University of Illinois **Spring 2026**

ECE 313: Problem Set 1

Due: Friday, January 30 at 06:59:00 p.m.
Reading: *ECE 313 Course Notes,* Chapter 1

Note on reading: For most sections of the course notes there are short answer questions at the end of the chapter. We recommend that after reading each section you try answering the short answer questions. Do not hand these in; answers to the short answer questions are provided in the appendix of the notes.

Note on turning in homework: Homework is assigned on a weekly basis on Fridays, and is due by 7 p.m. on the following Friday. You must upload handwritten homework to Gradescope. Alternatively, you can typeset the homework in LaTeX. However, no additional credit will be awarded to typeset submissions. No late homework will be accepted.

Please write on the top right corner of the first page:
NAME
NETID
SECTION
PROBLEM SET #

Page numbers are encouraged but not required. Five points will be deducted for improper headings. Please assign your uploaded pages to their respective question numbers while submitting your homework on Gradescope. **5 points will be deducted for incorrectly assigned page numbers.**

1. [**Defining a set of outcomes**]
   Ten balls, numbered zero through nine, are initially in a bag. Four balls are drawn out, one at a time, without replacement.
   (a) Define a sample space Ω describing the possible outcomes of this experiment. To be definite, suppose the order the four balls are drawn out is *unimportant.* Explain how the elements of your set correspond to outcomes of the experiment.
   (b) What is the cardinality of Ω?

2. [**Using set theory to calculate probabilities of events**]
   Suppose A and B are two events defined on a probability space with $P(A) = 3/4$ and $P(B) = 1/2$.
   (a) If $B \subset A$, calculate $P(AB)$.
   (b) If $A \cup B = \Omega$, calculate $P(AB)$.

3. [**Displaying outcomes in a two event Karnaugh map**]
   Two fair dice are rolled. Let A be the event where the sum is 4 and B be the event where at least one of the numbers rolled is strictly less than 3. Suppose that order matters.
   (a) Display the outcomes in a Karnaugh map.
   (b) Determine $P(AB)$.

4. [**A Karnaugh map for three events**]
   There are 100 individuals in a country club that offers three sports activities: yoga, running, and zumba.

- 6 members don't participate in any sports.
- 10 members participate in all three.
- 14 members don't participate in yoga and running.
- 28 members participate in zumba and running.
- 44 members do not like yoga.
- 65 members like to run.

How many members participate in yoga but do not run?

5. **[Selecting socks at random with a twist]**
Suppose there are eight socks in a bag which can be grouped into four pairs: $\{BR1, BR2\}$, $\{BG1, BG2\}$, $\{GR1, GR2\}$, or $\{GG1, GG2\}$. The socks of each pair have the same color (Red or Green) and are gender specific (Boy or Girl), e.g., $BR1$ is a red colored boy's socks. Suppose there are two boys and two girls present, and one at a time in a fixed order, they each draw two socks out of the bag, without replacement. Suppose all socks feel the same, so when two socks are drawn from the bag, all possibilities have equal probability. Let M be the event that each person draws a pair of socks that match in color and their gender.

   (a) Define a sample space Ω for this experiment. Suppose that the order that the people draw the socks doesn't matter–all that is recorded is which two socks each person selects.

   (b) Determine $|\Omega|$, the cardinality of Ω.

   (c) Determine the number of outcomes in M.

   (d) Find $P(M)$.

   (e) Find a short way to calculate $P(M)$ that doesn't require finding $|M|$ and $|\Omega|$. (Hint: Write $P(M)$ as one over an integer. Factor the integer and think about the probability of each step of the drawing procedure.)

6. **[Two more poker hands]**
Suppose five cards are drawn from a standard 52 card deck of playing cards, as described in Example 1.4.3, with all possibilities being equally likely.

   (a) *FLUSH* is the event that all five cards have the same suit. Find $P(FLUSH)$.

   (b) *TWO PAIRS* is the event there are two different pairs in the five cards. Find $P(TWO\ PAIRS)$. (Hint: *FULL HOUSE* and *FOUR OF A KIND* should not count as TWO PAIRS)

7. **[Fishing]**
Harry's backyard pond contains 6 goldfish and 4 catfish. All fish are equally likely to be caught.

   (a) Suppose that Harry catches a total of 5 fish (no fish are thrown back). Let G be the event that Harry catches exactly 3 goldfish. What is the probability $P(G)$?

   (b) Assume event G occurs in the first catch and suppose that all 5 fish are returned to the pond. Harry starts fishing again. This time he catches a total of 3 fish. Let A be the event that among the caught set of 3 fish, exactly 2 goldfish are included that were also caught in the first catch. (Assume that fish do not learn from experience.) Find $P(A)$.