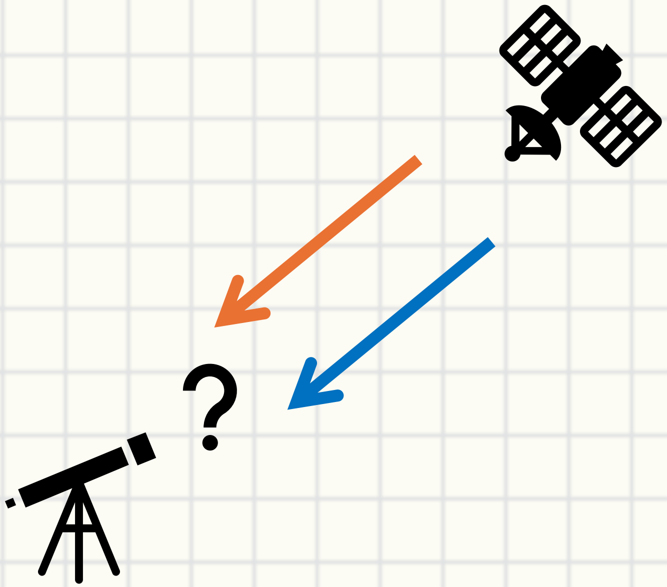
$H_0 : p = \frac{1}{2}$ coin

Example

Receive on-off keying (OOK) signal from a deep space Tx.

X: # of photons observe from a telescope

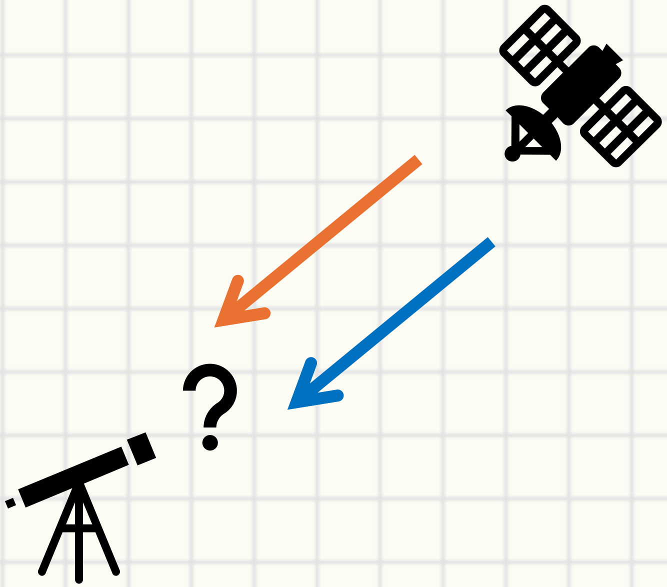
- $\lambda = 6$ If it's ON
- $\lambda = 2$ If it's OFF
- $\frac{\pi_0}{\pi_1} = 5$
- Describe ML and MAP rule, compute
  - p_{false_alarm}
  - p_{miss}
  - p_e



$H_1: Pois(\lambda = 6)$

$H_0: Pois(\lambda = 2)$

Example



$H_1: Pois(\lambda = 6)$

$H_0: Pois(\lambda = 2)$

Union Bound/ Reliability

Motivation and Definition

Reliability

- How likely a system will fail?
  - Each subsystem fail with probability
  - If sub-systems fail in some pattern, the system fails

Union Bound

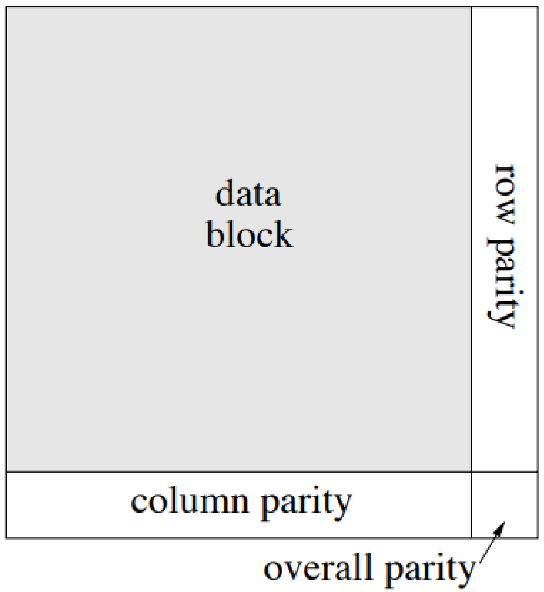
- Bounds for of small probability events
- $P(A \cup B) \leq$
- $P(A_1 \cup A_2 \cup \cdots \cup A_m) \leq$
- Bound is at most far from the actual value

Example – Array codes

Consider a code that can detect up to 3 bits error in 64 bit blocks

- Let Y denotes # error bits
- Solve $P\{Y = 4\}$

0	1	1	1	1	0	1	1
1	0	0	0	1	0	0	0
1	0	1	0	0	1	0	1
0	1	0	0	1	1	0	1
1	0	1	1	1	0	1	1
0	1	1	1	1	1	0	1
0	1	1	1	0	0	1	0
1	0	1	0	1	1	1	1



Example – Array codes

Consider a code that can detect up to 3 bits error in 64 bit blocks

- Let Y denotes # error bits
- Bound $P\{Y \geq 4\}$

Midterm #1 Until Here!